BESTSELLER

For **2023 EXAM**

OSWAAL BOOKS®

LEARNING MADE SIMPLE

Only with this Book FREE ACCESS TO OSWAAL360.COM Only with this Book

CBSE 10 SAMPLE QUESTION PAPERS

CLASS 10

MATHEMATICS (STANDARD)

Strictly based on CBSE Board Sample Paper 2023 issued on 16-9-2022

EXAM

Design of the question paper issued by CBSE

Divisions	Typologies of Questions	No. of Questions	Marks	Total
Section A	MCQs + Ass. & Reasons	Q1 - Q18 Q19 - Q20	1 × 18 1 × 2	20 Marks
Section B	SATQ-I	Q21 - Q25	2 × 5	10 Marks
Section C	SATQ-II	Q26 - Q31	3 × 6	18 Marks
Section D	LATQ	Q32 - Q35	5 × 4	20 Marks
Section E	Case based Questions	Q36 - Q38	4 ×3	12 Marks
Total		38 Questions		80 Marks

Get the Oswaal Edge

1 **100% Updated** with 2023 Board SQP - Fully Solved

2 **Valuable Exam Insights** with 2023 SQP Analysis

3 **Crisp Revision** with On Tips Notes, Mind Maps & Mnemonics

4 **Extensive Practice** with 500+ Questions & 1000+ Concepts

5 **100% Exam Readiness** with 'Highly Probable' SQPs (with a success rate of more than 85% in 2022)

BESTSELLER

For 2023 EXAM

OSWAAL BOOKS®

LEARNING MADE SIMPLE

Only with this Book
FREE ACCESS TO OSWAAL360.COM
Only with this Book

CBSE 10 SAMPLE QUESTION PAPERS CLASS 10

MATHEMATICS (STANDARD)

Strictly based on CBSE Board Sample Paper 2023 issued on 16-9-2022

Design of the question paper issued by CBSE

Divisions	Typologies of Questions	No. of Questions	Marks	Total
Section A	MCQs + Ass. & Reasons	Q1 - Q18 Q19 - Q20	1 × 18 1 × 2	20 Marks
Section B	SATQ-I	Q21 - Q25	2 × 5	10 Marks
Section C	SATQ-II	Q26 - Q31	3 × 6	18 Marks
Section D	LATQ	Q32 - Q35	5 × 4	20 Marks
Section E	Case based Questions	Q36 - Q38	4 ×3	12 Marks
Total		**38 Questions**		**80 Marks**

Get the Oswaal Edge

1. **100% Updated** with 2023 Board SQP - Fully Solved
2. **Valuable Exam Insights** with 2023 SQP Analysis
3. **Crisp Revision** with On Tips Notes, Mind Maps & Mnemonics
4. **Extensive Practice** with 500+ Questions & 1000+ Concepts
5. **100% Exam Readiness** with 'Highly Probable' SQPs (with a success rate of more than 85% in 2022)

17th EDITION

YEAR 2022-23

ISBN "978-93-5634-328-3"

SYLLABUS COVERED

CENTRAL BOARD OF
SECONDARY EDUCATION
DELHI

PUBLISHED BY

OSWAAL BOOKS & LEARNING PVT. LTD.

1/11, Sahitya Kunj, M.G. Road,
Agra - 282002, (UP) India

1010, Cambourne Business Centre
Cambridge, Cambridgeshire
CB 236DP, United kingdom

0562-2857671

contact@oswaalbooks.com

www.OswaalBooks.com

TABLE OF CONTENTS

❑❑

Scratch the Unique 'CODE' which is given at the back of the Title to access the solutions for Self Assessment Papers.

CBSE CIRCULAR 2022-23

केन्द्रीय माध्यमिक शिक्षा बोर्ड

CENTRAL BOARD OF SECONDARY EDUCATION

आज़ादी का अमृत महोत्सव

CBSE/DIR(ACAD)/2022/ **Dated: 20.05.2022**

Circular No. ACAD-57/2022

All Heads of schools affiliated to CBSE

Subject : Assessment and Evaluation Practices of the Board for the Session 2022-23

National Education Policy 2020 has affirmed the need to move from rote to competency-based learning. This will equip the learners with key competencies to meet the challenges of the 21st century proactively. Accordingly, the Board has taken multiple steps towards the implementation of Competency Based Education (CBE) in schools. These range from aligning assessment to CBE, development of exemplar resources for teachers and students on CBE pedagogy and assessment and continued teacher capacity building.

In this context the Board has released Circular No. Acad-05/2019 dated 18.01.2019; Circular No. Acad-11/2019 dated 06.03.2019; Circular No. Acad-18/2020 dated 16.03.2020; Circular No. Acad-32/2020 dated 14.05.2020 and Circular No. Acad-31/2020 dated 22.04.2021. In continuation to these circulars, the Board is initiating further corresponding changes in the Examination and Assessment practices for the year 2022-23 to align assessment to Competency Based Education. Therefore, in the forthcoming sessions a greater number of Competency Based Questions or questions that assess application of concepts in real-life/ unfamiliar situations will be part of the question paper.

The changes for classes IX-XII (2022-23) internal year-end/Board Examination are as detailed:

(Classes IX-X)		
Year End Examina-tion/ Board Examination (Theory)	(2021-22) Existing (As per Special Scheme of Assessment for Board Examination – Circular No. Acad-51/2021 dated 05.07.2021)	(2022-23) Modified (Annual Scheme)
Composition	• Term I – Multiple Choice Question including case based and assertion reasoning type MCQs – 100% (30% questions competency based) • Term II – Case based/ Situation based, Open Ended- short answer/long answer questions (30% questions competency based)	• Competency Based Questions would be minimum 40% These can be in the form of Multiple Choice Questions, Case based Questions, Source based Integrated Questions or any other types. • Objective Type Questions will be 20% • Remaining 40% short
Composition	• Term I – Multiple Choice Question including case based and assertion reasoning type MCQs – 100% (30% questions competency based) • Term II – Case based/ Situation based, Open Ended- short answer/long answer questions (30% questions competency based)	• Competency Based Questions would be minimum 40% These can be in the form of Multiple Choice Questions, Case based Questions, Source based Integrated Questions or any other types. • Objective Type Questions will be 20% • Remaining 40% short answer/long answer questions (as per existing pattern)
Internal Assessment : No change Internal Assessment: End of year examination = 20:80		
Year End Examination/ Board Examination (Theory)		

Curriculum document released by the Board vide circular No.Acad-50/2022 dated 28th April, 2022 and the forthcoming Sample Question Papers may be referred for the details of changes in the QP design of individual subjects.

(Dr. Joseph Emmanuel)
Director (Academics)

SYLLABUS

MATHEMATICS STANDARD

Class - X (Code No. 041)

Latest Syllabus issued by CBSE dated 21st April 2022 (CBSE cir no. Acad-48/2022) for Academic Year 2022-23 (Annual Examination)

Unit No.	Unit Name	Marks
I	Number Systems	06
II	Algebra	20
III	Coordinate Geometry	06
IV	Geometry	15
V	Trigonometry	12
VI	Mensuration	10
VII	Statistics Probability	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

1. REAL NUMBERS (15) Periods

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$.

UNIT II: ALGEBRA

1. POLYNOMIALS (8) Periods

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (15) Periods

Pair of linear equations in two variables and graphical method of their solution, consistency/ inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS (15) Periods

Standard form of a quadratic equation $ax^2 + bx + c = 0$, ($a \neq 0$). Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS (10) Periods

Motivation for studying Arithmetic Progression. Derivation of the n^{th} term and sum of first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

1. COORDINATE GEOMETRY (15) Periods

Review : Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

SYLLABUS

UNIT IV: GEOMETRY

1. TRIANGLES (15) Periods

Definitions, examples, counter examples of similar triangles.

1. **(Prove)** If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. **(Motivate)** If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. **(Motivate)** If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. **(Motivate)** If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. **(Motivate)** If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES (10) Periods

Tangent to a circle at, point of contact

1. **(Prove)** The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. **(Prove)** The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY (10) Periods

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES (15) Periods

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: (10) Periods

Angle of Elevation, Angle of Depression.

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, 60°.

UNIT VI : MENSURATION

1. AREAS RELATED TO CIRCLES (12) Periods

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90°and 120° only).

2. SURFACE AREAS AND VOLUMES (12) Periods

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII : STATISTICS AND PROBABILITY

1. STATISTICS (18) Periods

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY (10) Periods

Classical definition of probability. Simple problems on finding the probability of an event.

❑❑

SYLLABUS

MATHEMATICS-Standard

QUESTION PAPER DESIGN

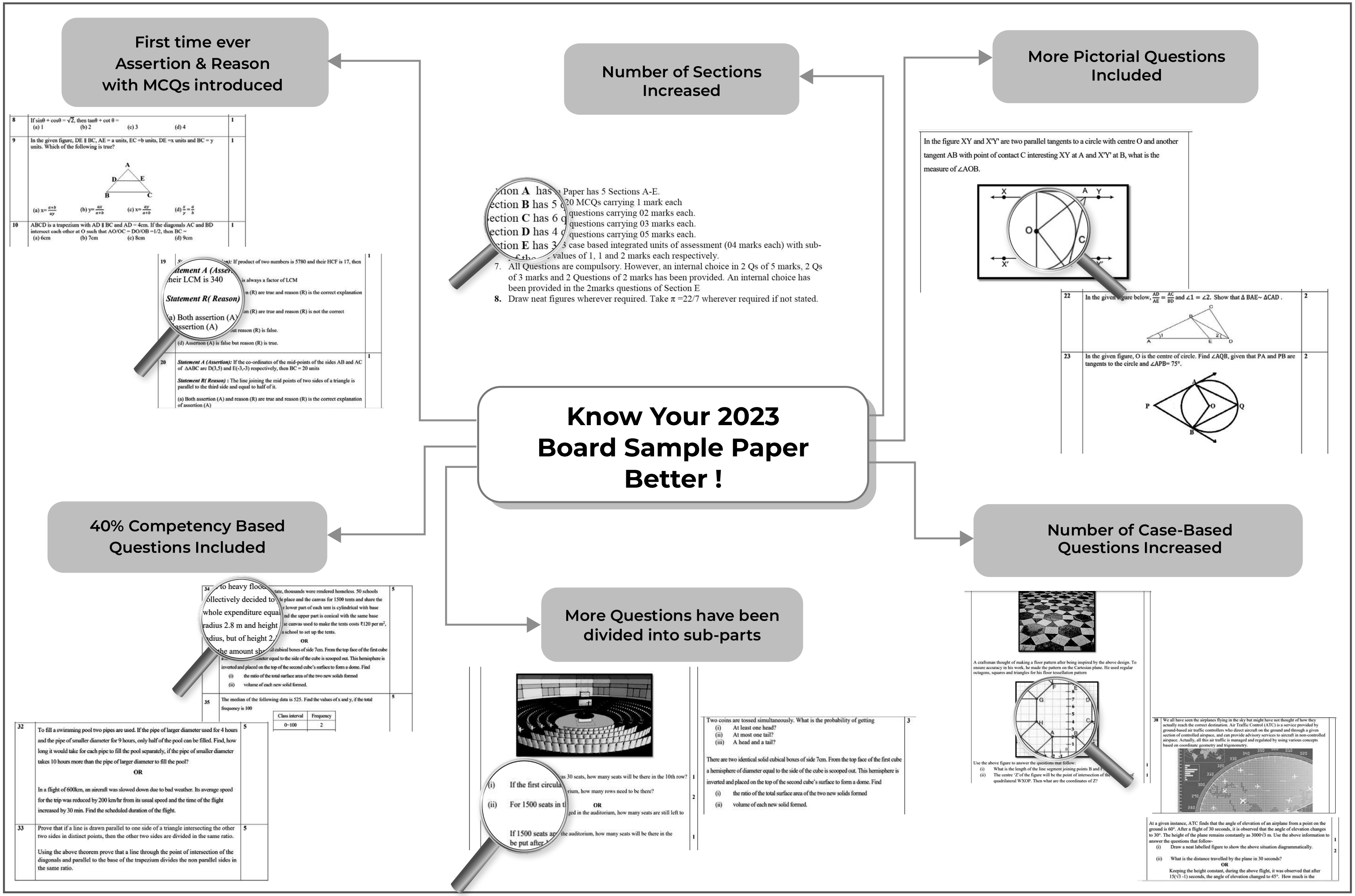
CLASS – X (2022-23)

Time : 3 Hours **Max. Marks : 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1.	**Remembering :** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding :** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas.	43	54
2.	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3.	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations. **Evaluating :** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating :** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions.	18	22
	Total	**80**	**100**

INTERNAL ASSESSMENT	**20 Marks**
● Pen Paper Test and Multiple Assessment (5+5)	10 Marks
● Portfolio	05 Marks
● Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

❑❑


Know Your 2023
Board Sample Paper
Better !
First time ever
Assertion & Reason
with MCQs introduced
Number of Sections
Increased
More Pictorial Questions
Included
40% Competency Based
Questions Included
More Questions have been
divided into sub-parts
Number of Case-Based
Questions Increased

Solved Paper, 2021-22

MATHEMATICS (STANDARD)

Term-II, Delhi Set-I

Series : PPQQC/2	Question Paper Code No. 30/2/1

Time : 2 Hours | **Max. Marks : 40**

General Instructions :

(i) This question paper contains of 14 questions. All questions are compulsory.

(ii) This question paper divided into four sections – A, B, C and D.

(iii) Section A contains 6 questions (Q No. 1 to 6) of 2 marks each. Internal choice has been provided in two questions.

(iv) Section B contains 4 questions (Q No. 7 to 10) of 3 marks each. Internal choice has been provided in one question.

(v) Section C contains 4 questions (Q No. 11 to 14) of 4 marks each. An internal choice has been provided in one question. It also contains two case study board questions.

(vii) Use of calculator is not permitted.

SECTION-A

Question Numbers 1 to 6 carry 2 marks each.

1. Solve the quadratic equation : $x^2 + 2\sqrt{2}x - 6 = 0$ for x.

2. **(a)** Which term of the $A.P.$ $-\frac{11}{2}, -3, -\frac{1}{2}$ .... is $\frac{49}{2}$?

OR

(b) Find a and b so that the numbers

a, 7, b, 23 are in $A.P.$

3. A solid piece of metal in the form of a cuboid of dimensions 11 cm × 7 cm × 7 cm is melted to form 'n' number of solid spheres of radii $\frac{7}{2}$ cm each. Find the value of n.

4. **(a)** In Fig. 1, AB is diameter of a circle centered at O. BC is tangent to the circle at B. If OP bisects the chord AD and $\angle AOP = 60°$, then find $\angle C$.

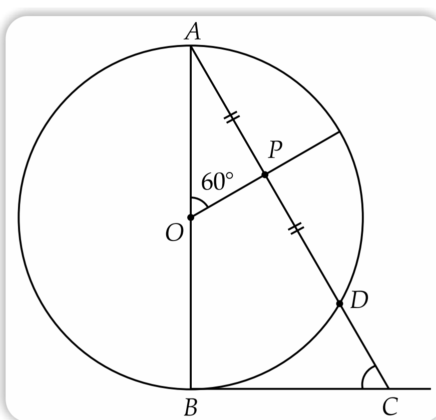


Fig. 1

OR

(b) In Fig. 2, XAY is a tangent to the circle centered at O. If $\angle ABO = 40°$, then find $\angle BAY$ and $\angle AOB$.

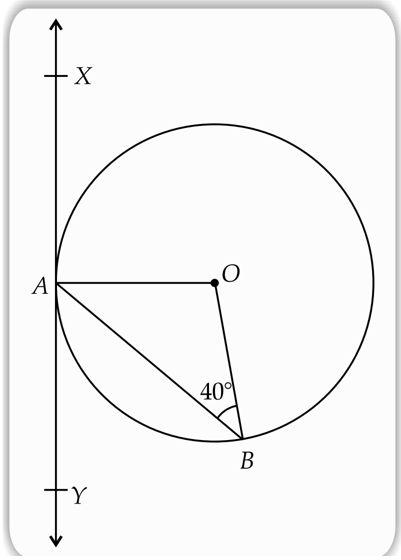


Fig. 2

5. If mode of the following frequency distribution is 55, then find the value of x.

Class	0 - 15	15 - 30	30 - 45	45 - 60	60 - 75	75 - 90
Frequency	10	7	x	15	10	12

6. Find the sum of first 20 terms of an $A.P.$ whose n^{th} term is given as $a_n = 5 - 2n$.

SECTION-B

Question Numbers 7 to 10 carry 3 marks each.

7. Draw two concentric circles of radii 2 cm and 3 cm. From a point on the outer circle, construct a pair of tangents to the inner circle.

8. In Fig. 3. AB is tower of height 50 m. A man standing on its top, observes two cars on the opposite sides of the tower with angles of depression 30° and 45° respectively. Find the distance between the two cars.

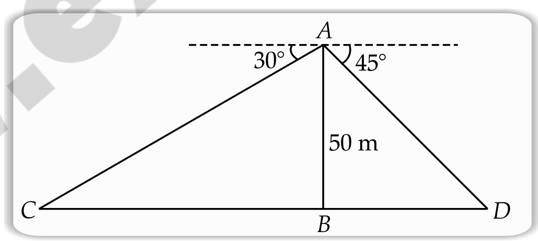


Fig. 3

9. (a) The mean of the following frequency distribution is 25. Find the value of f.

Class	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Frequency	5	18	15	f	6

OR

(b) Find the mean of the following data using assumed mean method :

Class	0 - 5	5 - 10	10 - 15	15 - 20	20 - 25
Frequency	8	7	10	13	12

10. Heights of 50 students of class X of a school are recorded and following data is obtained :

Height (in cm)	130 - 135	135 - 140	140 - 145	145 - 150	150 - 155	155 - 160
Number of Students	4	11	12	7	10	6

Find the median height of the students.

SECTION-C

Question Numbers 11 to 14 carry 4 marks each.

11. In Fig. 4. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q meet at a point T. Find the length of TP.

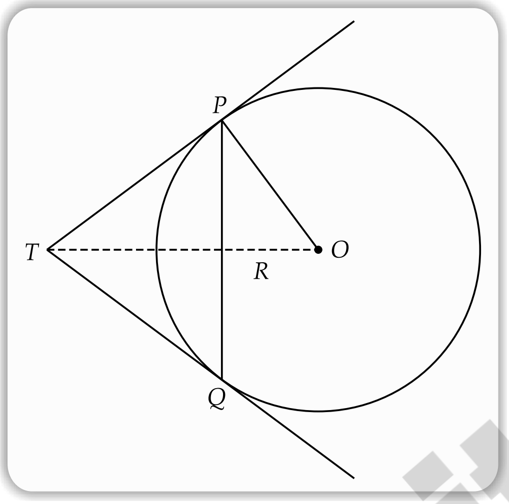


Fig. 4

12. **(a)** A 2-digit number is such that the product of its digits is 24. If 18 is subtracted from the number, the digits interchange their places. Find the number.

OR

(b) The difference of the squares of two numbers is 180. The square of the smaller number is 8 times the greater number. Find the two numbers.

13. **Case Study-1 :**

Kite Festival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below, show three kites flying together.

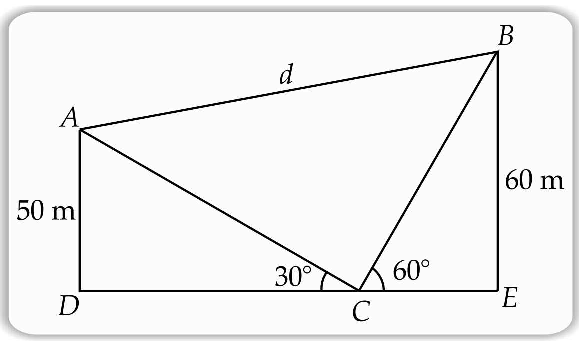


Fig. 5

In Fig. 5, the angles of elevation of two kites (Points A and B) from the hands of a man (Point C) are found to be 30° and 60° respectively. Taking $AD = 50$ m and $BE = 60$ m, find.

(1) the lengths of strings used (take them straight) for kites A and B as shown in the figure. **2**

(2) the distance 'd' between these two kites. **2**

14. **Case Study-2**

A 'circus' is a company of performers who put on shows of acrobats, clowns etc. to entertain people started around 250 years back, in open fields, now generally performed in tents.
One such 'Circus Tent' is shown below.

Fig. 6

The tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of cylindrical part are 9 m and 30 m respectively and height of conical part is 8 m with same diameter as that of the cylindrical part, then find

(1) the area of the canvas used in making the tent; **3**

(2) the cost of the canvas bought for the tent at the rate ₹ 200 per sq m, if 30 sq m canvas was wasted during stitching. **1**

□□□

SOLUTIONS

Term-II, Delhi Set-I

Series : PPQQC/2

Question Paper Code No. 30/2/1

SECTION-A

1. Given quadratic equation is :

$$x^2 + 2\sqrt{2}x - 6 = 0$$

$$\Rightarrow \quad x^2 + 3\sqrt{2}\,x - \sqrt{2}\,x - 6 = 0$$

$$\Rightarrow \quad x(x + 3\sqrt{2}) - \sqrt{2}\,(x - 3\sqrt{2}) = 0$$

$$\Rightarrow \quad (x + 3\sqrt{2})(x - \sqrt{2}) = 0$$

$$\Rightarrow \quad x + 3\sqrt{2} = 0 \text{ or } x - \sqrt{2} = 0$$

$$\Rightarrow \quad x = -3\sqrt{2} \text{ or } x = \sqrt{2}$$

2. **(a)** Given A.P. is :

$$-\frac{11}{2}, -3, -\frac{1}{2}, \ldots$$

Here, first term, $a = -\frac{11}{2}$

common difference, $d = -3 - \left(-\frac{11}{2}\right)$

$$= -3 + \frac{11}{2} = \frac{5}{2}$$

According to question,

$$a_n = \frac{49}{2}$$

$$\Rightarrow \quad \frac{49}{2} = a + (n-1)d$$

[since, $a_n = a + (n - 1)d$]

or, $\frac{49}{2} = -\frac{11}{2} + (n-1)\frac{5}{2}$

or, $\frac{49}{2} + \frac{11}{2} = (n-1)\frac{5}{2}$

or, $30 = (n-1)\frac{5}{2}$

or, $n - 1 = \frac{60}{5}$

or, $n = 12 + 1 = 13$

Hence, 13^{th} term of $A.P.$ is $\frac{49}{2}$.

OR

(b) Given, numbers a, 7, b, 23 are in A.P. .

$\therefore \quad 7 - a = b - 7 = 23 - b$

[$A.P.$ has equal common difference]

By equating, $b - 7 = 23 - b$

$\Rightarrow \quad 2b = 30$

$\Rightarrow \quad b = 15$

Now, equating $7 - a = b - 7$

$\Rightarrow \quad 7 - a = 15 - 7$

[Putting the value of a]

$\Rightarrow \quad -a = 1$

$\Rightarrow \quad a = -1$

Hence, $a = -1$ and $b = 15$.

