Ahmed Badr

First order D.E $(y' = f(x,y))$

* Exact D.E.

$$M\,dx + N\,dy = 0 \longrightarrow \text{I}$$

1. If $(M_y = N_x) \Rightarrow$ Exact

General solution is $\boxed{F(x,y) = c}$

Solve $\begin{cases} \text{i} & F_x = M \\ \text{ii} & F_y = N \end{cases} \xrightarrow{\text{get}} F(x,y)$

2. If $(M_y \neq N_x) \Rightarrow$ Non. Exact

i. if $\dfrac{M_y - N_x}{+N} = f(x) \Rightarrow \mu = e^{\int f(x)\,dx}$

ii. if $\dfrac{M_y - N_x}{M} = \Rightarrow \mu = e^{-\int g(y)\,dy}$

Multiply I by $\mu \Rightarrow$ Exact

$$* \ \frac{dy}{dx} = -\frac{y x^{y-1}}{x^y \ln x} = f(x,y) \qquad \text{Exact D.E.}$$

$$\Longrightarrow \underbrace{(y x^{y-1})}_{M} dx + \underbrace{(x^y \ln x)}_{N} dy = 0$$

Exercise P. 22

4) $y x^{y-1} dx + x^y \ln x \, dy = 0$

$M = y x^{y-1}$ $\qquad$ $N = x^y \ln x$

$M_y = \{x^{y-1} + y \cdot x^{y-1} \ln x\}$ $\qquad$ $N_x = \{y x^{y-1} \ln x + x^y \cdot \frac{1}{x}\}$

Equal

$\therefore$ Exact D.E

(i) $\frac{\partial F}{\partial x} = y x^{y-1} \Longrightarrow F = y\left(\frac{x^y}{y}\right) + h(y)$

$F = \int y x^{y-1} \partial x$

(ii) $\frac{\partial F}{\partial y} = N \Longrightarrow x^y \ln x + h'(y) = x^y \ln x$

$\Longrightarrow h'(y) = 0$

$\Longrightarrow h(y) = 0$

$\therefore$ General Solution

$$\boxed{x^y = c}$$

(13) $\frac{y}{x}dx + (y^2 - \ln x)\,dy = 0$

$M = \frac{y}{x}$, $N = y^2 - \ln x$

$M_y = \frac{1}{x}$ $\quad N_x = -\frac{1}{x}$

→ Non. Exact ←

$\frac{M_y - N_x}{M} = \frac{2/x}{y/x} = \frac{2}{y} = g(y)$

$\Longrightarrow \mu = e^{-\int \frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$

D.E $* \mu$

$\frac{1}{xy}dx + (1 - \frac{\ln x}{y^2})\,dy = 0$ (Exact)

(i) $\frac{\partial F}{\partial x} = \frac{1}{xy} \Longrightarrow F = \frac{1}{y}\cdot \ln x + h(y)$

(ii) $\frac{\partial F}{\partial y} = N \Longrightarrow \frac{-1}{y^2}\cdot \ln x + h'(y) = 1 - \frac{\ln x}{y^2}$

$\Longrightarrow h'(y) = 1$

$h(y) = y$

General Sol. $\frac{1}{y}\ln x + y = c$

P.g 22 No. (9) & (10) Canceled

(7) $2y^2(x+y^2)\,dx + xy(x+6y^2)\,dy = 0$

$M = 2y^2x + 2y^4 \qquad N = x^2y + 6xy^3$

$M_y = 4xy + 8y^3 \qquad N_x = 2yx + 6y^3$

Non. Exact

(i) $\dfrac{M_y - N_x}{M} = \dfrac{2xy + 2y^3}{y(2xy + 2y^3)} = \dfrac{1}{y} = g(y)$

$\mu = e^{-\int \frac{1}{y} dy} = e^{-\ln y} = \dfrac{1}{y}$

D.E. * μ

$(2xy + 2y^3)\,dx + (x^2 + 6xy^2)\,dy = 0$

(i) $\dfrac{\partial F}{\partial x} = 2xy + 2y^3 \Longrightarrow F = x^2y + 2y^3x + h(y)$

(ii) $x^2 + 6xy^2 + h'(y) = x^2 + 6xy^2$

$\Longrightarrow h'(y) = 0$

$\Longrightarrow h(y) = 0$

General sol.

$x^2y + 2y^3x = c$

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First Order D.E. ($y' = f(x,y)$)

Linear D.E.

$$y' + P(x) \cdot y = Q(x)$$

* Find $\mu = e^{\int P(x)\,dx}$

* Solution

$$\mu y = \int \mu Q(x)\,dx + c$$

Exercise P. 30

5) $y' = 2x(x^2+y) \Rightarrow y' - 2xy = 2x^3$

$P(x) = -2x$ & $Q(x) = 2x^3$

$$\mu = e^{\int -2x\,dx} = e^{-x^2}$$

Sol.

$$e^{-x^2} y = \int \underbrace{2x^3 \cdot e^{-x^2}}\,dx + c$$

$u = x^2$ $dv = 2xe^{-x^2} \cdot dx$

$du = 2x\,dx$ $\xleftarrow{-\int}$ $v = -e^{-x^2}$