3. We know that, volume of cuboid $= l \times b \times h$

volume of sphere $= \frac{4}{3}\pi r^3$

Given, $l = 11$ cm

$b = 7$ cm,

$h = 7$ cm and $r = \frac{7}{2}$ cm

Here,

volume of cuboid $= n \times$ volume of sphere

or, $11 \times 7 \times 7 = n \times \frac{4}{3}\pi\left(\frac{7}{2}\right)^3$

or, $11 \times 7 \times 7 = n \times \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

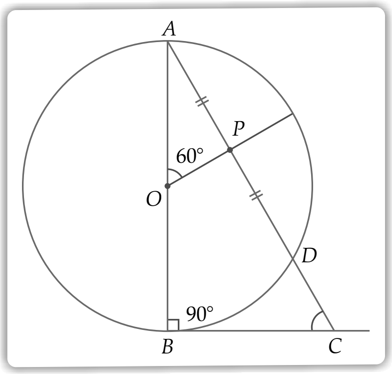
or, $n = \dfrac{11 \times 7 \times 7 \times 3 \times 7 \times 2 \times 2 \times 2}{4 \times 22 \times 7 \times 7 \times 7}$

or, $n = 3$

4. **(a)** Given, OP bisect the chord AD.

$\therefore$ $OP \perp AD$

$\angle P = 90°$ and $\angle B = 90°$



$\angle BOP = 180° - 60° = 120°$

$\angle P = 90°$

$\therefore$ OP is bisect the chord AD, radius to bisect the chord at 90°.

Now, in quad. $BOPC$, applying angle sum property

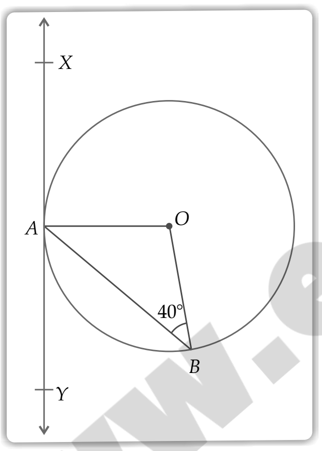
$\angle P + \angle B + \angle O + \angle C = 360°$

or, $90° + 90° + 120° + \angle C = 360°$

or, $\angle C = 360° - 300° = 60°$

OR

(b)



Given, $\angle ABO = 40°$

$\angle XAO = 90°$

(Angle between radius and tangent)

$OA = OB$ (Radii of same circle)

$\Rightarrow$ $\angle OAB = \angle OBA$

$\therefore$ $\angle OAB = 40°$

Now, applying linear pair of angles property, we get

$$\angle BAY + \angle OAB + \angle XAO = 180°$$

$$\Rightarrow \angle BAY + 40° + 90° = 180°$$

$$\Rightarrow \angle BAY + 130° = 180°$$

$$\Rightarrow \angle BAY = 180° - 130° = 50°$$

Now, in ΔAOB,

$$\angle AOB + \angle OAB + \angle OBA = 180°$$

or, $\angle AOB + 40° + 40° = 180°$

or, $\angle AOB = 180° - 80° = 100°$

5. Given :

Mode of frequency distribution = 55

So, modal class is 45 – 60.

Lower limit (l) = 45

Class interval (h) = 15

Also, $f_0 = 15, f_1 = x$ and $f_2 = 10$

$$\text{Mode} = l + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times h$$

$$\Rightarrow 55 = 45 + \left(\frac{15 - x}{30 - x - 10}\right) \times 15$$

$$\Rightarrow 55 - 45 = \frac{15(15 - x)}{30 - x - 10}$$

$$\Rightarrow 10\,(30 - x - 10) = 225 - 15x$$

$$\Rightarrow 300 - 10x - 100 = 225 - 15x$$

$$\Rightarrow 5x = 25$$

$$\Rightarrow x = 5$$

6. Given, $a_n = 5 - 2n$

for $n = 1$, $a_1 = 5 - 2(1) = 3$

$n = 2$, $a_2 = 5 - 2(2) = 1$

$\therefore$ Common difference $= 1 - (3) = -2$

Sum of first n terms :

$$S_n = \frac{n}{2}\,[2a + (n - 1)d]$$

$\therefore$ Sum of first 20 terms is :

$$S_n = \frac{20}{2}\,[2(3) + (20 - 1)(-2)]$$

$$= 10\,(6 - 38)$$

$$= 10 \times (-32) = -320$$

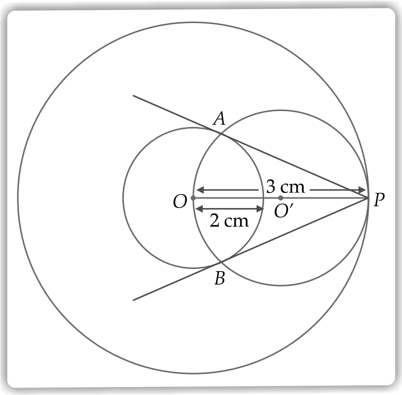
Hence, sum of first 20 terms is – 320.

SECTION-B

7. Steps of Construction :

1. Draw a circle with radius 2 cm and centre O.
2. Draw another circle with radius 3 cm and centre O.
3. Take a point P on the circumference of the larger circle and join P to O.
4. Now, taking OP as diameter and draw another circle which intersect the smaller circle at two points let say A and B.
5. Join A to P and B to P.

Hence, required tangents are AP and BP.

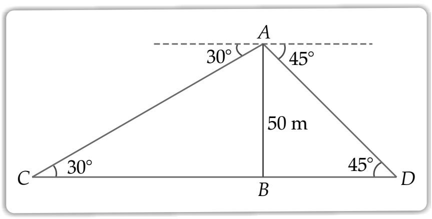


8. In ΔABC, $\angle B = 90°$

$$\tan 30° = \frac{AB}{CB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50}{CB}$$

$$\Rightarrow \quad CB = 50\sqrt{3} \text{ m}$$



In ΔABD,

$$\tan 45° = \frac{AB}{BD}$$

$$\Rightarrow \quad 1 = \frac{50}{BD}$$

$$\Rightarrow \quad BD = 50 \text{ m}$$

$$\therefore \quad CD = CB + BD = 50\sqrt{3} + 50$$

or, $CD = 50(\sqrt{3} + 1)$

or, $CD = 50(1.732 + 1)$

or, $CD = 50 \times 2.732$

or, $CD = 136.6$ m

9. (a) Given, mean = 25

Class Interval	Mid-point x_i	Frequency f_i	$f_i x_i$
0–10	5	5	25
10–20	15	18	270
20–30	25	15	375
30–40	35	f	$35f$
40–50	45	6	270
		$\Sigma f_i = 44 + f$	$\Sigma f_i x_i = 940 + 35f$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 25 = \frac{940 + 35f}{44 + f}$$

$$\Rightarrow \quad 25 \times 44 + 25f = 940 + 35f$$

$$\Rightarrow \quad 10f = 1100 - 940$$

$$\Rightarrow \quad 10f = 160$$

$$\Rightarrow \quad f = 16$$

OR

(b)

Class Interval	Mid-point (x)	Frequency (f)	$d = x - A$	fd
0–5	2.5	8	– 10	– 80
5–10	7.5	7	– 5	– 35
10–15	12.5 = A	10	0	0
15–20	17.5	13	5	65
20–25	22.5	12	10	120
		$\Sigma f = 50$		$\Sigma fd = 70$

Here, assumed mean, $A = 12.5$

Now, $\text{Mean} = A + \frac{\Sigma fd}{\Sigma f}$

$$= 12.5 + \frac{70}{50}$$

$$= 12.5 + 1.4$$

$$= 13.9$$

10.

Height (in cm)	No. of Students (f)	Cumulative Frequency (cf)
130–135	4	4
135–140	11	15
140–145	12	27 → Median Class
145–150	7	*34*
150–155	10	44
155–160	6	50
	$N = \Sigma f = 50$	

Since, $N = 50$ is an even number.

So, $\frac{N}{2} = \frac{50}{2} = 25$ and median class is 140 – 145.

$l = 140, h = 5, c = 15, f = 12$ (given)

Now, $\text{Median} = l + h\left(\frac{\frac{N}{2} - c}{f}\right)$

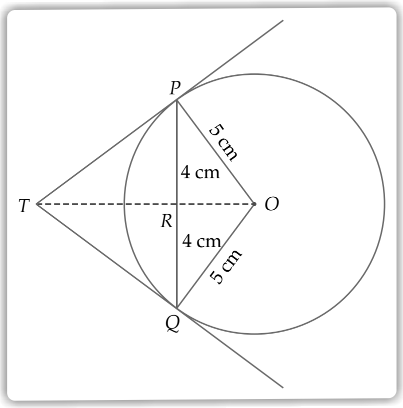
$$= 140 + 5\left(\frac{25 - 15}{12}\right)$$

$$= 140 + \left(\frac{5 \times 10}{12}\right)$$
$$= 140 + 4.167$$
$$= 144.167$$

Hence, median height of the students is 144.167 cm.

SECTION-C

11. Here, TP and TQ are the tangents from point T upon the circle. So, ΔTPQ is an isosceles triangle and TO is the angle bisector of $\angle PTO$.



$\therefore$ $OT \perp PQ$

$\therefore$ OT bisects PQ

$PR = RQ = 4$ cm

Now, $OR = \sqrt{OP^2 - PR^2}$

$= \sqrt{5^2 - 4^2} = 3$ cm

Now, $\angle TPR + \angle RPO = 90°$ ...(i)

($\because$ $\angle TPO = 90°$ angle between radius and tangent)

and $\angle TPR + \angle PTR = 90°$ ...(ii)

from eqs (i) and (ii), we get

$\angle RPO = \angle PTR$

Thus, Right $\Delta TRP \cong$ Right ΔPRO

(By AA rule of similarity)

$\therefore$ $\frac{TP}{PO} = \frac{RP}{RO}$

$\Rightarrow$ $\frac{TP}{5} = \frac{4}{3}$

$\Rightarrow$ $TP = \frac{20}{3}$ cm = 6.67 cm.

12. **(a)** Let the ten's digit be x and one's digit be y.

The number will be $10x + y$.

Given, product of digits is 24

$\therefore$ $xy = 24$

or, $y = \frac{24}{x}$ ...(i)

Given that when 18 is subtracted to the number, the digits interchange their places.

$\therefore$ $10x + y - 18 = 10y + x$

or, $9x - 9y = 18$ ...(ii)

Substituting y from eq (i) in eq (ii), we get

$$9x - 9\left(\frac{24}{x}\right) = 18$$

or, $x - \frac{24}{x} = 2$

or, $x^2 - 24 - 2x = 0$

or, $x^2 - 2x - 24 = 0$

or, $x^2 - 6x + 4x - 24 = 0$

or, $x(x-6) + 4(x-6) = 0$

or, $(x-6)(x+4) = 0$

or, $x - 6 = 0$ and $x + 4 = 0$

or, $x = 6$ and $x = -4$

Since, the digit cannot be negative, so, $x = 6$

Substiting $x = 6$ in eq (i), we get

$$y = \frac{24}{6} = 4$$

$\therefore$ The number $= 10(6) + 4 = 60 + 4 = 64$

OR

(b) Let the greater number be x.

The square of the smaller number is 8 times of the greater number $= 8x$

Given, the difference of squares of two numbers is 180.

$\therefore$ $x^2 - 8x = 180$

$\Rightarrow$ $x^2 - 8x - 180 = 0$

$\Rightarrow$ $x^2 - 18x + 10x - 180 = 0$

$\Rightarrow$ $x(x-18) + 10(x-18) = 0$

$\Rightarrow$ $(x-18)(x+10) = 0$

$\Rightarrow$ $(x-18) = 0$ or $(x+10) = 0$

$\Rightarrow$ $x = 18$ or $x = -10$

Since, number cannot be negative. So, $x = 18$

Now, square of smaller number

$= 8x$

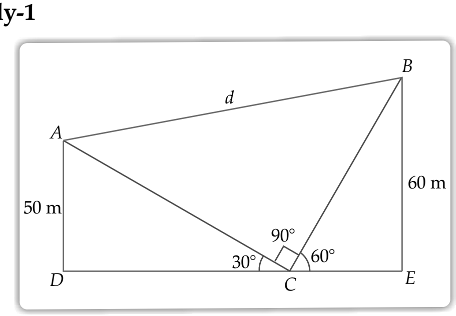
$= 8 \times 18$

$= 144$

$\therefore$ smaller number $= \sqrt{144} = 12$

Hence, smaller number is 12 and greater number is 18.

13. **Case study-1**



(1) In ΔADC, $\angle D = 90°$

$$\sin 30° = \frac{AD}{AC}$$

$\therefore \quad \dfrac{1}{2} = \dfrac{50}{AC}$

or, $\quad AC = 100$ m ...(i)

In ΔBEC, $\quad \angle E = 90°$

$\sin 60° = \dfrac{BE}{BC}$

$\therefore \quad \dfrac{\sqrt{3}}{2} = \dfrac{60}{BC}$

or, $\quad BC = \dfrac{120}{\sqrt{3}} = 40\sqrt{3}$ m ...(ii)

Hence, the length of strings used for kites A and B are 100 m and $40\sqrt{3}$ m, respectively.

(2) Here, $\angle DCA + \angle ACB + \angle BCE = 180°$

(Angles in straight line)

$\therefore \quad 30° + \angle ACB + 60° = 180°$

or, $\quad \angle ACB = 180° - 90° = 90°$

Now, in right ΔACB,

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad d^2 = (100)^2 + (40\sqrt{3})^2$

[from eq (i) and eq (ii)]

$\Rightarrow \quad d^2 = 10{,}000 + 4{,}800$

$\Rightarrow \quad d^2 = 14800$

$\Rightarrow \quad d = 20\sqrt{37}$ cm

Hence, distance between two kites A and B is $20\sqrt{37}$ cm.

14. Case study-2

(1) For cylinder,

height = 9 m, diameter = 30 m $\Rightarrow$ radius $= \dfrac{30}{2}$ = 15 m.

For cone,

height = 8 m, radius = 15 m

$\therefore$ slant height, $l = \sqrt{(8)^2 + (15)^2}$

$= \sqrt{64 + 225}$

$= \sqrt{289}$

$= 17$ m

Area of canvas required = $C.S.A$ of cylinder + $C.S.A.$ of cone

$= 2\pi rh + \pi rl$

$= \pi r\,(2h + l)$

$= \dfrac{22}{7} \times 15\ (2 \times 9 + 17)$

$= \dfrac{22}{7} \times 15 \times 35$

$= 22 \times 15 \times 5$

$= 1650$ sq m.

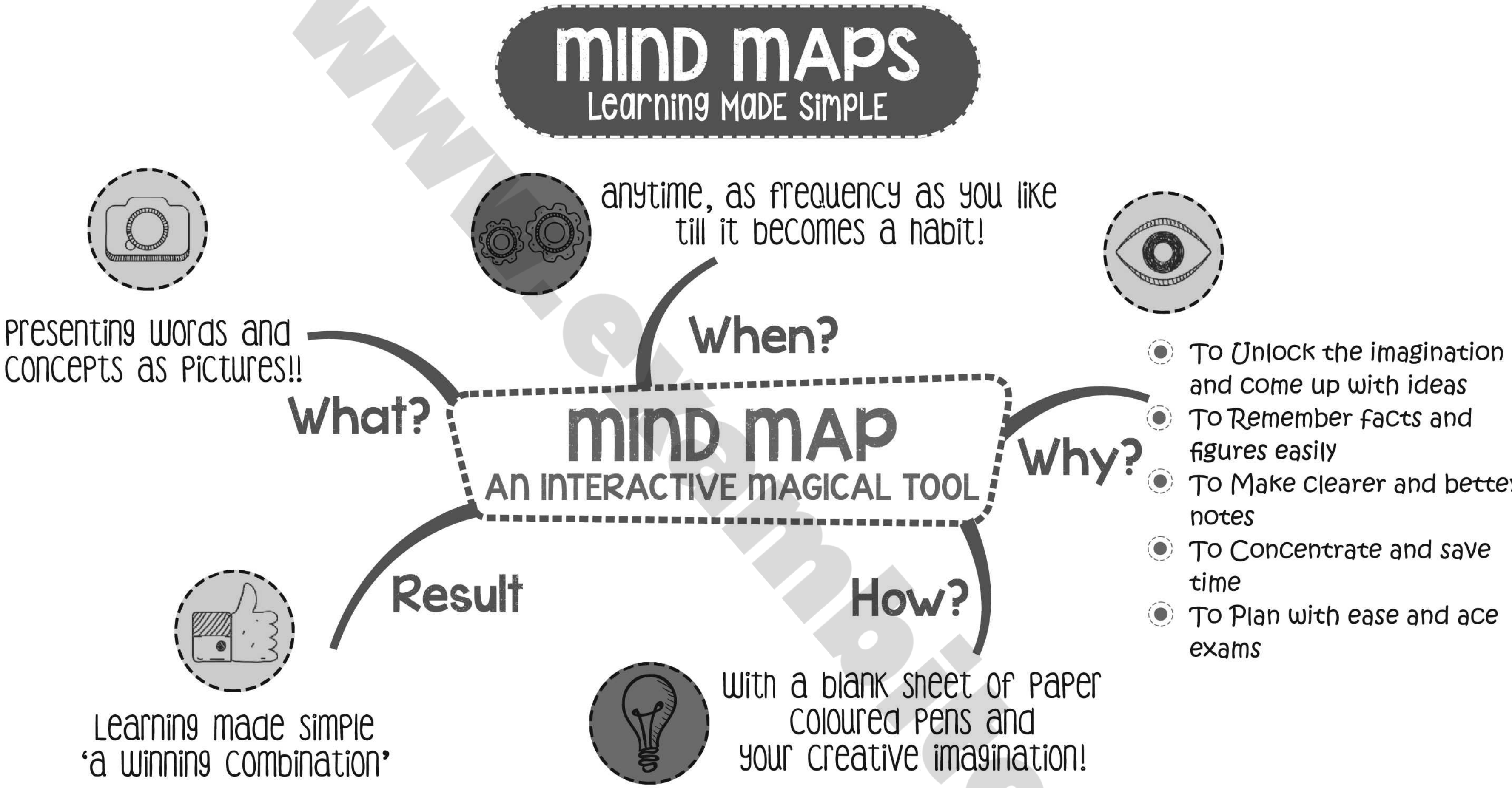
(2) The cost of the canvas = (Area of canvas required + area of canvas wasted during stitching) × 200

$= (1650 + 30) \times 200$

$= 1680 \times 200$

= ₹ 3,36,000

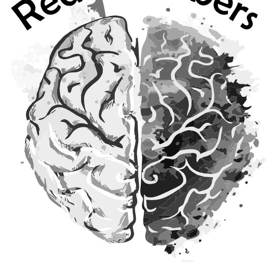
❑❑



What are Associations?

It's a technique connecting the core concept at the Centre to related concepts or ideas. Associations spreading out straight from the core concept are the First Level of Association. Then we have a Second Level of Association emitting from the first level and the chronology continues. The thickest line is the First Level of Association and the lines keep getting thinner as we move to the subsequent levels of association. This is exactly how the brain functions, therefore these Mind Maps. Associations are one powerful memory aid connecting seemingly unrelated concepts, hence strengthening memory.

Real Numbers



Theorems

1. Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer

2. $\sqrt{2}, \sqrt{3}$ are irrational number

3. Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form, $\frac{p}{q}$ where p & q are coprime, the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers

4. Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which terminates.

5. Let $x = \frac{p}{q}$ be a rational number, such that the prime factorisation of q is not of the form of $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating and recurring

Prime Factorization Method

To obtain H.C.F. & L.C.M.
For any two positive integers, a and b
H.C.F. (a, b) × L.C.M. $(a, b) = a \times b$
For Example
$f(x) = 3x^2y$
and $g(x) = 6xy^2$
H.C.F. $= 3xy$
and L.C.M. $= 6x^2y^2$

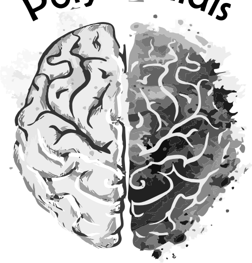
Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur

Composite Number $x = p_1 \times p_2 \times p_3 \ldots \times p_n$, where $p_1, p_2, \ldots, p_n$ are prime numbers

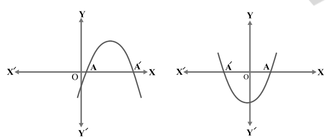
Trace the Mind Map
First Level · Second Level · Third Level

Polynomials



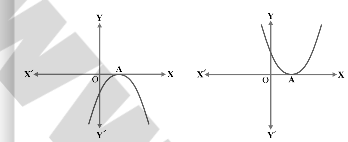
Zeroes of Polynomial : Graphically

Case 1 : Graph cuts X-axis at 2 points



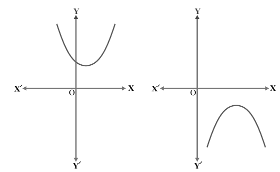
Number of Zeroes = 2

Case 2 : Graph cuts X-axis at exactly one point



Number of Zero = 1

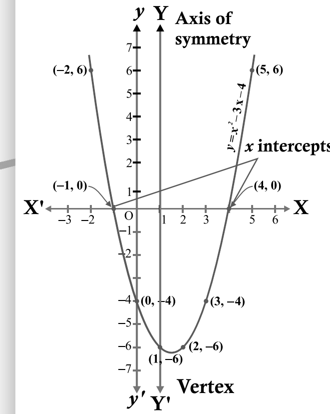
Case 3 : Graph does not cut X-axis



Number of Zero = 0

Graphical Representation of Quadratic Polynomial

Parabola



$y = x^2 - 3x - 4$

Degree of Polynomial

Highest power of x in Polynomial, $p(x)$

Types

Polynomial	Degree	General Form
Linear	1	$f(x) = ax+b$ $a \neq 0$
Quadratic	2	$f(x) = ax^2+bx+c$ $a \neq 0$
Cubic	3	$f(x) = ax^3+bx^2+cx+d$ $a \neq 0$

Relationship Between Zeroes and Coefficients of Quadratic Polynomials

Quadratic

Quadratic

If α and β are zeroes of Quadratic Polynomial $ax^2 + bx + c$

Then, Sum of zeroes

$$\alpha + \beta = -\frac{b}{a}$$

Product of zeroes

$$\alpha\beta = \frac{c}{a}$$

Cubic

Cubic

If α, β and γ are zeroes of Cubic Polynomial $ax^3 + bx^2 + cx + d$

Then, Sum of zeroes,

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

Sum of products of the zeroes taken two at a time

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

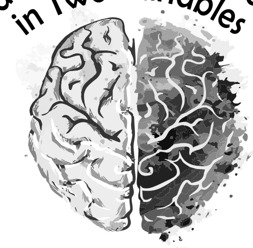
Product of zeroes

$$\alpha\beta\gamma = -\frac{d}{a}$$

Trace the Mind Map

- First Level
- Second Level
- Third Level

Pair of Linear Equations in Two Variables



Algebraic Methods

By Substitution

Solve : $7x - 15y = 2$ … (i)
$x + 2y = 3$ … (ii)

Solution : From equation (ii), $x = 3 - 2y$
Substitute this value of x in eq. (i)
$7(3 - 2y) - 15y = 2$

$-29y = -19 \Rightarrow y = \frac{19}{29}$

Now, from $x = 3 - 2y$

$x = 3 - 2\left(\frac{19}{29}\right) = \left(\frac{49}{29}\right)$

By Elimination

Solve : $x + 3y = 6$ … (i)
$2x - 3y = 12$ … (ii)
Now, Adding equation (i) and (ii)
$3x = 18$ or $x = 6$
Again, from (i) ×2– (ii)
$3y = 0$ or, $y = 0$
Hence, $x = 6$, $y = 0$

General Form

$a_1x + b_1y + c_1 = 0$,
$a_2x + b_2y + c_2 = 0$
where $a_1, b_1, c_1, a_2, b_2, c_2$ are real numbers.

Solution Graphically

Each solution (x, y), corresponds to a point on the line representing the equation and vice-versa

Graphical Representation

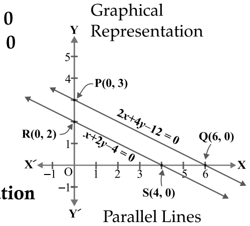
Pair of Lines : $x + 2y - 4 = 0$
$2x + 4y - 12 = 0$

$\frac{a_1}{a_2} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{2}{4}, \frac{c_1}{c_2} = \frac{-4}{-12}$

Compare the Ratios : $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Algebraic Interpretation : No solution *i.e.*, Inconsistent

Graphical Representation



Parallel Lines

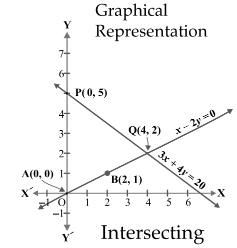
Pair of Lines : $x - 2y = 0$
$3x + 4y - 20 = 0$

$\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-2}{4}, \frac{c_1}{c_2} = \frac{0}{-20}$

Compare the Ratios: $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Algebraic Interpretation : Exactly one solution *i.e.*, consistent (unique)

Graphical Representation



Intersecting Lines

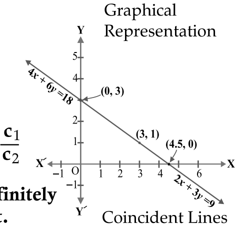
Pair of Lines : $2x + 3y - 9 = 0$
$4x + 6y - 18 = 0$

$\frac{a_1}{a_2} = \frac{2}{4}, \frac{b_1}{b_2} = \frac{3}{6}, \frac{c_1}{c_2} = \frac{-9}{-18}$

Compare the Ratios : $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

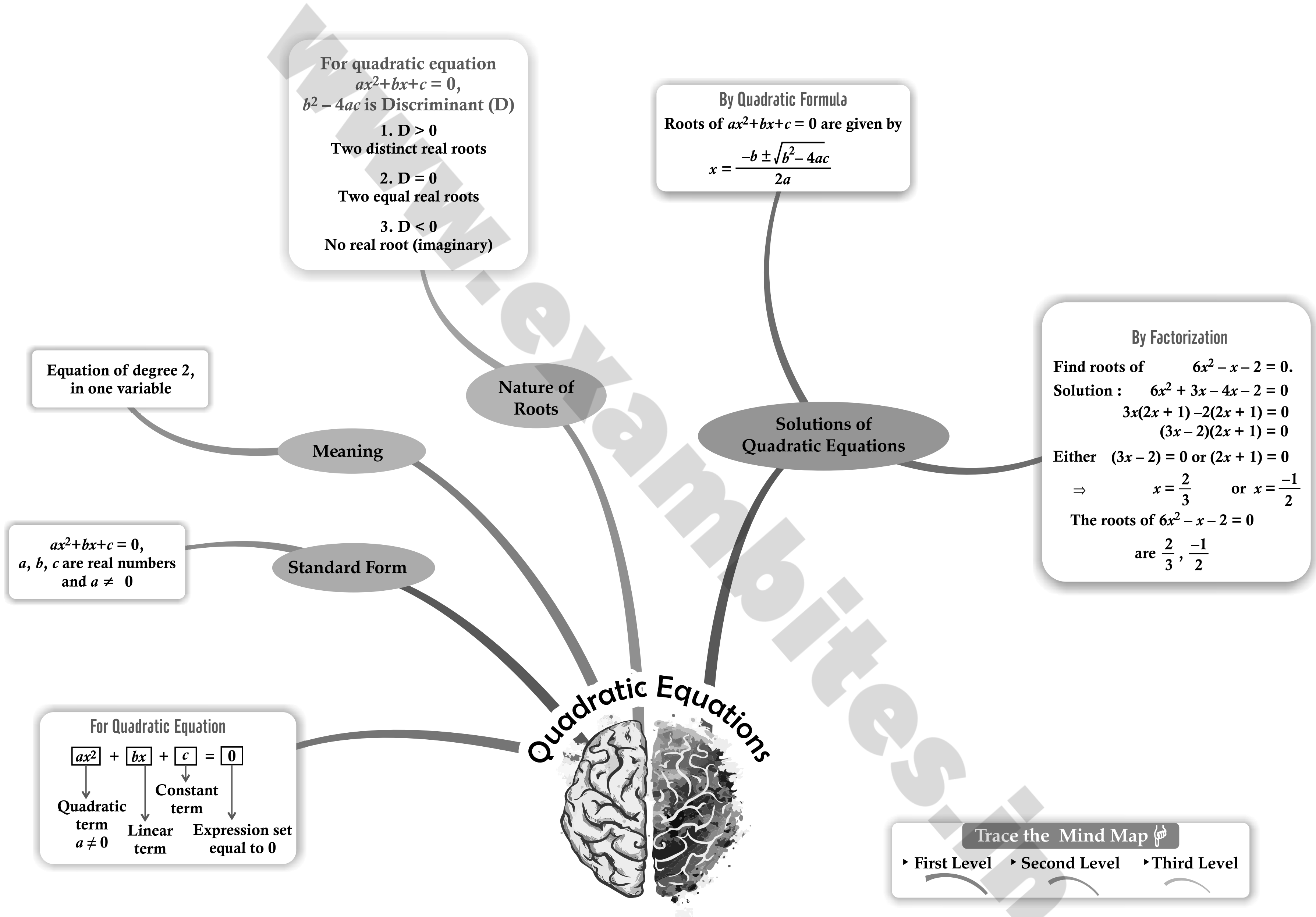
Algebraic Interpretation = Infinitely many solutions *i.e.*, consistent.

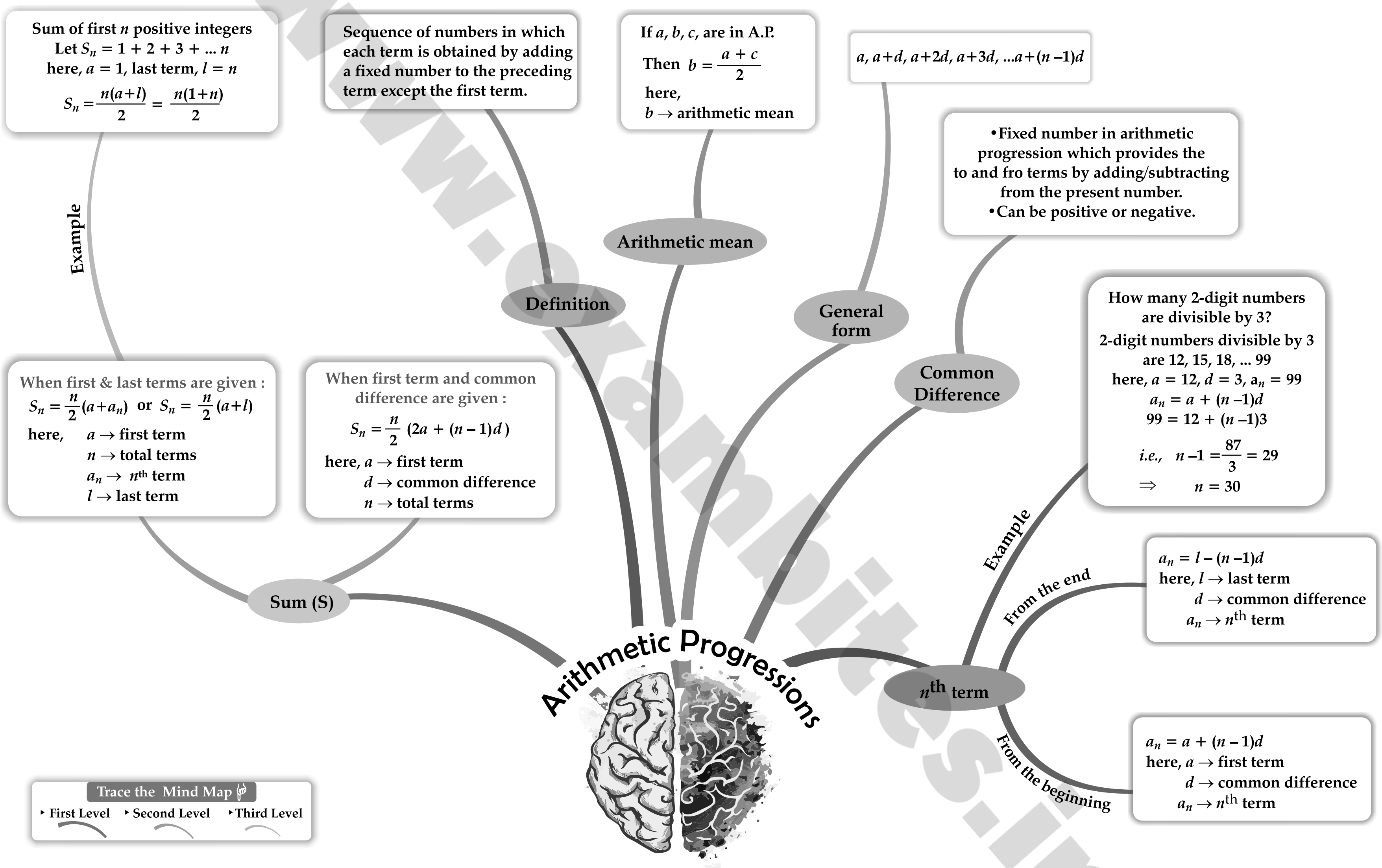
Graphical Representation



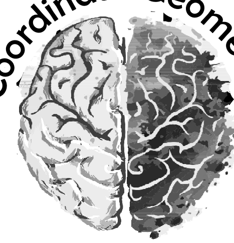
Coincident Lines

Trace the Mind Map
First Level · Second Level · Third Level

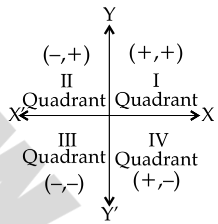

Quadratic Equations
Meaning
Equation of degree 2, in one variable
Standard Form
$ax^2+bx+c=0$, a, b, c are real numbers and $a \neq 0$
For Quadratic Equation
$\boxed{ax^2} + \boxed{bx} + \boxed{c} = \boxed{0}$
Quadratic term $a \neq 0$
Linear term
Constant term
Expression set equal to 0
Nature of Roots
For quadratic equation $ax^2+bx+c=0$, b^2-4ac is Discriminant (D)
1. D > 0
Two distinct real roots
2. D = 0
Two equal real roots
3. D < 0
No real root (imaginary)
Solutions of Quadratic Equations
By Quadratic Formula
Roots of $ax^2+bx+c=0$ are given by
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
By Factorization
Find roots of $6x^2 - x - 2 = 0$.
Solution : $6x^2 + 3x - 4x - 2 = 0$
$3x(2x+1) - 2(2x+1) = 0$
$(3x-2)(2x+1) = 0$
Either $(3x-2) = 0$ or $(2x+1) = 0$
$\Rightarrow$ $x = \frac{2}{3}$ or $x = \frac{-1}{2}$
The roots of $6x^2 - x - 2 = 0$
are $\frac{2}{3}, \frac{-1}{2}$
Trace the Mind Map
First Level
Second Level
Third Level


Arithmetic Progressions
Definition
Sequence of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
Arithmetic mean
If a, b, c, are in A.P.
Then $b = \frac{a+c}{2}$
here,
$b \rightarrow$ arithmetic mean
General form
$a, a+d, a+2d, a+3d, ...a+(n-1)d$
Common Difference
•Fixed number in arithmetic progression which provides the to and fro terms by adding/subtracting from the present number.
•Can be positive or negative.
n^{th} term
Example
How many 2-digit numbers are divisible by 3?
2-digit numbers divisible by 3 are 12, 15, 18, ... 99
here, $a = 12$, $d = 3$, $a_n = 99$
$a_n = a + (n-1)d$
$99 = 12 + (n-1)3$
i.e., $n - 1 = \frac{87}{3} = 29$
$\Rightarrow$ $n = 30$
From the end
$a_n = l - (n-1)d$
here, $l \rightarrow$ last term
$d \rightarrow$ common difference
$a_n \rightarrow n^{th}$ term
From the beginning
$a_n = a + (n-1)d$
here, $a \rightarrow$ first term
$d \rightarrow$ common difference
$a_n \rightarrow n^{th}$ term
Sum (S)
When first term and common difference are given :
$S_n = \frac{n}{2}(2a + (n-1)d)$
here, $a \rightarrow$ first term
$d \rightarrow$ common difference
$n \rightarrow$ total terms
When first & last terms are given :
$S_n = \frac{n}{2}(a+a_n)$ or $S_n = \frac{n}{2}(a+l)$
here, $a \rightarrow$ first term
$n \rightarrow$ total terms
$a_n \rightarrow n^{th}$ term
$l \rightarrow$ last term
Example
Sum of first n positive integers
Let $S_n = 1 + 2 + 3 + ... n$
here, $a = 1$, last term, $l = n$
$S_n = \frac{n(a+l)}{2} = \frac{n(1+n)}{2}$
Trace the Mind Map
First Level
Second Level
Third Level

Coordinate Geometry

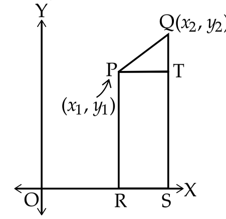


Meaning



Distance formula

$PQ = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$



Example

Are the following points vertices of a square : (1, 7), (4, 2), (–1, –1), (–4, 4)?

A = (1, 7); B = (4, 2); C = (–1, –1); D = (–4, 4)

$AB = \left|\sqrt{(1-4)^2 + (7-2)^2}\right| = \sqrt{34}$ Unit

$BC = \left|\sqrt{(4+1)^2 + (2+1)^2}\right| = \sqrt{34}$ Unit

$CD = \left|\sqrt{(-1+4)^2 + (-1-4)^2}\right| = \sqrt{34}$ Unit

$DA = \left|\sqrt{(1+4)^2 + (7-4)^2}\right| = \sqrt{34}$ Unit

$AC = \left|\sqrt{(1+1)^2 + (7+1)^2}\right| = \sqrt{68}$ Unit

$BD = \left|\sqrt{(4+4)^2 + (2-4)^2}\right| = \sqrt{68}$ Unit

Since, AB = BC = CD = DA and AC = BD.
All four sides and diagonals are equal
Hence, ABCD is a square

Section formula

Example

Find point of trisection of line segment AB, A(2, –2) and B(–7, 4)

A P Q B
(2, –2) (–7, 4)

Coordinate of P $= \left(\frac{1(-7)+2(2)}{1+2}, \frac{1(4)+2(-2)}{1+2}\right)$

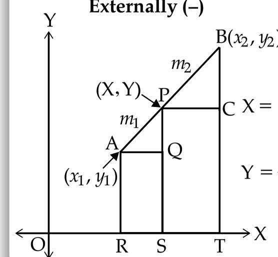
$= (-1, 0)$

Coordinate of Q

$Q = \left(\frac{2(-7)+1(2)}{2+1}, \frac{2(4)+1(-2)}{2+1}\right)$

$= (-4, 2)$

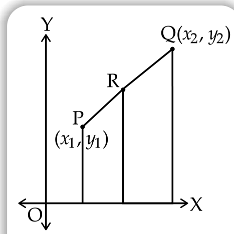
Internally (+)
Externally (–)



$X = \frac{m_1x_2 \pm m_2x_1}{m_1 \pm m_2}$

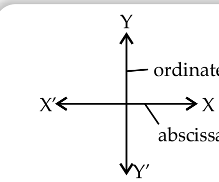
$Y = \frac{m_1y_2 \pm m_2y_1}{m_1 \pm m_2}$

Mid-point of a Line Segment



$R\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Coordinate axes

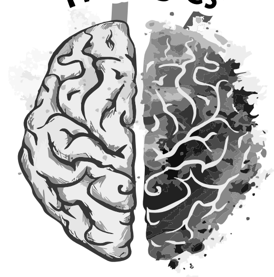


Horizontal = X–axis (Abscissa)
Vertical = Y–axis (Ordinate)

Trace the Mind Map

- **First Level**
- **Second Level**
- **Third Level**

Triangles



Theorems

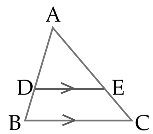
Summary

In ΔABC, let $DE \parallel BC$. Then,

(i) $\frac{AD}{DB} = \frac{AE}{EC}$

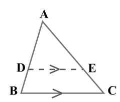
(ii) $\frac{AB}{DB} = \frac{AC}{EC}$

(iii) $\frac{AD}{AB} = \frac{AE}{AC}$



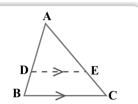
1. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

If, $DE \parallel BC$ then $\frac{AD}{DB} = \frac{AE}{EC}$

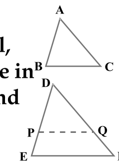


2. If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If $\frac{AD}{DB} = \frac{AE}{EC}$ then, $DE \parallel BC$



3. If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportion) and hence the two triangles are similar.(AAA criterion)

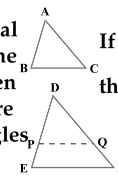


If $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

then, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$

$\Delta ABC \sim \Delta DEF$

4. If in two triangles, sides of one triangle are proportional to (*i.e.*, in the same ratio of) the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similiar.(SSS criterion)

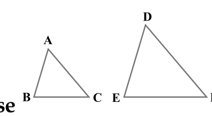


If $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

then, $\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

$\Delta ABC \sim \Delta DEF$

5. If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.(SAS criterion)



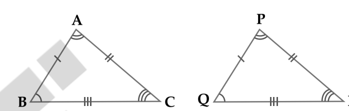
If $\frac{AB}{DE} = \frac{AC}{DF}$ & $\angle A = \angle D$

then, $\Delta ABC \sim \Delta DEF$

Similarity

(i) Corresponding angles are equal

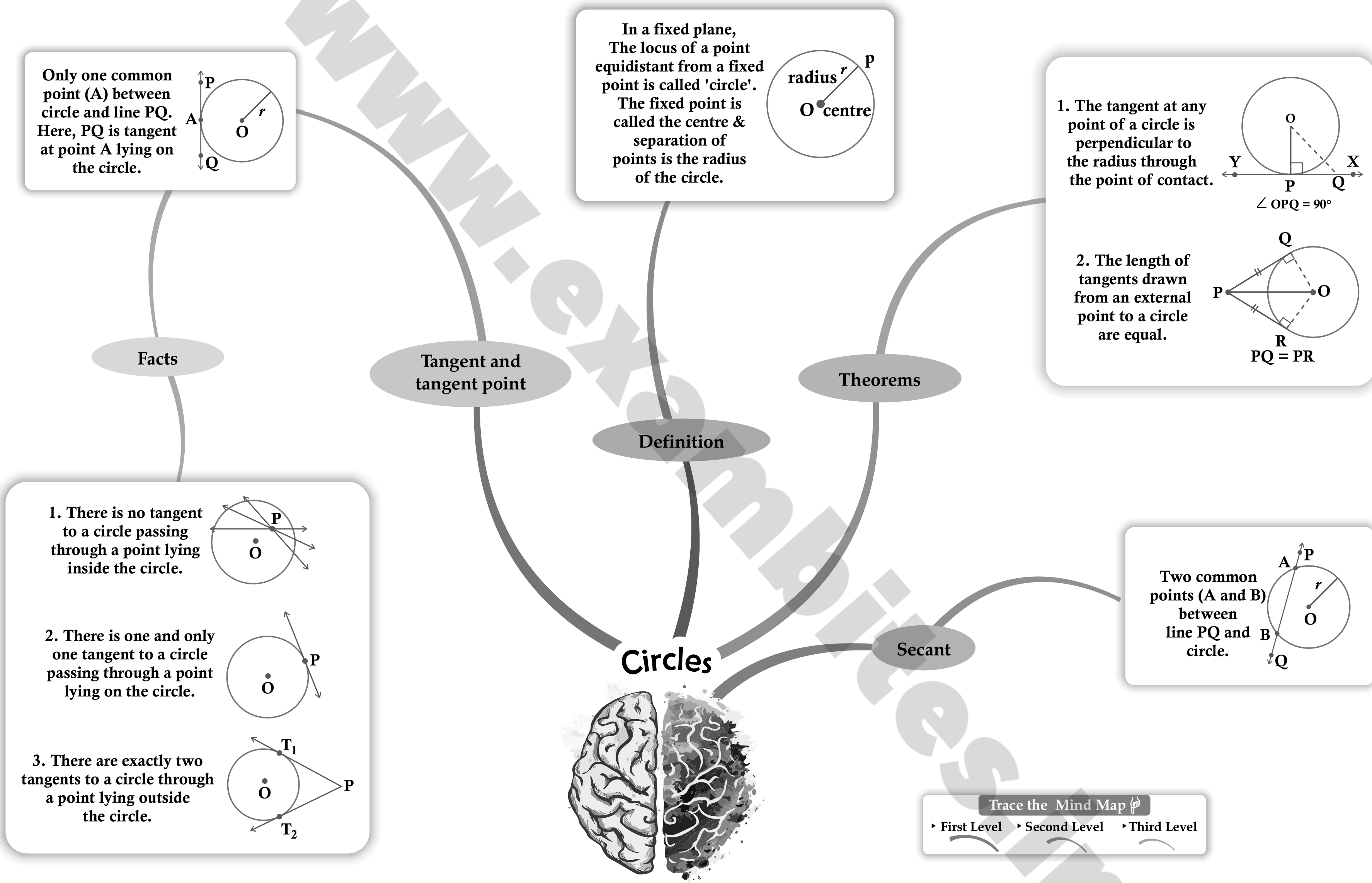
(ii) Corresponding sides are in the same ratio



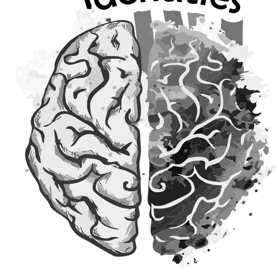
$\Delta ABC \sim \Delta PQR$

Trace the Mind Map

- First Level
- Second Level
- Third Level


Circles
Definition
In a fixed plane, The locus of a point equidistant from a fixed point is called 'circle'. The fixed point is called the centre & separation of points is the radius of the circle.
radius
r
p
O
centre
Theorems
1. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
O
Y
P
Q
X
∠ OPQ = 90°
2. The length of tangents drawn from an external point to a circle are equal.
P
Q
O
R
PQ = PR
Secant
Two common points (A and B) between line PQ and circle.
A
P
r
O
B
Q
Tangent and tangent point
Only one common point (A) between circle and line PQ. Here, PQ is tangent at point A lying on the circle.
P
A
O
r
Q
Facts
1. There is no tangent to a circle passing through a point lying inside the circle.
P
O
2. There is one and only one tangent to a circle passing through a point lying on the circle.
P
O
3. There are exactly two tangents to a circle through a point lying outside the circle.
T1
O
P
T2
Trace the Mind Map
First Level
Second Level
Third Level

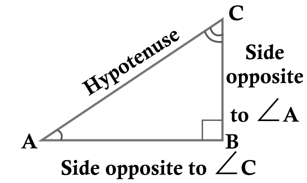
Trigonometry and Trigonometric Identities



Trigonometry

Study of relationships between the sides & angles of a triangle

Trigonometric Ratios



Sine of $\angle A = \frac{BC}{AC}$

Cosine of $\angle A = \frac{AB}{AC}$

Tangent of $\angle A = \frac{BC}{AB}$

Cotangent of $\angle A = \frac{AB}{BC}$

Secant of $\angle A = \frac{AC}{AB}$

Cosecant of $\angle A = \frac{AC}{BC}$

Note: How to learn the relation

"Some people have" $\sin\theta = \frac{P}{H}$

"Curly Brown Hair" $\cos\theta = \frac{B}{H}$

"through proper Brushing" $\tan\theta = \frac{P}{B}$

Values of trigonometric ratios between 0° to 90°

$\angle A$	0°	30°	45°	60°	90°
sin A	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos A	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan A	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	(∞) Not defined
cot A	(∞) Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
sec A	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	(∞) Not defined
cosec A	(∞) Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

Relation of tan with sin & cos

Express tan A and cos A in terms of sin A

Solution : Since, $\cos^2 A + \sin^2 A = 1$

$\cos^2 A = 1 - \sin^2 A$ *i.e.* $\cos A = \sqrt{1 - \sin^2 A}$

$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

Trigonometric Identities

$0 \le A \le 90°$ $\cos^2 A + \sin^2 A = 1$

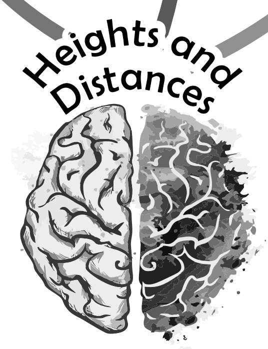
$0 \le A < 90°$ $1 + \tan^2 A = \sec^2 A$

$0 < A \le 90°$ $\cot^2 A + 1 = \text{cosec}^2 A$

Trace the Mind Map

First Level · Second Level · Third Level

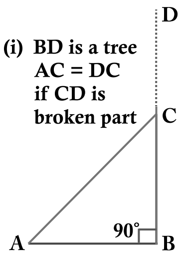
Heights and Distances



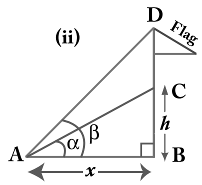
Examples of Distance

Height / Length of an object

(i) BD is a tree
AC = DC
if CD is
broken part



(ii)



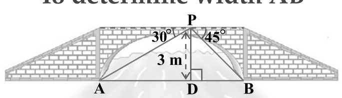
Find flag length

$\tan \alpha = \frac{h}{x}$ ...(i)

$\tan \beta = \frac{h+DC}{x}$ ...(ii)

and solve it

To determine width AB



From figure, AB = AD + DB

In right ΔAPD, $\angle A = 30°$, $\angle D = 90°$

$\tan 30° = \frac{PD}{AD}$ *i.e.*, $AD = 3\sqrt{3}$ m

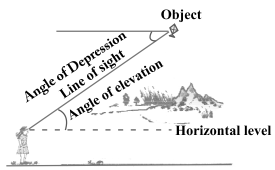
In right ΔBPD, $\angle B = 45°$, $\angle D = 90°$

$\tan 45° = \frac{PD}{BD}$ *i.e.*, BD = 3

$\therefore$ $AB = (3\sqrt{3} + 3)$ m $= 3(\sqrt{3} + 1)$ m

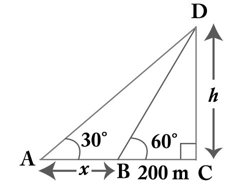
Measuring Angles

Angle of Elevation is equal to Angle of Depression



Examples Height

Distance between two objects



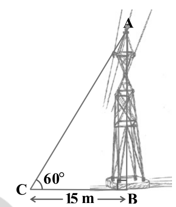
Find x and h

$\tan 60° = \frac{h}{200}$ ...(i)

$\tan 30° = \frac{h}{x+200}$ ...(ii)

and solve both equations

To determine height of object AB



In ΔABC, $\angle B = 90°$, $\angle C = 60°$

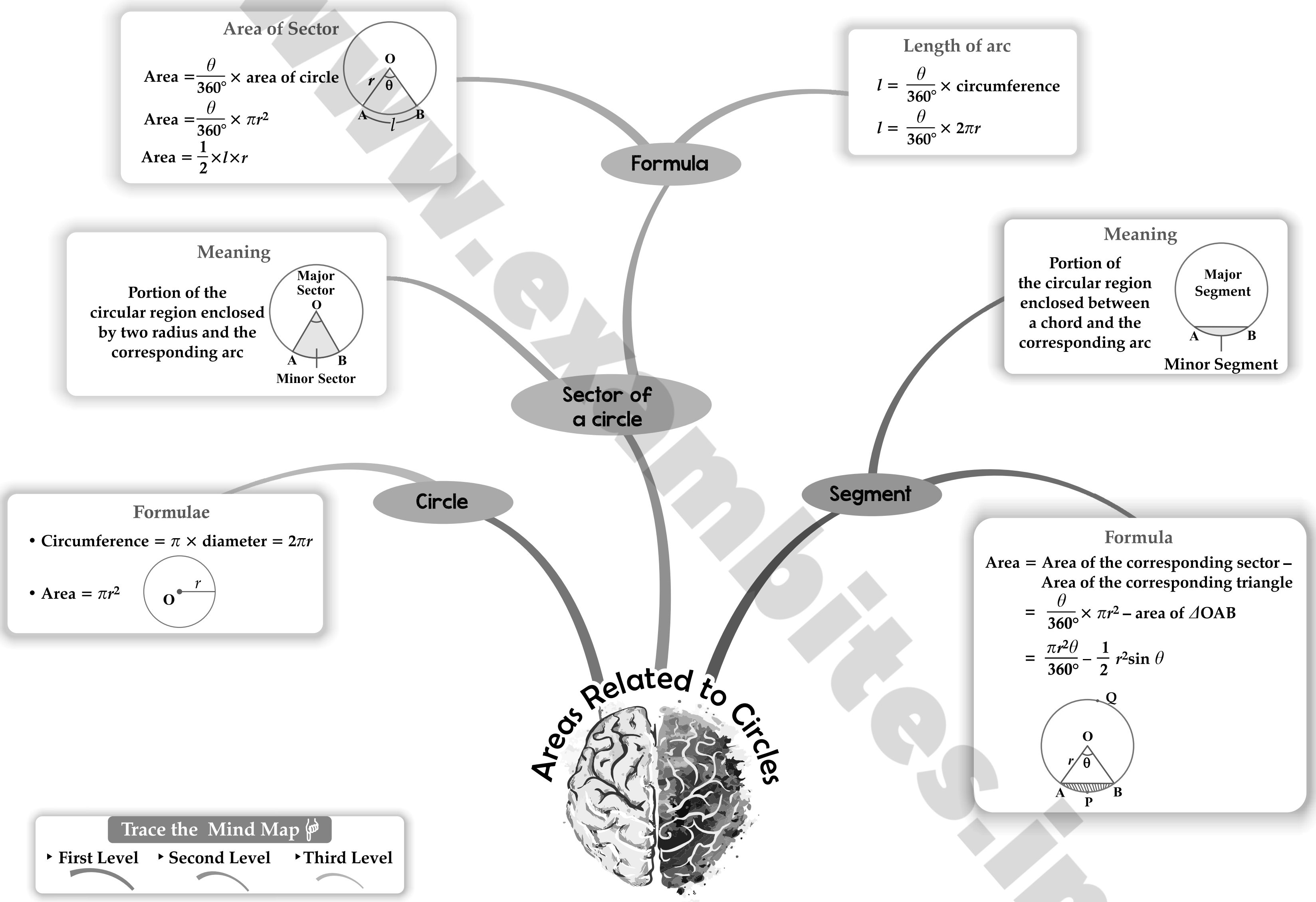
Here, $\tan 60° = \frac{AB}{BC}$

$\sqrt{3} = \frac{AB}{15}$

i.e., $AB = 15\sqrt{3}$ m

Trace the Mind Map

- First Level
- Second Level
- Third Level


Areas Related to Circles
Circle
Formulae
• Circumference = π × diameter = $2\pi r$
• Area = πr^2
O
r
Sector of a circle
Meaning
Portion of the circular region enclosed by two radius and the corresponding arc
Major Sector
O
A
B
Minor Sector
Formula
Area of Sector
Area = $\frac{\theta}{360°}$ × area of circle
Area = $\frac{\theta}{360°} \times \pi r^2$
Area = $\frac{1}{2} \times l \times r$
O
r
θ
A
B
l
Length of arc
$l = \frac{\theta}{360°}$ × circumference
$l = \frac{\theta}{360°} \times 2\pi r$
Segment
Meaning
Portion of the circular region enclosed between a chord and the corresponding arc
Major Segment
A
B
Minor Segment
Formula
Area = Area of the corresponding sector – Area of the corresponding triangle
$= \frac{\theta}{360°} \times \pi r^2$ – area of ΔOAB
$= \frac{\pi r^2\theta}{360°} - \frac{1}{2} r^2 \sin\theta$
Q
O
r
θ
A
P
B
Trace the Mind Map
First Level
Second Level
Third Level

Name of solid	Volume	Total surface Area	Lateral surface Area
Cube	$V = a^3$	TSA = $6a^2$	LSA = $4a^2$
Cuboid	$V = l \times b \times h$	TSA = $2(lb + bh + hl)$	LSA = $2h(l + b)$
Cylinder	$V = \pi r^2 h$	TSA = $2\pi r(h + r)$	CSA = $2\pi rh$
Hollow cylinder ($R > r$)	$V = \pi(R^2 - r^2)h$	TSA = $2\pi(R + r)(h + R - r)$	$2\pi(R+r)h$
Cone	$V = \frac{1}{3}\pi r^2 h$	TSA = $\pi r(l + r)$	CSA = πrl
Sphere	$V = \frac{4}{3}\pi r^3$	TSA = $4\pi r^2$	CSA = $4\pi r^2$
Hemisphere	$V = \frac{2}{3}\pi r^3$	TSA = $3\pi r^2$	CSA = $2\pi r^2$

Sum of surface areas of the faces of solid

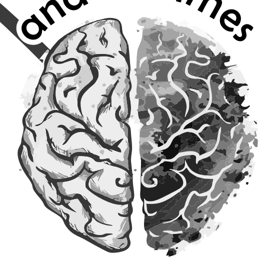
Surface Area

Formulas

Combination of Solids

Volume

Quantity of 3-D space enclosed by a hollow/closed solid



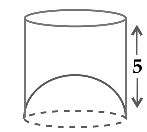
Surface Areas and Volumes

Trace the Mind Map

- **First Level**
- **Second Level**
- **Third Level**

Example

Given: Inner diameter of the Cylindrical glass = 5 cm
Height = 5 cm



Find: Actual capacity of cylindrical glass.

Solution : Apparent capacity of the glass = $\pi r^2 h$

$= 3.14 \times 2.5 \times 2.5 \times 5 \text{ cm}^3$

$= 98.125 \text{ cm}^3$

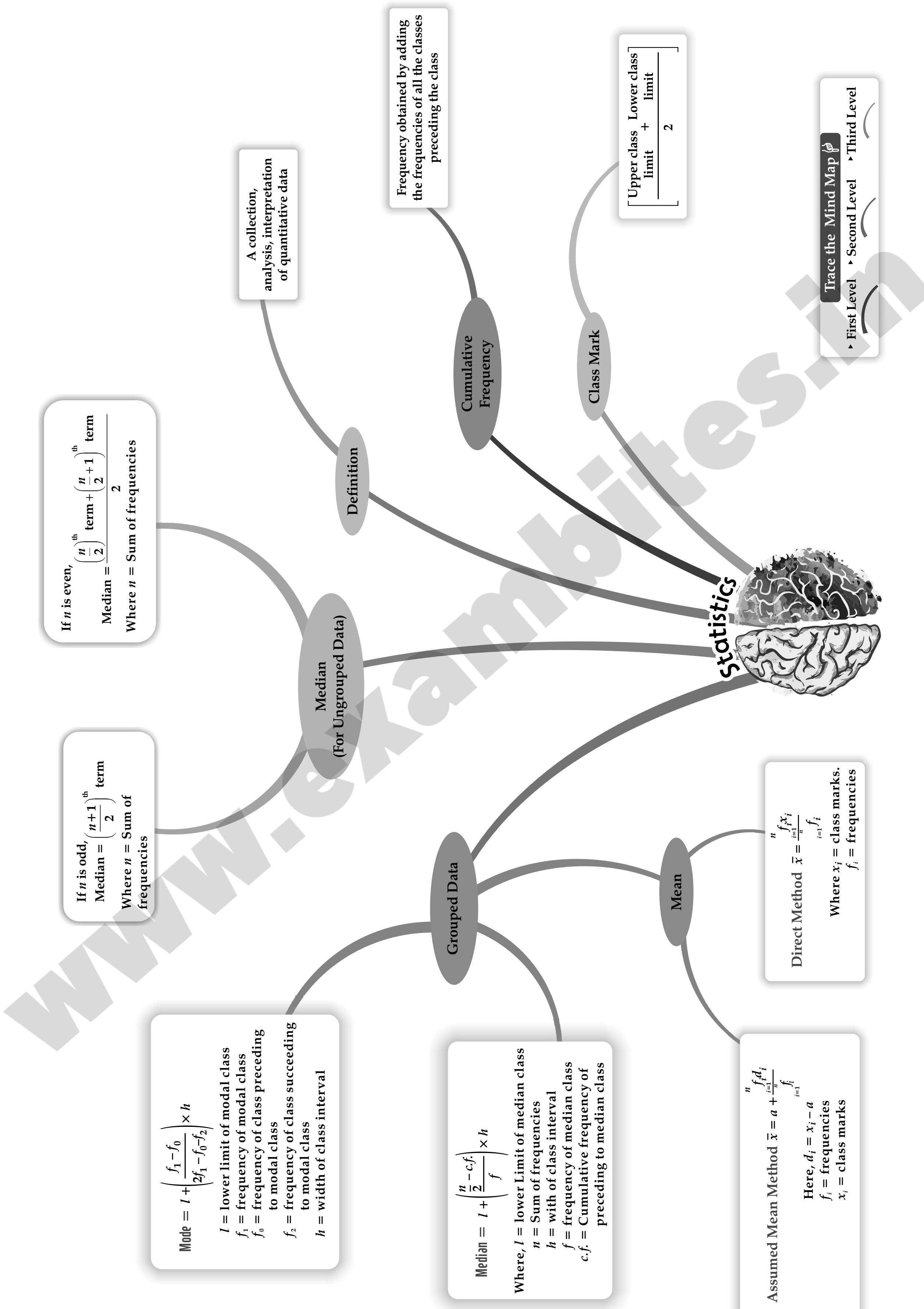
Volume of hemisphere = $\frac{2}{3}\pi r^3$, if r = 2.5 cm

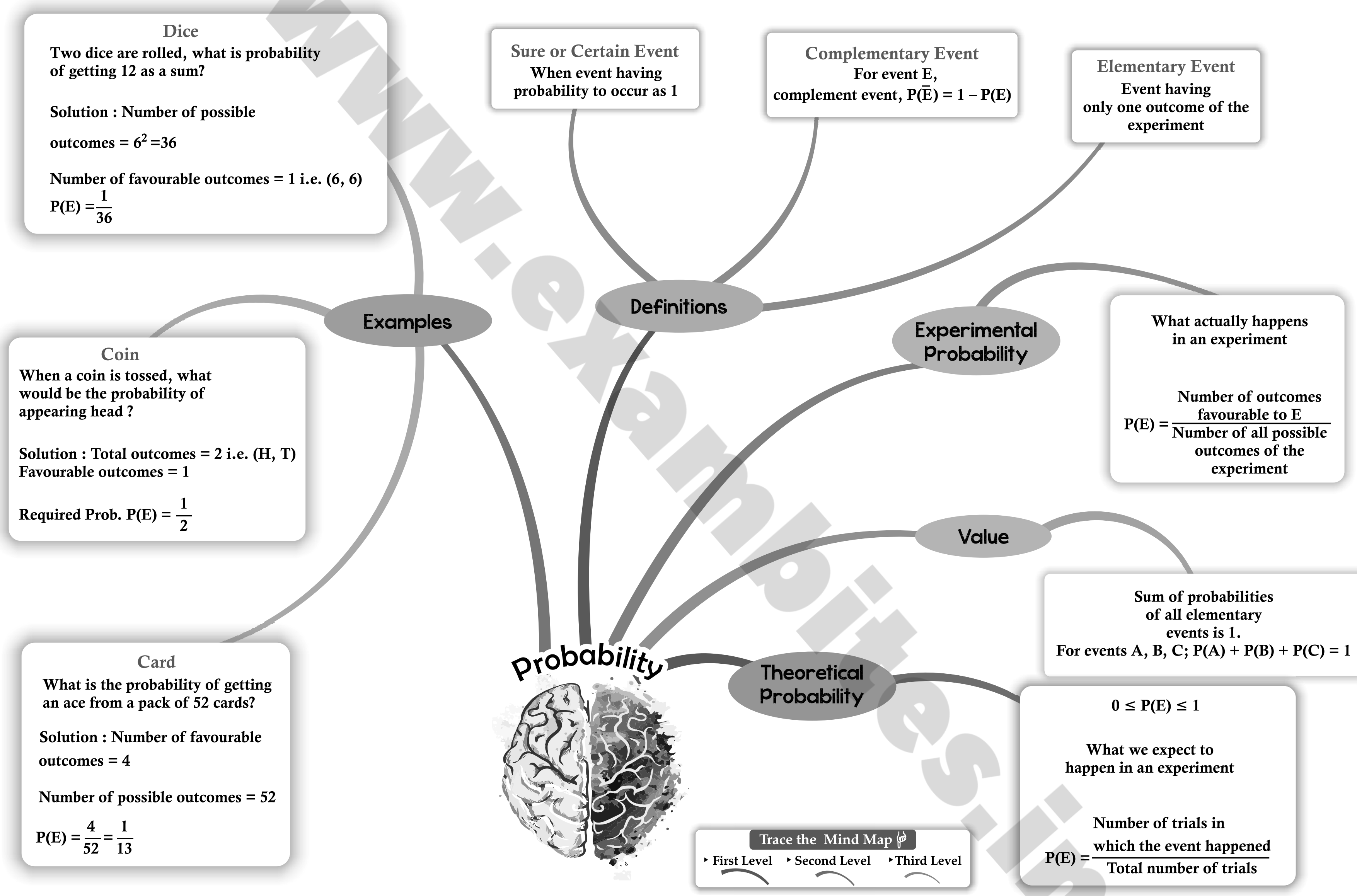
$= \frac{2}{3} \times 3.14 \times (2.5)^3 \text{ cm}^3 = 32.71 \text{ cm}^3$

Actual capacity = Apparent capacity − Volume of hemisphere

$= 98.125 - 32.71$

$= 65.42 \text{ cm}^3$


Statistics
Definition
A collection, analysis, interpretation of quantitative data
Cumulative Frequency
Frequency obtained by adding the frequencies of all the classes preceding the class
Class Mark
$\left[\dfrac{\text{Upper class limit} + \text{Lower class limit}}{2}\right]$
Trace the Mind Map
First Level
Second Level
Third Level
Median (For Ungrouped Data)
If n is even, Median $= \dfrac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}$
Where n = Sum of frequencies
If n is odd, Median $= \left(\dfrac{n+1}{2}\right)^{th}$ term
Where n = Sum of frequencies
Grouped Data
Mean
Direct Method $\bar{x} = \dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$
Where x_i = class marks. f_i = frequencies
Assumed Mean Method $\bar{x} = a + \dfrac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$
Here, $d_i = x_i - a$
f_i = frequencies
x_i = class marks
Median $= l + \left(\dfrac{\frac{n}{2} - c.f}{f}\right) \times h$
Where, l = lower Limit of median class
n = Sum of frequencies
h = with of class interval
f = frequency of median class
$c.f.$ = Cumulative frequency of preceding to median class
Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$
l = lower limit of modal class
f_1 = frequency of modal class
f_0 = frequency of class preceding to modal class
f_2 = frequency of class succeeding to modal class
h = width of class interval


Probability
Examples
Dice
Two dice are rolled, what is probability of getting 12 as a sum?
Solution : Number of possible outcomes = $6^2 = 36$
Number of favourable outcomes = 1 i.e. (6, 6)
$P(E) = \frac{1}{36}$
Coin
When a coin is tossed, what would be the probability of appearing head ?
Solution : Total outcomes = 2 i.e. (H, T)
Favourable outcomes = 1
Required Prob. $P(E) = \frac{1}{2}$
Card
What is the probability of getting an ace from a pack of 52 cards?
Solution : Number of favourable outcomes = 4
Number of possible outcomes = 52
$P(E) = \frac{4}{52} = \frac{1}{13}$
Definitions
Sure or Certain Event
When event having probability to occur as 1
Complementary Event
For event E, complement event, $P(\bar{E}) = 1 - P(E)$
Elementary Event
Event having only one outcome of the experiment
Experimental Probability
What actually happens in an experiment
$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Number of all possible outcomes of the experiment}}$
Value
Sum of probabilities of all elementary events is 1.
For events A, B, C; $P(A) + P(B) + P(C) = 1$
Theoretical Probability
$0 \le P(E) \le 1$
What we expect to happen in an experiment
$P(E) = \frac{\text{Number of trials in which the event happened}}{\text{Total number of trials}}$
Trace the Mind Map
First Level
Second Level
Third Level

ON TIPS NOTES

Note-making is a skill that we use in many walks of life: at school, at university and in the world of work. However, accurate note-making requires a thorough understanding of concepts. We, at Oswaal, have tried to encapsulate all the chapters from the given syllabus into the following **ON TIPS NOTES**. These notes will not only facilitate better understanding of concepts, but will also ensure that each and every concept is taken up and every chapter is covered in totality. So, go ahead and use these to your advantage… go get the **OSWAAL ADVANTAGE !!**

CHAPTER 1: Real Numbers

- **Fundamentals:**
  1. A non-negative integer 'p' is said to be divisible by an integer 'q' if there exists an integer 'd' such that:
  $$p = qd.$$
  2. ± 1 divides every non-zero integer.
  3. 0 does not divide any integer.
- **Fundamental Theorem of Arithmetic:**
  Every composite number can be expressed as a product of primes, and this factorisation is unique except for the order in which the prime factors occur.
  Important Theorems:
  1. Let p be a prime number and a be a positive integer. If p divides a^2, then p divides a.
  2. Consider two positive integers a and b, then LCM × HCF = $a \times b$.

  HCF and LCM:
  1. HCF (a, b): Product of the smallest power of each common prime factor in the numbers.
  2. LCM (a, b): Product of the greatest power of each prime factor, involved in the numbers.
  3. HCF (a, b) × LCM (a, b) = $a \times b$, for any two positive integers a and b.

CHAPTER 2: Polynomials

- **Fundamentals:**
  An algebraic equation of the form,
  $$p(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots\ldots + a_{n-1}x + a_n$$
  is called polynomial, provided it has no negative exponent for any variable.
  where, $a_0, a_1, a_2, \ldots\ldots a_{n-1}, a_n$ are constants (real numbers); $a_0 \neq 0$.
- **Degree of polynomial:** n is called the degree (highest power of variable x). If $n = 1$ then polynomial is called linear polynomial.
  General form: $ax + b$ where, $a \neq 0$
  If $n = 2$ then polynomial is called quadratic polynomial.
  General form: $ax^2 + bx + c$ where, $a \neq 0$
  If $n = 3$ then polynomial is called cubic polynomial.
  General form: $ax^3 + bx^2 + cx + d$ where, $a \neq 0$
- **Zeroes of polynomial:** For polynomial $p(x)$, the value of x for which $p(x) = 0$, is called zero(es) of polynomial.
  Linear polynomial can have atmost 1 root (zero).
  Quadratic polynomial can have atmost 2 roots (zeroes).
  Cubic polynomial can have atmost 3 roots.

- **Relationship between zeroes and coefficient of polynomial:**
  - Zero of linear polynomial $ax + b$ is given by $x = \frac{-b}{a} = \frac{-(\text{Constant term})}{(\text{Coefficient of } x)}$.
  - If α and β are zeroes of the quadratic polynomial $ax^2 + bx + c$, then

    Sum of zeroes, $\alpha + \beta = \frac{-b}{a} = -\frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$.

    Product of zeroes, $\alpha\beta = \frac{c}{a} = \frac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$.
  - If α, β and γ are the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, then

    Sum of zeroes, $\alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{Coefficient of } x^2)}{(\text{Coefficient of } x^3)}$.

    Product of roots taken two at a time

    $$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{(\text{Coefficient of } x)}{(\text{Coefficient of } x^3)}.$$

    Product of zeroes, $\alpha\beta\gamma = \frac{-d}{a} = \frac{-(\text{Constant term})}{(\text{Coefficient of } x^3)}$.

CHAPTER 3: Pair of Linear Equations in Two Variables

- **Fundamentals:**

1. The general form of linear equation in one variable is: $ax + b = 0, (a \neq 0)$.
2. The general form of linear equation in two variables is: $ax + by + c = 0$.
   where, a, b, c are real coefficients; ($a^2 + b^2 \neq 0$ i.e., a and b are not both zero).
3. Every linear equation gives a straight line graph. Every point lying on this line is a solution of linear equation.
4. Two linear equations which are in the same two variables x and y simultaneously are called pair of linear equations in two variables.
5. The general form of pair of linear equation in two variables is:

   $$a_1x + b_1y + c_1 = 0$$

   and

   $$a_2x + b_2y + c_2 = 0$$

   where, $a_1, b_1, c_1, a_2, b_2, c_2$ are all real coefficients.
6. A pair of values of x and y satisfying each one of the equations is called solution of the system.

- **Graphical and Algebraic Interpretation:**

Pair of linear equations	Algebraic condition	Graphical interpretation	Algebraic interpretation	consistency
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Unique solution	Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions	Dependent Consistent
	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution	Inconsistent

Note: A pair of linear equations having atleast one solution is called consistent otherwise inconsistent

- **Methods of Solving Pair of Linear Equations:**
  (A) Graphical method
  (B) Algebraic methods
  (i) Substitution method
  (ii) Elimination method

Graphical Method:

Step 1: Plot both the linear equations on the same graph.
Step 2: Find the intersecting point on graph, if the lines are intersecting.
Step 3: Intersecting point is the required solution.
Step 4: In step 2, if the lines are coincident, then there are infinitely many solutions—each point on the line being a solution.
Step 5: In step 2, if the lines are parallel, then the pair of equations has no solution.

Substitution Method:

Step 1: From one equation, find the value of one variable in terms of other variable.
Step 2: Substitute the value of variable obtained in step 1 in the other equation, you will get the equation in one variable.
Step 3: Solve the equation in one variable and find the value of variable.
Step 4: Substitute the value of the variable so obtained in step 3 in any equation, you will get equation in unknown variable.
Step 5: Solve this equation in one variable and find the value of this variable.

Elimination Method:

Step 1: First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.
Step 2: Then add or subtract one equation from other so that one variable gets eliminated and resultant equation will become an equation in one variable.
Step 3: Solve the equation in one variable and find the value of the variable.
Step 4: Substitute the value of the variable so obtained in step 3 in any equation, you will get equation in unknown variable.
Step 5: Solve this equation in one variable and find the value of this variable now.

Equations Reducible to a pair of linear equations in two variable: There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.

CHAPTER 4 : Quadratic Equations

- **Fundamentals:**
  1. General form of quadratic equation is: $ax^2 + bx + c = 0$, where $a \neq 0$.
  2. a is co-efficient of x^2, b is co-efficient of x, c is called constant term.
  3. Equation of the form $ax^2 + c = 0$ is called pure quadratic equation.
  4. The value of variable satisfying equation is called root of that equation.
  5. Quadratic equation has atmost two roots.
- **Methods of Solving Quadratic Equations:**
  (a) Factorisation method (splitting the middle term)
  (c) Quadratic formula method (Sridharacharya formula)

Factorisation Method:

Step 1: Resolve the equation in factor using splitting the middle term method,
i.e,: $ax^2 + bx + c = (Ax + B)(Cx + D)$

Step 2: Put both factors equal to zero,
$Ax + B = 0$ and $Cx + D = 0$
Therefore, $x = -\frac{B}{A}$ and $x = -\frac{D}{C}$ are two roots.

- **Quadratic Formula Method:**
  Direct formula to calculate the roots is given as:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Proof: Use steps given in completing the square method.
Discriminant: It is denoted by 'D' and given by:

$$D = b^2 - 4ac$$

- **Nature of Roots of Quadratic Equation:**
  First of all find D.
  - If $D > 0$, roots are real and unequal.
  - If $D = 0$, roots are real and equal.
  - If $D < 0$, roots are imaginary.

CHAPTER 5: Arithmetic Progressions

- **Sequence:** An arrangement of numbers which has a pattern, which can suggest the successor of every number in the arrangement.
  Examples of Arithmetic Progressions:
- **Arithmetic Progression (AP):** It is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.
  (i) 3, 5, 7, 9, 11..........
  (ii) – 8, – 5, – 2, 1, 4, 7........
  (iii) 6, 1, – 4, – 9, – 14.........
  (iv) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$ .... Yes it is AP, because it can be written as below
  $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, 5\sqrt{2}$ ....
  (v) $3 + \sqrt{2}, 3 + 2\sqrt{2}, 3 + 3\sqrt{2}, 3 + 4\sqrt{2}, 3 + 5\sqrt{2}$ ....

  This fixed number is called common difference, denoted by 'd'. It can be positive or negative.
  So, general form of an AP is given by:
  $a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots\ldots\ldots, a + (n - 1)d.$
  where, a = first term, d = common difference and $a + (n - 1)d = n^{\text{th}}$ term.
  If t_1, t_2, t_3 .... be the I, II, III...... terms of an AP.
  then, $t_2 - t_1 = d$
  $t_3 - t_2 = d$
  $t_4 - t_3 = d$ and so on.....................
  Hence, It can be written:
  $t_2 - t_1 = t_3 - t_2$
  $2t_2 = t_1 + t_3$
  Conclusion: If three numbers a, b and c are in AP then:
  $2b = a + c.$
- **Important Formulae:**
  nth term of an AP $\quad T_n = a + (n - 1)d$

  Sum of the n terms of an AP is: $S_n = \frac{n}{2}[2a + (n - 1)d]$

  Also: $S_n = \frac{n}{2}(a + l)$

  where, l = last term, i.e., $l = a + (n - 1)d$.

1. n^{th} term from the end of an AP: $(l - (n-1)d)$
2. $t_n = (S_n - S_{n-1})$

- **Tips:**
  1. To an AP if we (i) add (ii) subtract (iii) multiply or (iv) divide each term by the same number, the resulting sequence would always be an AP.
  2. Whenever you be asked to take three numbers which are in AP, always take: $a - d, a, a + d$.
  3. Whenever you be asked to take four numbers which are in AP, always take: $(a - 3d), (a - d), (a + d), (a + 3d)$
  4. Whenever you be asked to take five numbers which are in AP, always take: $(a - 2d), (a - d), (a), (a + d), (a + 2d)$

- **Proof of "Sum of n terms of an AP"**

We know that general form of an AP is given by:

$a, a + d, a + 2d, a + 3d, a + 4d, \ldots\ldots\ldots, a + (n-1)d.$

$\Rightarrow S_n = a + (a + d) + (a + 2d) + (a + 3d) + \ldots\ldots\ldots + [a + (n-2)d] + [a + (n-1)d]$ (i)

Now write the above equation in reverse order:

$\Rightarrow S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots\ldots\ldots + (a + 3d) + (a + 2d) + (a + d) + a$ (ii)

Adding the corresponding terms of eq (i) & (ii), we get

$\Rightarrow 2S_n = [2a + (n-1)d] + [2a + (n-1)d] + [2a + (n-1)d] \ldots\ldots\ldots [2a + (n-1)d]$

$\Rightarrow 2S_n = n[2a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$

Also, it can be written as: $S_n = \frac{n}{2}[a + a + (n-1)d]$

$\Rightarrow S_n = \frac{n}{2}[a + l]$ [where, last term, $l = a + (n-1)d$] **Hence Proved.**

- **Some important key points:**

1. We know, $a_n = a + (n-1)d$

$a_n = a + nd - d$

$a_n = (a - d) + nd$

i.e., Linear equation denotes general term where,

(i) co-efficient of n is common difference 'd'

(ii) constant term is $(a - d)$.

2. We know, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_n = na + \frac{n}{2}(n-1)d$

$S_n = na + n^2\frac{d}{2} - n\frac{d}{2}$

$S_n = n\left(a - \frac{d}{2}\right) + n^2\frac{d}{2}$

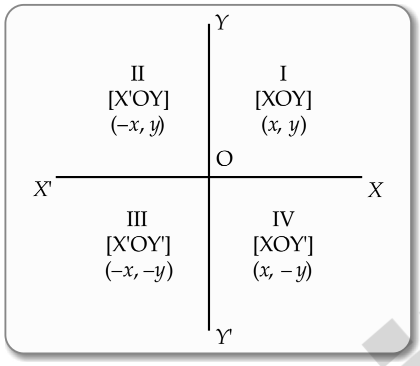
i.e., Quadratic equation denotes sum to n terms where,

(i) Co-efficient of n^2 is $\frac{d}{2}$

(ii) Co-efficient of n is $\left[a - \frac{d}{2}\right]$

CHAPTER 6: Coordinate Geometry

- **Fundamentals:**
  (i) Distance of any point from the y-axis is called x co-ordinate or **abscissa**.
  (ii) Distance of any point from the x-axis is called y co-ordinate or **ordinate**.
  (iii) Origin: (0, 0)
  (iv) Point on x-axis: $(x, 0)$.
  (v) Point on y-axis: $(0, y)$
  (vi) There are four quadrants in a co-ordinate plane:



- **Distance Formula:**
  Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

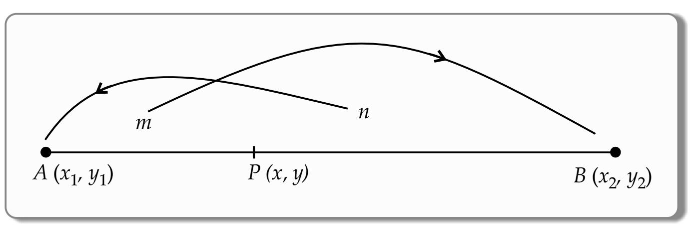
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Corollary: Distance of point $A(x, y)$ from origin is $\sqrt{x^2 + y^2}$

- **Tips:** Co-ordinates will form:
  1. Rhombus, if all the four sides are equal.
  2. Square, if all the four sides and diagonals are equal.
  3. Parallelogram, if opposite sides are equal.
  4. Rectangle, if opposite sides and diagonals are equal.
  5. Right triangle, if it follows Pythagoras theorem.
  6. Collinearity condition. [A, B, C are collinear if $AB + BC = AC$]
- **Section Formula:**
  Co-ordinates of the point $P(x, y)$, dividing the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by:

$$x = \frac{mx_2 + nx_1}{m+n}, \; y = \frac{my_2 + ny_1}{m+n}$$

- **How to remember the section formula?**



- **Corollary:** If $P(x, y)$ is the mid-point, therefore $m : n = 1 : 1$

$$x = \frac{x_2 + x_1}{2}, \; y = \frac{y_2 + y_1}{2}$$

- **Tips:** If the ratio in which P divides AB is not given then we take assumed ratio as $k : 1$.

CHAPTER 7: Triangles

- **Fundamentals:**

  Similar figures: Two figures of same shape are said to be similar. if:
  1. Their corresponding angles are equal.
  2. Their corresponding sides are proportional.

  Examples:
  1. All circles
  2. All squares
  3. All equilateral triangles
  4. All congruent triangles.

- **Criterion for similarity of two triangles**

  (i) SSS Similarity: If the corresponding sides of two triangles are proportional, then triangles are similar.
  (ii) AAA Similarity: If the corresponding angles of two triangles are equal, then triangles are similar.
  (iii) SAS Similarity: If the pair of corresponding sides of two triangles are proportional and the included angle is equal, then triangles are similar.

- **Statements of theorems**

  1. Basic proportionality theorem: If a line is drawn parallel to one side of a triangle to intersect the other two sides at distinct points, the other two sides are divided in the same ratio.

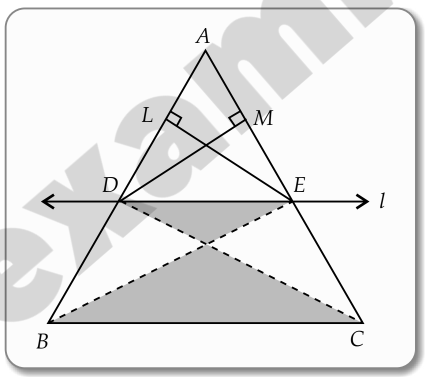
  2. Converse of basic proportionality theorem: If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

Proof of Theorems:

1. Basic proportionality theorem (Thales theorem):

Statement: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: A ΔABC and line 'l' parallel to BC intersect AB at D and AC at E.



To Prove: $$\frac{AD}{DB} = \frac{AE}{EC}$$

Construction: Join BE and CD. Draw $EL \perp AB$ and $DM \perp AC$.

Proof: We know that areas of the triangles on the same base and between same parallel lines are equal, hence we have

$$\text{area } (\Delta BDE) = \text{area } (\Delta CDE) \quad \text{(i)}$$

Now, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}\times AD \times EL}{\frac{1}{2}\times DB \times EL} = \frac{AD}{DB} \quad \text{(ii)}$$

Put value from (i) in (ii), we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{AD}{DB} \quad \text{(iii)}$$

Again, we have

$$\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta CDE} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad \text{(iv)}$$

On comparing equations (iii) and (iv), we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence Proved.

Corollary:

(i) $\frac{AB}{DB} = \frac{AC}{EC}$ (ii) $\frac{DB}{AD} = \frac{EC}{AE}$

(iii) $\frac{AB}{AD} = \frac{AC}{AE}$ (iv) $\frac{DB}{AB} = \frac{EC}{AC}$

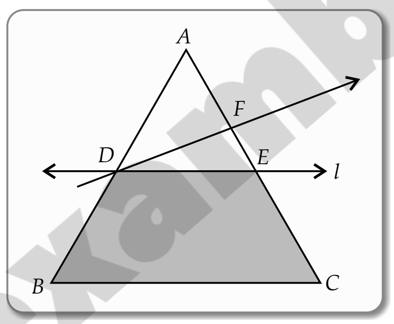
(v) $\frac{AD}{AB} = \frac{AE}{AC}$

2. Converse of Basic Proportionality:

Statement: If a line divides any two sides of a triangle in the same ratio, the line must be parallel to the third side.

Given: A ΔABC and line '*l*' intersecting the sides *AB* at *D* and *AC* at *E* such that:

$$\frac{AD}{DB} = \frac{AE}{EC}$$



To Prove: $l \parallel BC$.

Proof: Let us suppose that the line l is not parallel to *BC*.

Then through *D*, there must be any other line which must be parallel to *BC*.

Let $DF \parallel BC$, such that $E \neq F$.

Since, $DF \parallel BC$ (by supposition)

$$\frac{AD}{DB} = \frac{AF}{FC} \quad \text{(Basic proportionality theorem) (i)}$$

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{(Given) (ii)}$$

Comparing (i) and (ii), we get

$$\frac{AF}{FC} = \frac{AE}{EC}$$

Adding 1 to both sides, we get

$$\frac{AF}{FC} + 1 = \frac{AE}{EC} + 1$$

$$\Rightarrow \quad \frac{AF + FC}{FC} = \frac{AE + EC}{EC}$$

$$\Rightarrow \quad \frac{AC}{FC} = \frac{AC}{EC}$$

$\Rightarrow \qquad \dfrac{1}{FC} = \dfrac{1}{EC}$

$\Rightarrow \qquad FC = EC$

This shows that E and F must coincide, but it contradicts our supposition that $E \neq F$ and $DF \parallel BC$. Hence, there is one and only line, $DE \parallel BC$, i.e.,

$$\boxed{l \parallel BC}$$

Hence Proved.

CHAPTER 8: Circles

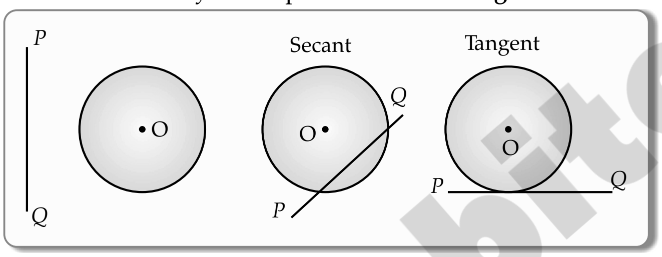
- **Fundamentals:**

Consider a circle $C(O, r)$ and a line PQ. There can be three possibilities given below:

(i) Non intersecting line w.r.t. circle

(ii) A line intersects circle in two distinct points, this line is called a **Secant**.

(iii) A line which intersects circle exactly at one point is called a **Tangent**.



From a point P inside a circle, the number of tangents drawn to the circle = 0.

From a point P on a circle, the number of tangents drawn to the circle = 1.

From a point P outside the circle, the number of tangents drawn to the circle = 2.

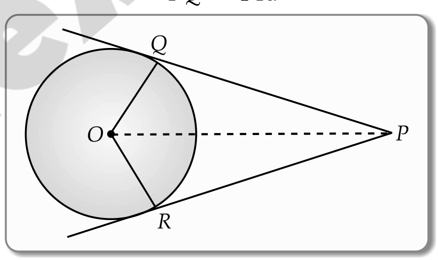
The distance between two parallel tangents drawn is equal to the diameter of the circle.

Theorem 1: The tangent at any point of a circle is perpendicular to the radius through the point of contact.

Theorem 2: The lengths of two tangents from an external point to a circle are equal.

Given: A circle $C(O, r)$ and two tangents say PQ and PR from an external point P.

To prove: $PQ = PR$.



Construction: Join OQ, OR and OP.

Proof: In ΔOQP and ΔORP

$OQ = OR$ (radii of the same circle)

$OP = OP$ (Common)

$\angle Q = \angle R$ = each 90° (The tangent at any point of a circle is perpendicular to the radius through the point of contact)

Hence, $\Delta OQP \cong \Delta ORP$ (By RHS criterion)

$\therefore \quad PQ = PR$ (By c.p.c.t.)

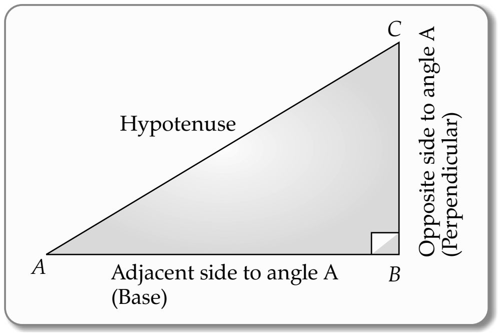
Hence Proved.

Theorem 3: Tangents are equally inclined on the line segment joining external point and centre.

Theorem 4: Tangent subtend equal angle at the centre.

CHAPTER 9: Introduction to Trigonometry and Trigonometric Identities

- Trigonometry is the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles.
- **Trigonometric Ratios:** The values of the ratios of the sides of any right triangle with respect to any angle (other than 90°) are called trigonometric ratios of that angle. For example: In right $\angle ABC$, the ratios of the sides of the triangle with respect to $\angle A$ are called trigonometric ratios of $\angle A$.



There are six different trigonometric ratios as follows:

1. Sine A $= \dfrac{\text{Opposite side to angle } A}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

2. Cosine A $= \dfrac{\text{Adjacent side to angle } A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{\text{Base}}{\text{Hypotenuse}}$

3. Tangent A $= \dfrac{\text{Opposite side to angle } A}{\text{Adjacent side to angle } A} = \dfrac{BC}{AB} = \dfrac{\text{Perpendicular}}{\text{Base}}$

4. Cosecant A $= \dfrac{\text{Hypotenuse}}{\text{Opposite side to angle } A} = \dfrac{AC}{BC} = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

5. Secant A $= \dfrac{\text{Hypotenuse}}{\text{Adjacent side to angle } A} = \dfrac{AC}{AB} = \dfrac{\text{Hypotenuse}}{\text{Base}}$

6. Cotangent A $= \dfrac{\text{Adjacent side to angle } A}{\text{Opposite side to angle } A} = \dfrac{AB}{BC} = \dfrac{\text{Base}}{\text{Perpendicular}}$

Tips:

1. sin A is written for sine A.
2. cos A is written for cosine A.
3. tan A is written for tangent A.
4. cosec A is written for cosecant A.
5. sec A is written for secant A.
6. cot A is written for cotangent A.

Relation between Trigonometric ratios:

$\sin\theta = \dfrac{1}{\operatorname{cosec}\theta}$ OR $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ OR $\sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ OR $\cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ OR $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

- **Trigonometric ratios of some specific angles:**

In this part, we will put values of angles as 0°, 30°, 45°, 60° and 90°, hence we will find ratios.

θ	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	1/2	0
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞
$\cot\theta$	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0
$\sec\theta$	1	$2/\sqrt{3}$	$\sqrt{2}$	2	∞
$\text{cosec}\,\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

- **How to remember trigonometric ratios of some specific angles?**

1. First of all learn only sin row. If you can't learn then follow the step to find $\sin\theta$ row:

θ	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	$\frac{\sqrt{0}}{\sqrt{4}}$	$\frac{\sqrt{1}}{\sqrt{4}}$	$\frac{\sqrt{2}}{\sqrt{4}}$	$\frac{\sqrt{3}}{\sqrt{4}}$	$\frac{\sqrt{4}}{\sqrt{4}}$
	0	1/2	$1/\sqrt{2}$	$\sqrt{3}/2$	1

2. For $\cos\theta$ row, write all the values of $\sin\theta$ row in reverse order, i.e., from right to left.

- **Fundamental Trigonometric Identities:**

There are three fundamental identities which can be written in six different ways.

1. $\sin^2\theta + \cos^2\theta = 1$

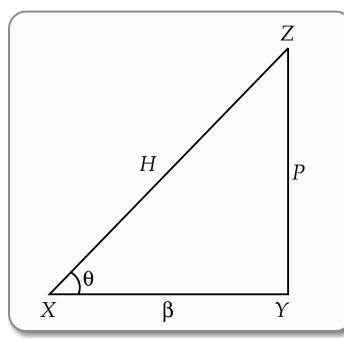
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

2. $\sec^2\theta - \tan^2\theta = 1$

$$\tan^2\theta = \sec^2\theta - 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$

3. $\text{cosec}^2\theta - \cot^2\theta = 1$

$$\cot^2\theta = \text{cosec}^2\theta - 1$$
$$\text{cosec}^2\theta = 1 + \cot^2\theta$$



Proof of first identity: $\sin^2\theta + \cos^2\theta = 1$

Let P denotes Perpendicular, B denotes Base, and H denotes Hypotenuse.

In ΔXYZ, ∠Y=90°, $\sin\theta = \frac{P}{H}$ and $\cos\theta = \frac{B}{H}$

$$\text{LHS} = \sin^2\theta + \cos^2\theta \quad \text{(putting values of } \sin\theta \text{ and } \cos\theta\text{)}$$

$$= \frac{P^2}{H^2} + \frac{B^2}{H^2}$$

$$\Rightarrow \qquad = \frac{P^2 + B^2}{H^2}$$

$$= \frac{H^2}{H^2}$$

Therefore, $= 1 =$ R.H.S. **Hence Proved.**

Similarly, other two identities can be proved.

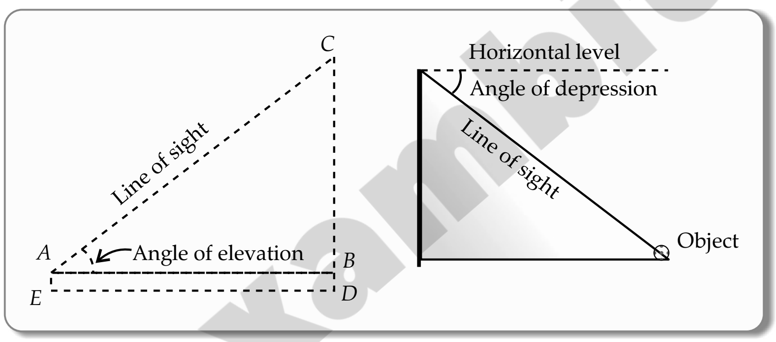
CHAPTER 10: Heights and Distances

- **Angle of Elevation:**

  The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

  The **angle of elevation** of the point viewed is the angle formed by the line of sight with the horizontal.

  When the point being viewed is above the horizontal level, *i.e.*, the case when we raise our head to look at the object is also shown in diagram that we have to assume a horizontal level at our eyes.



- **Angle of Depression:**

  The **angle of depression** of a point on the object being viewed is the angle formed by the line of sight with the horizontal.

  When the point is below the horizontal level, i.e.,the case when we lower our head to look at the point being viewed.

CHAPTER 11: Areas Related to Circles

- **Fundamentals:**

1. Circle is defined as the set of all those points which are at a constant distance from a fixed point. The fixed point is called **centre**.
2. The constant distance is called **radius**.
3. The longest chord passing through centre and whose end point lies on circle is called **diameter**.
4. Circles with same center are called **concentric circles**.
5. Perimeter of circle is called **circumference.**
6. π is defined as the ratio of circumference and diameter of circle.

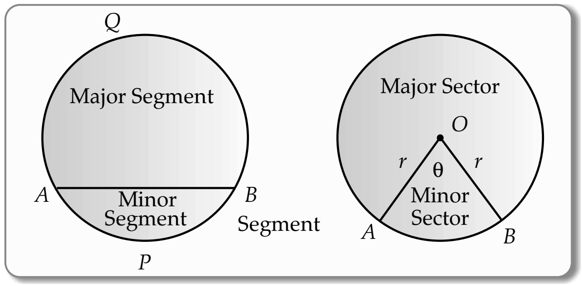
   i.e., $$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

   $\therefore$ $$\text{Circumference} = \pi \times \text{diameter}$$

$\Rightarrow$ Circumference $= 2\pi r$ [where, r is the radius of circle]

and Area of the Circle $= \pi r^2$

7. Arc, Chord, Segment, Sector of a Circle



(i) Arc: Any portion of circumference. i.e., APB is minor arc while AQB is major arc.

(ii) Chord: The line joining any two points on the circle. i.e., AB.

(iii) Segment: In figure, chord AB divides the circle in two segments i.e., $APBA$ (minor segment) and $AQBA$ (major segment).

(iv) Sector: The region bounded by the two radii AO and BO and arc AB is called sector of the circle.

8. Length of Arc: When sector angle $\angle AOB = \theta$. (where, θ is called central angle.)

We know that length of arc when sector angle ($\angle AOB = 360°$) is $2\pi r$

Length of arc when sector angle ($\angle AOB$=1°) is $= \dfrac{2\pi r}{360°}$

Length of arc when sector angle ($\angle AOB = \theta$) $= \dfrac{2\pi r \times \theta}{360°}$

$$\text{Length of arc } AB = 2\pi r \times \frac{\theta}{360°}$$

9. Area of Sector: When sector angle $\angle AOB = \theta$

We know that area of circle when sector angle ($\angle AOB = 360°$) is πr^2

Area of arc when sector angle ($\angle AOB = 1°$) is $\dfrac{\pi r^2}{360°}$

Area of arc when sector angle ($\angle AOB = \theta$) is $\dfrac{\pi r^2}{360°} \times \theta$

$\therefore$
$$\text{Area of Sector} = \pi r^2 \times \frac{\theta}{360°}$$

$$\text{Area of Sector} = \frac{1}{2} \times l \times r$$

l = length of arc

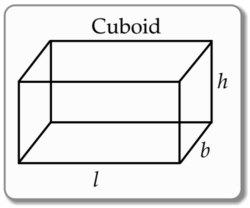
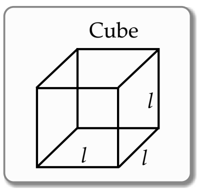
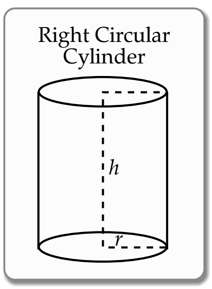
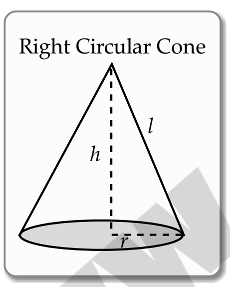
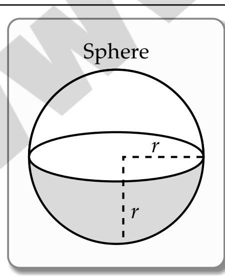
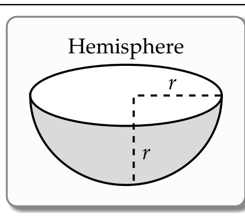
r = radius

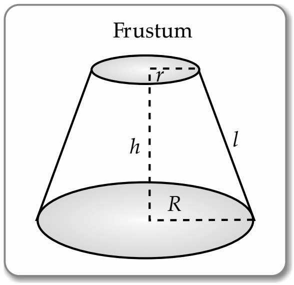
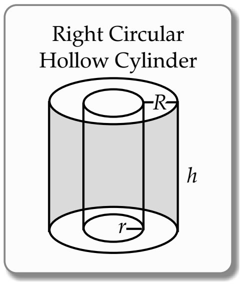
10. Perimeter of Segment (shaded) of a Circle: $AB + \text{arc}(APB)$

$$\text{Perimeter of segment} = \frac{2\pi r\theta}{360°} + 2r \sin \frac{\pi\theta}{360°}$$

CHAPTER 12: Surface Areas and Volumes

Fundamentals:

S. No.	Shape	CSA	TSA	Volume	Nomenclature
1.		$2(bh + hl)$	$2(lb+bh+hl)$	lbh	l = length b = breadth h = height
2.		$4l^2$	$6l^2$	l^3	l = length or side
3.		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = radius of base h = height
4.		πrl	$\pi r(r + l)$	$\frac{1}{3}(\pi r^2 h)$	r = radius of base h = height l = slant height $l = \sqrt{r^2 + h^2}$
5.		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}(\pi r^3)$	r = radius
6.		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}(\pi r^3)$	r = radius

7.		$\pi l(r + R)$	$\pi l(r + R)$ $+$ $\pi(r^2 + R^2)$	$\frac{1}{3}\pi h(r^2 + R^2 + rR)$	r = radius of smaller base R = radius of larger base h = height l = slant height $l = \sqrt{h^2 + (R - r)^2}$
8.		$2\pi h(r + R)$	$2\pi(r + R)$ $(h + R - r)$	$\pi h(R^2 - r^2)$	r = inner radius R = outer radius h = height

- **Tips:**

$$\text{Area} \times \text{Rate} = \text{Cost}$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$1\text{ m}^3 = 1000\text{ L}$$

$$1\text{ m}^3 = 1\text{ kL}$$

$$1\text{ L} = 1000\text{ cm}^3$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$1\text{ km} = 1000\text{ m} = 10^5\text{ cm}$$

$$1\text{ km}^2 = 10^6\text{ m}^2$$

$$1\text{ m} = 100\text{ cm}$$

$$1\text{ m}^2 = 10000\text{ cm}^2$$

$$1\text{ km/hr} = \frac{5}{18}\text{ m/s}$$

$$1\text{ km/hr} = \frac{50}{3}\text{ m/min}$$

$$\text{Shape of river} = \text{Cuboid}$$

$$1\text{ acre} \approx 4047\text{ m}^2$$

$$1\text{ hectare} = 10000\text{ m}^2$$

CHAPTER 13: Statistics

- **Fundamentals:**
  1. The word **statistics** is used in both singular as well as plural.
  2. In singular, it means "science of collection, presentation, analysis and interpretation of numerical data".

3. In plural, it means "numerical facts collected with definite purpose".
4. The number of times an observation occurs in the given data is called the frequency.
5. Frequency distribution is of two types:

   (i) Discrete frequency distribution.

   (ii) Continuous or grouped frequency distribution.
6. Class mark = (Lower limit + Upper Limit)/2.
7. The commonly used measures of central tendency are as follows:

   Arithmetic mean (MEAN), Geometric mean, Harmonic mean, Median and Mode.

(i) Relation between mean, median and mode:

$$3\,\text{Median} = \text{Mode} + 2\,\text{Mean}$$

(ii) Mean of Grouped Data : If x_1 , x_2 , x_3 , ……….. x_n, are observations with respective frequencies f_1 , f_2, f_3 ,……………. f_n, it means observation x_1 occurs f_1 times, observation x_2 occurs f_2 times and so on. Mean is denoted by $\bar{x}$.

There are three different ways to find the mean of a grouped data which are:

(a) Direct method.

(b) Assumed Mean method.

(c) Shortcut method (Step-deviation method).

Direct Method:

$$\text{Mean } \bar{x} = \frac{\text{Sum of all the observations}}{\text{No. of observations}}$$

$$\bar{x} = \frac{f_1x_1 + f_2x_2 + \dots\dots + f_nx_n}{f_1 + f_2 + \dots\dots + f_n}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

Assumed Mean Method:

$$\text{Mean } (\bar{x}) = a + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

where, a is any arbitrary value, chosen as assumed mean (somewhere in the middle of x_i), and $d_i = x_i - a$

(iii) Median of Grouped Data:

Condition I: When the data is discrete.

Step 1: Arrange data in ascending order.

Step 2: If the total frequency n is odd:

Then, $\left(\dfrac{n+1}{2}\right)^{th}$, observation is the median.

Step 3: If the total frequency n is even:

Then, mean of $\dfrac{n^{\text{th}}}{2}$ and $\left(\dfrac{n}{2}+1\right)^{\text{th}}$, observations is the median.

Condition II: When the data is continuous and in the form of frequency distribution :

Then,

$$\text{Median} = l + \left[\frac{\frac{n}{2} - c}{f}\right] \times h$$

Median class = The class whose cumulative frequency is greater than (nearest to) $\frac{n}{2}$.

where,

l = lower limit of median class

f = frequency of median class

h = class-size

n = number of observations

c = cumulative frequency of class preceding the median class.

(iv) Mode of Grouped Data: The class with maximum frequency is called the modal class.

$$\text{Mode} = l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$$

where,

l = lower limit of the modal class

h = class-size

f_1 = frequency of the modal class

f_0 = frequency of the class preceding the modal class

f_2 = frequency of the class succeeding the modal class

CHAPTER 14: Probability

- **Fundamentals:**
  1. **Experiment:** An operation which can produce some well defined outcomes.
  2. **Sample Space:** It is the total number of possible outcomes of a random experiment.
  3. **Event:** Any subset of sample space is called event.
  4. **Elementary Event:** Each outcome of any random experiment.
  5. **Sure Event (Certain event):** An event which always occurs whenever the random experiment is performed.
  6. **Impossible Event:** An event which never occurs whenever the random experiment is performed.
  7. **Favourable Event:** The cases which ensure the occurrence of an event.
  8. **Probability:** Probability $P(E)$ of an event E is defined as:

     $$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total Number of outcomes}}$$

     or $$P(E) = \frac{\text{Favourable Event}}{\text{Sample Space}}$$

  9. **Complement Events:** An event associated with a random experiment denoted by $P(\text{not } E)$ which happens only when E does not happen is called the complement of event E.

     $P(\bar{E})$ or $P(\text{not } E) = 1 - P(E)$

- **Tips:**
  1. Sum of the probabilities of all the elementary events of an experiment is 1.

     $P(E_1) + P(E_2) + P(E_3) + ... + P(E_n) = 1,$

2. Probability of sure event is 1.
3. Probability of an impossible event is 0.
4. Probability of any event lies between 0 and 1 (including 0 and 1) i.e.,

$$0 \le P(E) \le 1.$$

5. 52 cards are divided into 4 suits of 13 cards is each. The suits are:

SPADE ♠	**HEARTS** ♥
DIAMONDS ♦	**CLUBS** ♣

6. Out of 52 cards 26 are red in colour and 26 are black.
7. In each suit there is an Ace, a King, a Queen, a Jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.
8. King, Queen and Jack are called face cards.

■■■

Sample Question Paper-1

(Issued by Board dated 16th Sep. 2022)

Mathematics Standard (041)

Class- X

Session- 2022-23

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. Let a and b be two positive integers such that $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers. If HCF $(a, b) = p^mq^n$ and LCM $(a, b) = p^rq^s$, then $(m + n)(r + s) =$

(A) 15 **(B)** 30
(C) 35 **(D)** 72 **1**

2. Let p be a prime number. The quadratic equation having its roots as factors of p is

(A) $x^2 - px + p = 0$ **(B)** $x^2 - (p + 1)x + p = 0$
(C) $x^2 + (p + 1)x + p = 0$ **(D)** $x^2 - px + p + 1 = 0$ **1**

3. If α and β are the zeros of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is

(A) $-\frac{2}{3}$ **(B)** $\frac{2}{3}$
(C) $\frac{1}{3}$ **(D)** $-\frac{1}{3}$ **1**

4. If the system of equations $3x + y = 1$ and $(2k - 1)x + (k - 1)y = 2k + 1$ is inconsistent, then $k =$

(A) –1 **(B)** 0
(C) 1 **(D)** 2 **1**

5. If the vertices of a parallelogram PQRS taken in order are P(3,4), Q(-2,3) and R(-3,-2), then the coordinates of its fourth vertex S are

(A) (–2, –1) **(B)** (– 2, –3)
(C) (2, –1) **(D)** (1, 2) **1**

6. $\Delta ABC \sim \Delta PQR$. If AM and PN are altitudes of ΔABC and ΔPQR respectively and $AB^2 : PQ^2 = 4 : 9$, then AM : PN =

(A) 3 : 2 **(B)** 16 : 81

(C) 4 : 9 **(D)** 2 : 3 **1**

7. If $x \tan 60° \cos 60° = \sin 60° \cot 60°$, then $x =$

(A) cos 30° **(B)** tan 30°

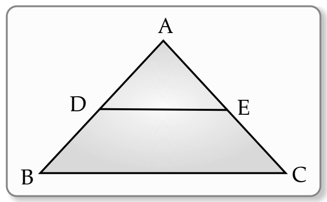
(C) sin 30° **(D)** cot 30° **1**

8. If $\sin\theta + \cos\theta = \sqrt{2}$, then $\tan\theta + \cot\theta =$

(A) 1 **(B)** 2

(C) 3 **(D)** 4 **1**

9. In the given figure, DE || BC, $AE = a$ units, $EC = b$ units, $DE = x$ units and $BC = y$ units. Which of the following is true?



(A) $x = \dfrac{a+b}{ay}$ **(B)** $y = \dfrac{ax}{a+b}$

(C) $x = \dfrac{ay}{a+b}$ **(D)** $\dfrac{x}{y} = \dfrac{a}{b}$ **1**

10. ABCD is a trapezium with AD || BC and $AD = 4$ cm. If the diagonals AC and BD intersect each other at O such that $\dfrac{AO}{OC} = \dfrac{DO}{OB} = \dfrac{1}{2}$, then BC =

(A) 6 cm **(B)** 7 cm

(C) 8 cm **(D)** 9 cm **1**

11. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then the length of each tangent is equal to

(A) $\dfrac{3\sqrt{3}}{2}$ cm **(B)** 3 cm

(C) 6 cm **(D)** $3\sqrt{3}$ cm **1**

12. The area of the circle that can be inscribed in a square of 6 cm is

(A) 36π cm^2 **(B)** 18π cm^2

(C) 12π cm^2 **(D)** 9π cm^2 **1**

13. The sum of the length, breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm. The total surface area of the cuboid is

(A) 48 cm^2 **(B)** 72 cm^2

(C) 96 cm^2 **(D)** 108 cm^2 **1**

14. If the difference of Mode and Median of a data is 24, then the difference of median and mean is

(A) 8 **(B)** 12

(C) 24 **(D)** 36 **1**

15. The number of revolutions made by a circular wheel of radius 0.25 m in rolling a distance of 11 km is

(A) 2800 **(B)** 4000

(C) 5500 **(D)** 7000 **1**

16. For the following distribution,

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of the lower limits of the median and modal class is

(A) 15 **(B)** 25

(C) 30 **(D)** 35 **1**

17. Two dice are rolled simultaneously. What is the probability that 6 will come up at least once?

(A) $\frac{1}{6}$ **(B)** $\frac{7}{36}$

(C) $\frac{11}{36}$ **(D)** $\frac{13}{36}$ **1**

18. If $5 \tan \beta = 4$, then $\frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} =$

(A) $\frac{1}{3}$ **(B)** $\frac{2}{5}$

(C) $\frac{3}{5}$ **(D)** 6 **1**

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

19. **Statement A (Assertion):** If product of two numbers is 5780 and their HCF is 17, then their LCM is 340.

Statement R(Reason): HCF is always a factor of LCM.

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true. **1**

20. **Statement A (Assertion):** If the co-ordinates of the mid-points of the sides AB and AC of ΔABC are D(3, 5) and E(–3, –3) respectively, then BC = 20 units.

Statement R(Reason): The line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

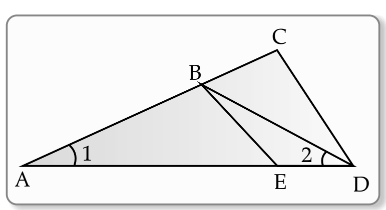
(D) Assertion (A) is false but reason (R) is true. **1**

Section - B

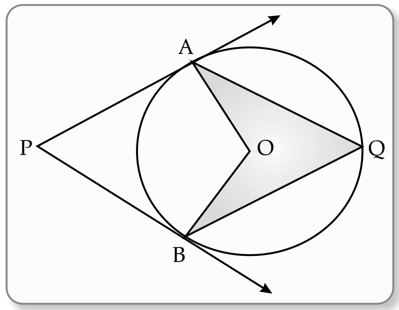
Section B consists of 5 questions of 2 marks each.

21. If $49x + 51y = 499$, $51x + 49y = 501$, then find the value of x and y. **2**

22. In the given figure below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$. Show that ΔBAE ~ ΔCAD . **2**



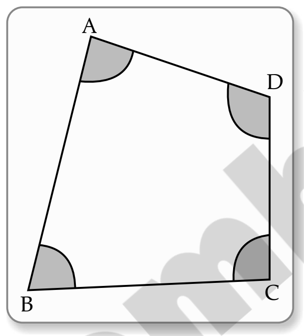
23. In the given figure, O is the centre of circle. Find $\angle AQB$, given that PA and PB are tangents to the circle and $\angle APB = 75°$. **2**



24. The length of the minute hand of a clock is 6 cm. Find the area swept by it when it moves from 7:05 p.m. to 7:40 p.m.

OR

In the given figure, arcs have been drawn of radius 7 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region. **2**



25. If $\sin(A + B) = 1$ and $\cos(A - B) = \frac{\sqrt{3}}{2}$, $0° < A + B \le 90°$ and A > B, then find the measures of angles A and B.

OR

Find an acute angle θ when $\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$

Section - C

Section C consists of 6 questions of 3 marks each.

26. Given that $\sqrt{3}$ is irrational,prove that $5 + 2\sqrt{3}$ is irrational. **3**

27. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of the polynomial $2x^2 - 5x - 3$, then find the values of p and q. **3**

28. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr ; it would have taken 6 hours more than the scheduled time. Find the length of the journey.

OR

Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 chocolates and the second lot at the rate of ₹ 1 per chocolate, and got a total of ₹ 400. If he had sold the first lot at the rate of ₹1 per chocolate, and the second lot at the rate of ₹ 4 for 5 chocolates, his total collection would have been ₹ 460. Find the total number of chocolates he had. **3**

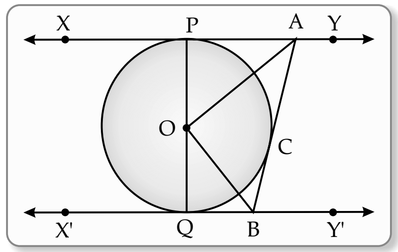
29. Prove the following that:

$$\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta} = \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$$ **3**

30. Prove that a parallelogram circumscribing a circle is a rhombus. **3**

OR

In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, what is the measure of $\angle AOB$.



31. Two coins are tossed simultaneously. What is the probability of getting

(i) At least one head?

(ii) At most one tail?

(iii) A head and a tail? **3**

Section - D

Section D consists of 4 questions of 5 marks each.

32. To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool?

OR

In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr from its usual speed and the time of the flight increased by 30 min. Find the scheduled duration of the flight. **5**

33. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.

Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non parallel sides in the same ratio. **5**

34. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and the upper part is conical with the same base radius, but of height 2.1 m. If the canvas used to make the tents costs ₹ 120 per m^2, find the amount shared by each school to set up the tents.

OR

There are two identical solid cubical boxes of side 7 cm. From the top face of the first cube a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find

(i) the ratio of the total surface area of the two new solids formed

(ii) volume of each new solid formed. **5**

35. The median of the following data is 525. Find the values of x and y, if the total frequency is 100

Class interval	Frequency
0−100	2
100−200	5
200−300	x
300−400	12
400−500	17
500−600	20
600−700	y

700−800	9
800−900	7
900−1000	4

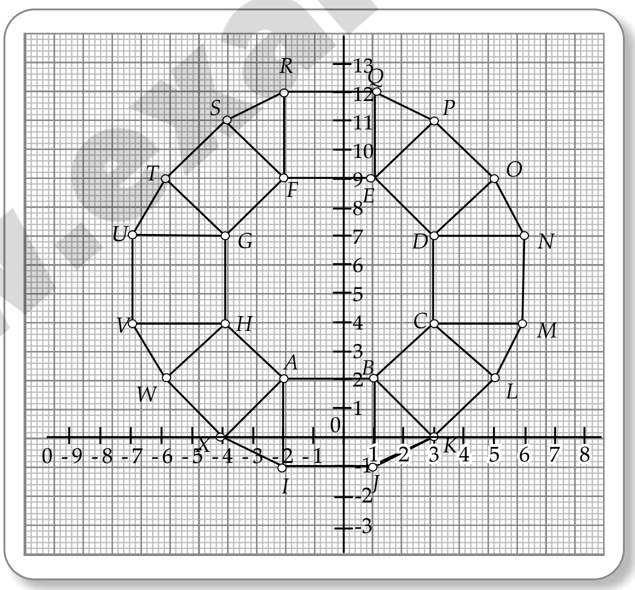
5

Section - E

Case study based questions are compulsory.

36. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern



Use the above figure to answer the questions that follow:

(i) What is the length of the line segment joining points B and F? **1**

(ii) The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z? **1**

(iii) What are the coordinates of the point on y-axis equidistant from A and G? **2**

OR

What is the area of Trapezium AFGH?

37. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

(i) If the first circular row has 30 seats, how many seats will be there in the 10^{th} row? **1**

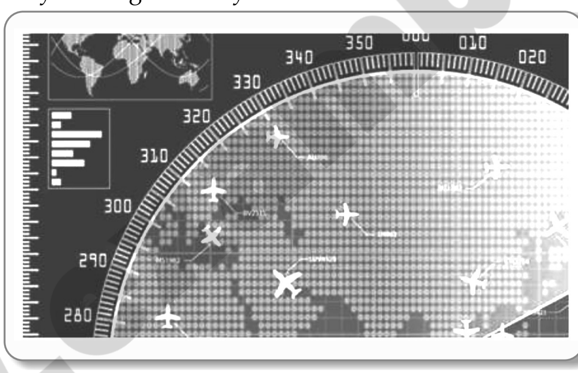
(ii) For 1500 seats in the auditorium, how many rows need to be there?

OR

If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10^{th} row? **2**

(iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row? **1**

38. We all have seen the airplanes flying in the sky but might have not thought of how they actually reach the correct destination. Air Traffic Control (ATC) is a service provided by ground-based air traffic controllers who direct aircraft on the ground and through a given section of controlled airspace, and can provide advisory services to aircraft in non-controlled airspace. Actually, all this air traffic is managed and regulated by using various concepts based on coordinate geometry and trigonometry.



At a given instance, ATC finds that the angle of elevation of an airplane from a point on the ground is 60°. After a flight of 30 seconds, it is observed that the angle of elevation changes to 30°. The height of the plane remains constantly as $3000\sqrt{3}$ m. Use the above information to answer the questions that follow-

(i) Draw a neat labelled figure to show the above situation diagrammatically.

(ii) What is the distance travelled by the plane in 30 seconds? **1**

OR

Keeping the height constant, during the above flight, it was observed that after $15(\sqrt{3}-1)$ seconds, the angle of elevation changed to 45°. How much is the distance travelled in that duration. **2**

(iii) What is the speed of the plane in km/hr. **1**

■■■

SOLUTIONS

Sample Question Paper-1

With CBSE Marking Scheme 2022-23
Mathematics Standard (041)

Section - A

1. **Option (C) is correct.**

Explanation: $a = p^3q^4$ and $b = p^2q^3$

$\text{HCF}(a, b) = p^2q^3$ ...(i)

and $\text{LCM}(a, b) = p^3q^4$ ...(ii)

But gives: $\text{HCF}(a, b) = p^mq^n$ and $\text{LCM}(a, b) = p^rq^s$

From eq. (i), $p^mq^n = p^2q^3$

So, $m = 2$ and $n = 3$

From eq. (iii), $p^rq^s = p^3q^4$

So, $r = 3$ and $s = 4$

$\therefore\ (m + n)(r + s) = (2 + 3)(3 + 4) = 35.$

2. **Option (B) is correct.**

Explanation: Factors of $p = p \times 1$

$\therefore$ Roots are p and 1.

The quadratic equation is:

$x^2 -$ (sum of roots) x + product of roots $= 0$

$\Rightarrow\ x^2 - (p + 1)x + p = 0$

3. **Option (B) is correct.**

Explanation:

Given, $f(x) = px^2 - 2x + 3p$

Since α and β are the zeroes of given polynomial.

$$\therefore\ \alpha + \beta = -\frac{(-2)}{p} = \frac{2}{p}$$

$$\text{and}\ \alpha\beta = \frac{3p}{p} = 3$$

$$\because\ \alpha + \beta = \alpha\beta \quad \text{(given)}$$

$$\therefore\ \frac{2}{p} = 3$$

$$\Rightarrow\ p = \frac{2}{3}$$

4. **Option (D) is correct.**

Explanation: $3x + y = 1$ ...(i)

and $(2k - 1)x + (k - 1)y = 2k + 1$ ...(ii)

Comparing eq. (i) with $a_1x + b_1y + c_1 = 0$ and eq. (ii) with $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 3, a_2 = 2k - 1, b_1 = 1, b_2 = k - 1, c_1 = -1$ and $c_2 = -(2k + 1)$

Since, system is inconsistent, then

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow\ \frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{-1}{-(2k+1)}$$

$$\text{Either}\ \frac{3}{2k-1} = \frac{1}{k-1} \text{ or } \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$\Rightarrow\ 3k - 3 = 2k - 1$ or $2k + 1 \neq k - 1$

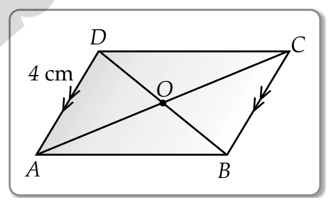
$\Rightarrow\ k = 2$ or $k \neq -2$

Hence, the value of k is 2.

5. **Option (C) is correct.**

Explanation: Given PQRS is a parallelogram and diagonal PR and QS bisect each other at O.

Let fourth vertex be S(x, y), then



mid point of SQ = mid point of PR

$$\left(\frac{x-2}{2}, \frac{y+3}{2}\right) = \left(\frac{3-3}{2}, \frac{4-2}{2}\right)$$

$$\Rightarrow\ \frac{x-2}{2} = \frac{3-3}{2}$$

$$\text{and}\ \frac{y+3}{2} = \frac{4-2}{2}$$

$\Rightarrow\ x - 2 = 0$

$\Rightarrow\ x = 2$

and $y + 3 = 2$

$\Rightarrow\ y = -1$

Hence, fourth vertex S are $(2, -1)$.

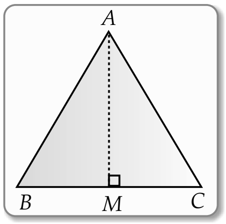
6. **Option (D) is correct.**

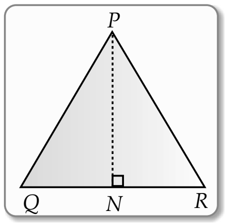
Explanation:

We have, $\Delta ABC \sim \Delta PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AM}{PN}$$

(corresponding sides of similar triangle)





But $\dfrac{AB^2}{PQ^2} = \dfrac{4}{9}$

or $\left(\dfrac{AB}{PQ}\right)^2 = \left(\dfrac{2}{3}\right)^2$

or $\dfrac{AB}{PQ} = \dfrac{2}{3}$

i.e., $\dfrac{AB}{PQ} = \dfrac{AM}{PN} = \dfrac{2}{3}$

Hence, $AM : PN = 2 : 3$.

7. **Option (B) is correct.**

Explanation:

$$x \tan 60° \cos 60° = \sin 60° \cot 60°$$

$\Rightarrow$ $x \times \sqrt{3} \times \dfrac{1}{2} = \dfrac{\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $x\sqrt{3} = 1$

$\Rightarrow$ $x = \dfrac{1}{\sqrt{3}}$

$\Rightarrow$ $x = \tan 30°$ $\left[\because \tan 30° = \dfrac{1}{\sqrt{3}}\right]$

8. **Option (B) is correct.**

Explanation:

$\because$ $\sin\theta + \cos\theta = \sqrt{2}$

Squaring on both sides, we get

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow$ $1 + 2\sin\theta\cos\theta = 2$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $2\sin\theta\cos\theta = 1$

$\Rightarrow$ $\sin\theta\cos\theta = \dfrac{1}{2}$

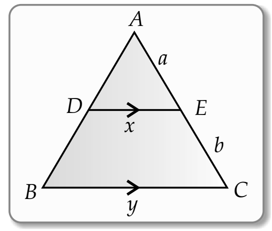
But $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= 2$

9. **Option (C) is correct.**

Explanation:



As $DE \parallel BC$

$\therefore$ $\dfrac{AE}{AC} = \dfrac{DE}{BC}$ [From BPT]

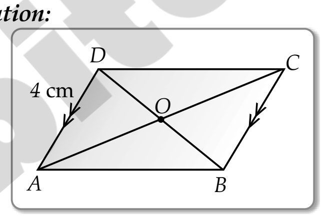
$\Rightarrow$ $\dfrac{a}{a+b} = \dfrac{x}{y}$

$\Rightarrow$ $x(a+b) = ay$

$\Rightarrow$ $x = \dfrac{ay}{a+b}$

10. **Option (C) is correct.**

Explanation:



$\because$ $AD \parallel BC$

and $\dfrac{AO}{OC} = \dfrac{DO}{OB} = \dfrac{1}{2}$

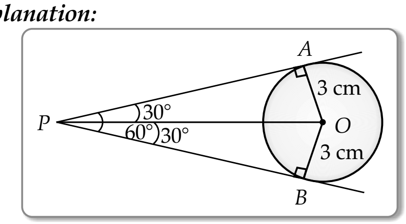
$\therefore$ $\dfrac{AD}{BC} = \dfrac{AO}{OC}$

$\Rightarrow$ $\dfrac{4}{BC} = \dfrac{1}{2}$

$\Rightarrow$ $BC = 8$ cm

11. **Option (D) is correct.**

Explanation:



Angle between two tangents = 60° (given)

$\therefore$ Tangents are equally inclined to each other

$\therefore$ $\angle OPA = \angle OPB = 30°$

and $\angle OAP = 90°$

(Angle between tangent and radius)

In ΔPAO, $\tan 30° = \dfrac{OA}{AP}$

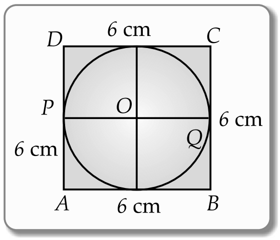
$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = \dfrac{3}{AP}$

$\Rightarrow$ $AP = 3\sqrt{3}$

Hence, the length of each tangent is $3\sqrt{3}$ cm

12. Option (D) is correct.

Explanation:



ABCD is a square of side 6 cm. PQ is a diameter of given circle such that

$$PQ = AB = 6 \text{ cm}$$

$$\therefore \quad \text{Radius } (r) = \frac{\text{Diameter}}{2}$$

$$= \frac{6}{2} = 3 \text{ cm}$$

$$\text{Area of the circle} = \pi r^2$$

$$= \pi(3)^2 = 9\pi \text{ cm}^2.$$

13. Option (C) is correct.

Explanation:

Given: $l + b + h = 6\sqrt{3}$ cm ...(i)

and the length of its diagonal $= 2\sqrt{3}$ cm

i.e., $\sqrt{l^2 + b^2 + h^2} = 2\sqrt{3}$

Squaring both sides, we get

$$l^2 + b^2 + h^2 = 12 \quad \text{...(ii)}$$

From eq. (i),

$$(l + b + h)^2 = (6\sqrt{3})^2$$

$\Rightarrow \quad l^2 + b^2 + h^2 + 2(lb + bh + hl) = 108$

$\Rightarrow \quad 12 + 2(lb + bh + hl) = 108$

[From eq. (iii)]

$\Rightarrow \quad 2(lb + bh + hl) = 96$

Hence, total surface area of the cuboid is 96 cm^2.

14. Option (B) is correct.

Explanation:

$\because$ mode – median = 24 (given)

$\therefore$ mode = 24 + median

Since, mode = 3 median – 2 mean

[By empirical relation]

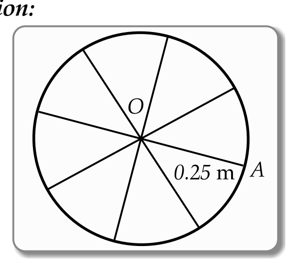
$\therefore$ 24 + median = 3 median – 2 mean

$\Rightarrow$ 2 median – 2 mean = 24

$\Rightarrow$ median – mean = 12

15. Option (D) is correct.

Explanation:



Since, radius of wheel (r) = 0.25 m

Total distance covered by a circular wheel

= 11 km = 11000 m

i.e., No. of revolutions $\times 2\pi r = 11000$

$$\Rightarrow \quad \text{No of revolutions} = \frac{11000 \times 7}{2 \times 22 \times 0.25}$$

$$= 7000$$

16. Option (B) is correct.

Explanation:

Class	Frequency (f)	c.f.
0 – 5	10	10
5 – 10	15	25
10 – 15	12	37
15 – 20	20	57
20 – 25	9	66
	$N = 66$	

Since, $N = 66$, then $\frac{N}{2} = 33$

and cumulative frequency greater than or equal to 33 lies in class 10 – 15

So, median class is 10 – 15

$\therefore$ Lower limit of median class is 10

and highest frequency is 20 lie in class 15 – 20

So, modal class is 15 – 20.

$\therefore$ Lower limit of modal class is 15.

Hence, sum of lower limits of the median and modal class is 10 + 15 = 25.

17. Option (C) is correct.

Explanation: Total possible outcomes, when two dice are thrown together = 6 × 6

i.e., $n(s) = 36$

Favourable outcomes are (1, 6), (2, 6), (3, 6), (4, 6) (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

i.e., P(6 will come up at least once)

$$= \frac{n(E)}{n(S)} = \frac{11}{36}$$

18. Option (A) is correct.

Explanation: $5 \tan \beta = 4$

$$\Rightarrow \quad \tan \beta = \frac{4}{5}$$

$$\therefore \quad \frac{5\sin\beta - 2\cos\beta}{5\sin\beta + 2\cos\beta} = \frac{5\tan\beta - 2}{5\tan\beta + 2}$$

[dividing cos β by Nr. and Dr.]

$$= \frac{5 \times \frac{4}{5} - 2}{5 \times \frac{4}{5} + 2} = \frac{2}{6} = \frac{1}{3}$$

19. Option (B) is correct.

Explanation:

Given: HCF = 17

and LCM = 340

$\therefore$ Product of HCF and LCM = 17 × 340

= 5780.

Here, it is given that the product of two numbers is 5780

So, it is clear that the product of two numbers is equal to the product of HCF and LCM.

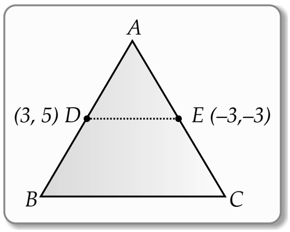
Hence, Assertion is true

For reason, HCF is always a factor of LCM which in true.

But it is not correct explanation of assertion.

20. **Option (A) is correct.**

Explanation:



For assertion: Distance between two points (x_1, x_1) and (x_2, y_2)

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\therefore \text{ Distance between } DE = \sqrt{(-3-3)^2 + (-3-5)^2}$$

$$= \sqrt{36+64} = \sqrt{100}$$

$$= 10 \text{ units}$$

By mid point theorem,

distance between BC = 2 × distance between DE

= 2 × 10

= 20 units.

So, assertion is true

For reason: By mid point theorem, the line joining the mid points of two sides of a triangle is parallel to the third side and equal to half of it.

So, reason is also true.

Section - B

21. Adding the two equations and dividing by 10, we get: $x + y = 10$ ½

Subtracting the two equations and dividing by –2, we get: $x - y = 1$ ½

Solving these two new equations, we get,

$$x = \frac{11}{2} \quad ½$$

$$y = \frac{9}{2} \quad ½$$

22. In ΔABC, $\angle 1 = \angle 2$

$\therefore$ $AB = BD$ …(i) ½

Given, $\frac{AD}{AE} = \frac{AC}{BD}$

Using equation (i), we get

$$\frac{AD}{AE} = \frac{AC}{AB} \quad \text{…(ii) ½}$$

In ΔBAE and ΔCAD, by equation (ii),

$$\frac{AC}{AB} = \frac{AD}{AE} \quad ½$$

$\angle A = \angle A$ (common)

$\therefore$ ΔBAE ~ ΔCAD ½

[By SAS similarity criterion]

23. $\angle PAO = \angle PBO = 90°$ ½

(angle between radius and tangent)

$\angle AOB = 105°$ ½

(By angle sum property of a triangle)

$$\angle AQB = \frac{1}{2} \times 105° = 52.5° \quad \mathbf{1}$$

(Angle at the remaining part of the circle is half the angle subtended by the arc at the centre)

24. We know that, in 60 minutes, the tip of minute hand moves 360°

In 1 minute, it will move $= \frac{360°}{60} = 6°$ ½

$\therefore$ From 7 : 05 pm to 7 : 40 pm i.e. 35 min, it will move through

= 35 × 6° = 210° ½

$\therefore$ Area of swept by the minute hand in 35 min

= Area of sector with sectorial angle θ of 210° and radius of 6 cm

$$= \frac{210}{360} \times \pi \times 6^2 \quad ½$$

$$= \frac{7}{12} \times \frac{22}{7} \times 6 \times 6$$

$$= 66 \text{ cm}^2 \quad ½$$

OR

Let the measure of ∠A, ∠B, ∠C and ∠D be θ_1, θ_2, θ_3 and θ_4 respectively

Required area = Area of sector with centre A
+ Area of sector with centre B
+ Area of sector with centre C
+ Area of sector with centre D ½

$$= \frac{\theta_1}{360} \times \pi \times 7^2 + \frac{\theta_2}{360} \times \pi \times 7^2 + \frac{\theta_3}{360} \times \pi \times 7^2 + \frac{\theta_4}{360} \times \pi \times 7^2 \quad ½$$

$$= \frac{(\theta_1 + \theta_2 + \theta_3 + \theta_4)}{360} \times \pi \times 7^2$$

$$= \frac{(360)}{360} \times \frac{22}{7} \times 7 \times 7 \quad ½$$

(By angle sum property of a triangle)

$= 154 \text{ cm}^2$ ½

25. $\sin(A + B) = 1 = \sin 90°$, so $A + B = 90°$ …(i) ½

$\cos(A - B) = \frac{\sqrt{3}}{2} = \cos 30°$, so $A - B = 30°$ …(ii) ½

From (i) & (ii) $\angle A = 60°$ ½

And $\angle B = 30°$ ½

OR

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

Dividing the numerator and denominator of LHS by $\cos\theta$, we get ½

$$\frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \quad ½$$

Which on simplification (or comparison) gives

$$\tan\theta = \sqrt{3} \quad ½$$

Or $\theta = 60°$ ½

Section - C

26. Let us assume $5+2\sqrt{3}$ is rational, then it must be in the form of $\frac{p}{q}$ where p and q are co-prime integers

and $q \neq 0$ **1**

i.e., $5+2\sqrt{3} = \frac{p}{q}$ ½

So $\sqrt{3} = \frac{p-5q}{2q}$ ...(i) ½

Since p, q, 5 and 2 are integers and $q \neq 0$, RHS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational. This is not possible. ½

This contradiction has arisen due to our wrong assumption that $5+2\sqrt{3}$ is rational. So, $5+2\sqrt{3}$ is irrational. ½

27. Let α and β be the zeros of the polynomial $2x^2-5x-3$

Then $\alpha + \beta = \frac{5}{2}$ ½

And $\alpha\beta = -\frac{3}{2}$ ½

Let 2α and 2β be the zeros $x^2 + px + q$

Then $2\alpha + 2\beta = -p$ ½

$2(\alpha + \beta) = -p$

$2\times\frac{5}{2} = -p$

So $p = -5$ ½

And $2\alpha \times 2\beta = q$ ½

$4\alpha\beta = q$

So $q = 4\times-\frac{3}{2}$

$= -6$ ½

28. Let the actual speed of the train be x km/hr and let the actual time taken be y hours.

Distance covered is xy km

If the speed is increased by 6 km/hr, then time of journey is reduced by 4 hours i.e., when speed is $(x + 6)$ km/hr, time of journey is $(y - 4)$ hours.

$\therefore$ Distance covered $= (x + 6)(y - 4)$

$\Rightarrow xy = (x + 6)(y - 4)$

$\Rightarrow -4x + 6y - 24 = 0$

$\Rightarrow -2x + 3y - 12 = 0$ ...(i) ½

Similarly $xy = (x - 6)(y + 6)$

$\Rightarrow 6x - 6y - 36 = 0$

$\Rightarrow x - y - 6 = 0$ ...(ii) ½

Solving (i) and (ii) we get $x = 30$ and $y = 24$ **1**

Putting the values of x and y in equation (i), we obtain

Distance $= (30 \times 24)$ km $= 720$ km.

Hence, the length of the journey is 720 km. ½

OR

Let the number of chocolates in lot A be x

And let the number of chocolates in lot B be y

$\therefore$ total number of chocolates $= x + y$

Price of 1 chocolate = ₹ $\frac{2}{3}$ so for x chocolates $= \frac{2}{3}x$

and price of y chocolates at the rate of ₹ 1 per chocolate $= y$.

$\therefore$ by the given condition

$$\frac{2}{3}x + y = 400$$

$\Rightarrow 2x + 3y = 1200$ ...(i) ½

Similarly $x + \frac{4}{5}y = 460$

$\Rightarrow 5x + 4y = 2300$ ...(ii) ½

Solving (i) and (ii) we get

$x = 300$ and $y = 200$

$\therefore x + y = 300 + 200 = 500$ **1**

So, Anuj had 500 chocolates. ½

29. LHS : $\dfrac{\frac{\sin^3\theta}{\cos^3\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} + \dfrac{\frac{\cos^3\theta}{\sin^3\theta}}{1+\frac{\cos^2\theta}{\sin^2\theta}}$ ½

$$= \dfrac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}} + \dfrac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$$

$$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta} \quad ½$$

$$= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta} \quad ½$$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} \quad ½$$

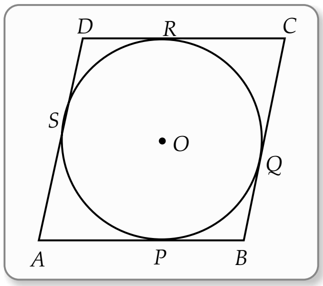
$$= \frac{1-2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta} \quad ½$$

$$= \frac{1}{\cos\theta \sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta \sin\theta}$$

$= \sec\theta \operatorname{cosec}\theta - 2\sin\theta\cos\theta$

= RHS ½

30.



Let ABCD be the rhombus circumscribing the circle with centre O, such that AB, BC, CD and DA touch the circle at points P, Q, R and S respectively.

We know that the tangents drawn to a circle from an exterior point are equal in length.

$\therefore \quad AP = AS \quad \ldots(1)$

$BP = BQ \quad \ldots(2)$

$CR = CQ \quad \ldots(3)$

$DR = DS \quad \ldots(4)$ **1**

Adding (1), (2), (3) and (4) we get

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$(AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\therefore \quad AB + CD = AD + BC \quad \ldots(5)$ **1**

Since $AB = DC$ and $AD = BC$

(opposite sides of parallelogram ABCD) ½

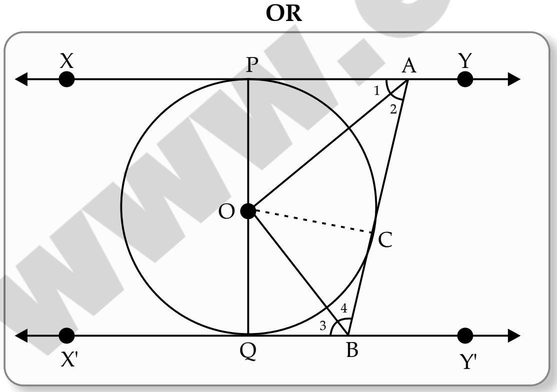
putting in (5) we get, $2AB = 2AD$

or $AB = AD$.

$\therefore \quad AB = BC = DC = AD$

Since a parallelogram with equal adjacent sides is a rhombus, so ABCD is a rhombus ½

OR



Join OC

In Δ OPA and Δ OCA

$OP = OC$ (radii of same circle)

$PA = CA$ **1**

(length of two tangents from an external point)

$AO = AO$ (Common)

Therefore, $\Delta OPA \cong \Delta OCA$ ½

(By SSS congruency criterion)

Hence, $\angle 1 = \angle 2$ (CPCT) ½

Similarly $\angle 3 = \angle 4$

$\angle PAB + \angle QBA = 180°$ ½

(co interior angles are supplementary as XY || X'Y')

$2\angle 2 + 2\angle 4 = 180°$

$\angle 2 + \angle 4 = 90 \quad \ldots(1)$ ½

$\angle 2 + \angle 4 + \angle AOB = 180°$ (Angle sum property)

Using (1), we get, $\angle AOB = 90°$

31. **(i)** P (At least one head) $= \frac{3}{4}$ **1**

(ii) P(At most one tail) $= \frac{3}{4}$ **1**

(iii) P(A head and a tail) $= \frac{2}{4} = \frac{1}{2}$ **1**

Section - D

32. Let the time taken by larger pipe alone to fill the tank $= x$ hours ½

Therefore, the time taken by the smaller pipe $= x + 10$ hours

Water filled by larger pipe running for 4 hours

$= \frac{4}{x}$ litres

Water filled by smaller pipe running for 9 hours

$= \frac{9}{x+10}$ litres

We know that $\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$ **1**

Which on simplification gives:

$x^2 - 16x - 80 = 0$ **1**

$x^2 - 20x + 4x - 80 = 0$

$x(x-20) + 4(x-20) = 0$

$(x+4)(x-20) = 0$

$x = -4, 20$ **1**

x cannot be negative.

Thus, $x = 20$ ½

$x + 10 = 30$ ½

Larger pipe would alone fill the tank in 20 hours and smaller pipe would fill the tank alone in 30 hours. ½

OR

Let the usual speed of plane be x km/hr ½

and the reduced speed of the plane be $(x - 200)$ km/hr

Distance = 600 km [Given]

According to the question,

(time taken at reduced speed) – (Schedule time)

= 30 minutes

= 0.5 hours. **1**

$$\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$$ **1**

Which on simplification gives:

$$x^2 - 200x - 240000 = 0$$
$$x^2 - 600x + 400x - 240000 = 0$$
$$x(x-600) + 400(x-600) = 0$$
$$(x-600)(x+400) = 0$$
$$x = 600 \text{ or } x = -400 \qquad 1$$

But speed cannot be negative. ½

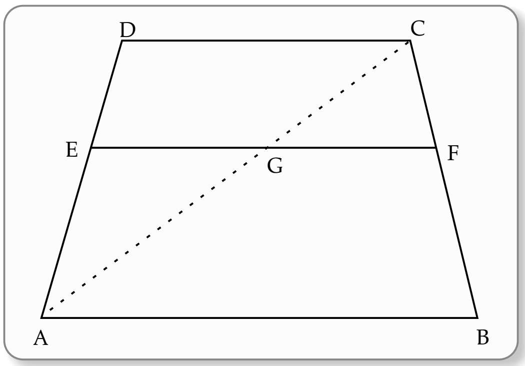
$\therefore$ The usual speed is 600 km/hr and ½

the scheduled duration of the flight is $\frac{600}{600}$ = 1 hour ½

33. For the Theorem:

Given, To prove, Construction and figure 1½

Proof



Let ABCD be a trapezium DC || AB and EF is a line parallel to AB and hence to DC.

To prove: $\frac{DE}{EA} = \frac{CF}{FB}$

Construction: Join AC, meeting EF in G.

Proof: In ΔABC, we have

GF || AB

$\frac{CG}{GA} = \frac{CF}{FB}$ [By BPT] …(1) ½

In ΔADC, we have

EG || DC (EF || AB & AB || DC)

$\frac{DE}{EA} = \frac{CG}{GA}$ [By BPT] …(2) ½

From (1) & (2), we get,

$\frac{DE}{EA} = \frac{CF}{FB}$ ½

34. Radius of the base of cylinder (r) = 2.8 m = Radius of the base of the cone (r)

Height of the cylinder (h) = 3.5 m

Height of the cone (H) = 2.1 m.

Slant height of conical part (l) = $\sqrt{r^2 + H^2}$

$= \sqrt{(2.8)^2 + (2.1)^2}$

$= \sqrt{7.84 + 4.41}$

$= \sqrt{12.25}$ 1

$= 3.5$ m 1

Area of canvas used to make tent

= *CSA* of cylinder + *CSA* of cone 1

$= 2 \times \pi \times 2.8 \times 3.5 + \pi \times 2.8 \times 3.5$ 1

$= 61.6 + 30.8$

$= 92.4 \text{ m}^2$ 1

Cost of 1500 tents at ₹ 120 per sq.m

$= 1500 \times 120 \times 92.4$

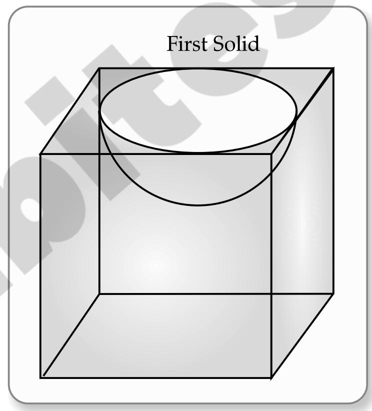
= ₹ 16,632,000

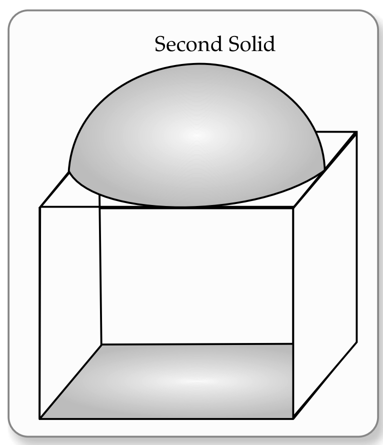
Share of each school to set up the tents

$= \frac{16632000}{50}$

= ₹ 332,640

OR





(i) SA for first new solid (S_1):

$6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2$

$= 294 + 77 - 38.5$ 1

$= 332.5 \text{ cm}^2$

SA for second new solid (S_2):

$6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2$ 1

$= 294 + 77 - 38.5$

$= 332.5 \text{ cm}^2$ 1

So $S_1 : S_2 = 1 : 1$

(ii) Volume for first new solid (V_1)

$$= 7 \times 7 \times 7 - \frac{2}{3}\pi \times 3.5^3$$

$$= 343 - \frac{539}{6} = \frac{1519}{6} \text{ cm}^3 \quad \mathbf{1}$$

Volume for second new solid (V_2)

$$= 7 \times 7 \times 7 + \frac{2}{3}\pi \times 3.5^3$$

$$= 343 + \frac{539}{6} = \frac{2597}{6} \text{ cm}^3 \quad \mathbf{1}$$

35. Median = 525, so Median Class = 500 – 600 ½

Class interval	Frequency	Cumulative Frequency
0–100	2	2
100–200	5	7
200–300	x	$7 + x$
300–400	12	$19 + x$
400–500	17	$36 + x$
500–600	20	$56 + x$
600–700	y	$56 + x + y$
700–800	9	$65 + x + y$
800–900	7	$72 + x + y$
900–1000	4	$76 + x + y$

1½

$$76 + x + y = 100$$

$$\Rightarrow \quad x + y = 24 \quad \ldots\text{(i)} \ \mathbf{1}$$

$$\text{Median} = l + \frac{\frac{n}{2} - cf}{f} \times h \quad ½$$

Since, $l = 500$, $h = 100$, $f = 20$, $cf = 36 + x$ and $n = 100$

Therefore, putting the value in the Median formula, we get;

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100 \quad ½$$

so $x = 9$

$y = 24 - x$ (from eq. (i))

$y = 24 - 9 = 15$ ½

Therefore, the value of $x = 9$ and $y = 15$. ½

36. (i) B(1, 2), F(–2, 9)

$$BF^2 = (-2-1)^2 + (9-2)^2$$

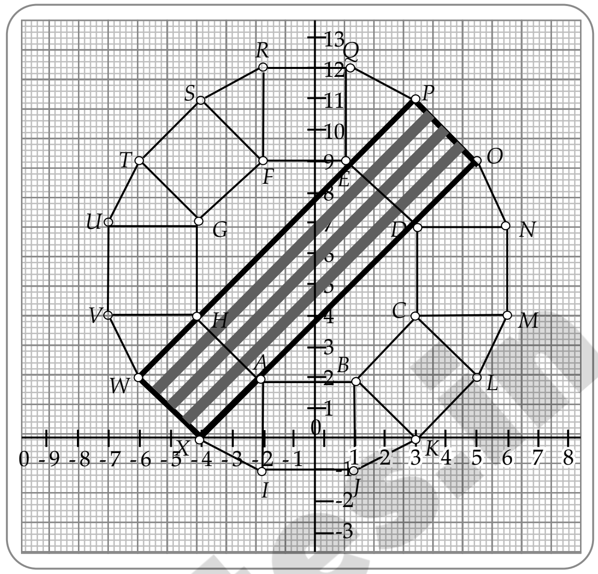
$$= (-3)^2 + (7)^2$$

$$= 9 + 49$$

$$= 58$$

So, $BF = \sqrt{58}$ units **1**

(ii)



W(–6, 2), X(–4, 0), O(5, 9), P(3,11) ½

Clearly WXOP is a rectangle

Point of intersection of diagonals of a rectangle is the mid point of the diagonals. So the required point is mid point of WO or XP

$$= \left(\frac{-6+5}{2}, \frac{2+9}{2}\right)$$

$$= \left(\frac{-1}{2}, \frac{11}{2}\right) \quad ½$$

(iii) A(–2, 2), G(–4, 7)

Let the point on y-axis be Z(0, y) ½

$$AZ^2 = GZ^2 \quad ½$$

$$(0 + 2)^2 + (y - 2)^2 = (0 + 4)^2 + (y - 7)^2$$

$$(2)^2 + y^2 + 4 - 4y = (4)^2 + y^2 + 49 - 14y$$

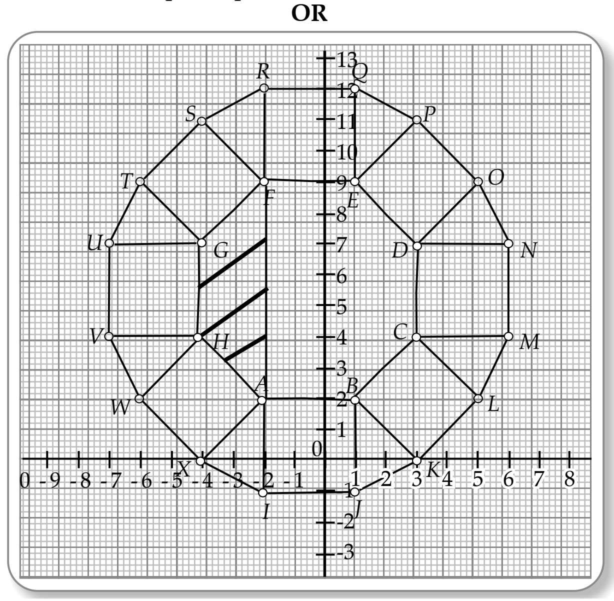
$$8 - 4y = 65 - 14y$$

$$10y = 57$$

So, $y = 5.7$

i.e. the required point is (0, 5.7)

OR



A(–2, 2), F(–2, 9), G(–4, 7), H(–4, 4)

Clearly $GH = 7 - 4 = 3$ units ½

$AF = 9 - 2 = 7$ units ½

So, height of the trapezium AFGH = 2 units

So, area of $AFGH = \frac{1}{2}(AF + GH) \times \text{height}$

$= \frac{1}{2}(7 + 3) \times 2$ ½

$= 10$ sq. units ½

37. **(i)** Since each row is increasing by 10 seats, so it is an AP with first term $a = 30$, and common difference $d = 10$. ½

So number of seats in 10th row

$= a_{10}$

$= a + 9d$

$= 30 + 9 \times 10 = 120$ ½

(ii) $S_n = \frac{n}{2}(2a + (n-1)d)$

$1500 = \frac{n}{2}(2 \times 30 + (n-1)10)$

$3000 = 50n + 10n^2$

$n^2 + 5n - 300 = 0$ ½

$n^2 + 20n - 15n - 300 = 0$

$(n + 20)(n - 15) = 0$ ½

Rejecting the negative value, $n = 15$ ½

OR

No. of seats already put up to the 10th row $= S_{10}$ ½

$S_{10} = \frac{10}{2}\{2 \times 30 + (10-1)10\}$ ½

$= 5(60 + 90) = 750$ ½

So, the number of seats still required to be put are $1500 - 750 = 750$ ½

(iii) If no. of rows = 17

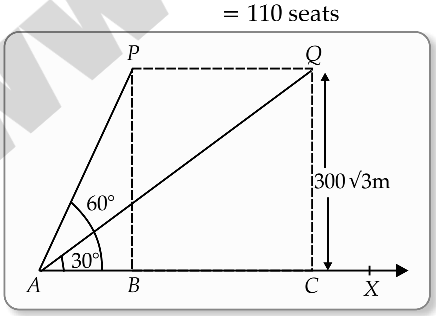
then the middle row is the 9th row ½

$a_8 = a + 8d$

$= 30 + 80$

$= 110$ seats ½

38. **(i)**



1

P and Q are the two positions of the plane flying at a height of $3000\sqrt{3}$ m.

A is the point of observation.

(ii) In ΔPAB, $\tan 60° = \frac{PB}{AB}$

Or $\sqrt{3} = \frac{3000\sqrt{3}}{AB}$

So $AB = 3000\text{m}$ 1

$\tan 30° = \frac{QC}{AC}$

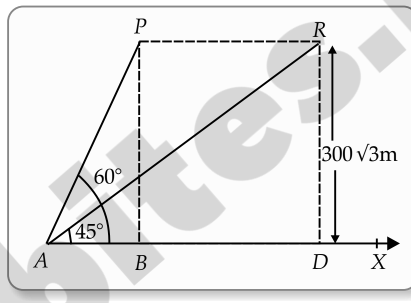
$\frac{1}{\sqrt{3}} = \frac{3000\sqrt{3}}{AC}$

$AC = 9000$ m

distance covered $= 9000 - 3000$ ½

$= 6000$ m. ½

OR



In ΔPAB, $\tan 60° = \frac{PB}{AB}$

Or $\sqrt{3} = \frac{3000\sqrt{3}}{AB}$

So $AB = 3000$ m ½

$\tan 45° = \frac{RD}{AD}$

$1 = \frac{3000\sqrt{3}}{AD}$ ½

$AD = 3000\sqrt{3}$ m

distance covered $= 3000\sqrt{3} - 3000$

$= 3000(\sqrt{3} - 1)$ m ½

(iii) speed $= \frac{6000}{30}$

$= 200$ m/s ½

$= 200 \times \frac{3600}{1000}$

$= 720$ km/hr ½

Alternatively: speed $= \frac{3000(\sqrt{3}-1)}{15(\sqrt{3}-1)}$ ½

$= 200$ m/s

$= 200 \times \frac{3600}{1000}$

$= 720$ km/hr ½

Sample Question Paper-2

Mathematics Standard (041)

Class-X

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is:
(A) 13 **(B)** 65 **(C)** 875 **(D)** 1,750

2. The greatest number which when divides 1251, 9377 and 15,628 leaves remainder 1, 2 and 3, respectively is:
(A) 575 **(B)** 450 **(C)** 750 **(D)** 625

3. The number of polynomials having zeroes as –2 and 5 is
(A) 1 **(B)** 2 **(C)** 3 **(D)** more than 3

4. If $2x + y = 23$ and $4x - y = 19$, then the values of $(5y - 2x)$ and $\left(\frac{y}{x} - 2\right)$ are:
(A) $30, \frac{5}{7}$ **(B)** $31, \frac{-5}{7}$ **(C)** $32, \frac{5}{7}$ **(D)** None of these

5. Which of the following is not a quadratic equation?
(A) $2(x-1)^2 = 4x^2 - 2x + 1$ **(B)** $2x - x^2 = x^2 + 5$
(C) $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$ **(D)** $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

6. Which term of the progression 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$ is the first negative term:
(A) 27^{th} term **(B)** 28^{th} term **(C)** 26^{th} term **(D)** 25^{th} term

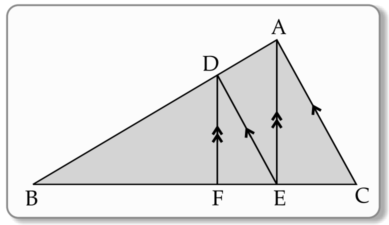
7. The distance of the point P (2, 3) from the x-axis is:
(A) 2 **(B)** 3 **(C)** 1 **(D)** 5

8. X-axis divides the join of (2, –3) and (5, 6) in the ratio.
(A) 1 : 2 **(B)** 2 : 1 **(C)** 2 : 5 **(D)** 5 : 2

9. If the radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is the tangent to the other circle is:

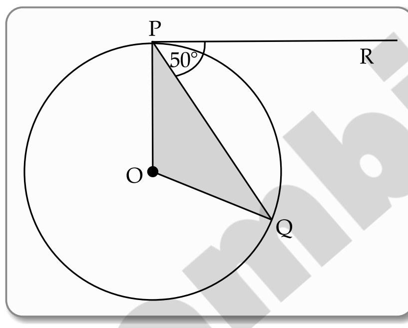
(A) 3 cm **(B)** 6 cm **(C)** 9 cm **(D)** 1 cm

10. In the figure below, DE || AC and DF || AE. Which of these is equal to $\frac{BF}{FE}$?



(A) $\frac{DF}{AE}$ **(B)** $\frac{BE}{EC}$ **(C)** $\frac{BA}{AC}$ **(D)** $\frac{FE}{EC}$

11. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to:



(A) 100° **(B)** 80° **(C)** 90° **(D)** 75°

12. Given that, $\sin \alpha = \frac{1}{2}$ and $\cos \beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is:

(A) 0° **(B)** 30° **(C)** 60° **(D)** 90°

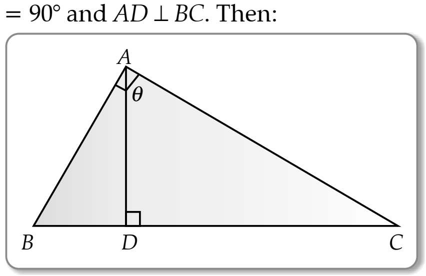
13. Given that, $\sin \theta = \frac{a}{b}$ then $\cos \theta$ is equal to:

(A) $\frac{b}{\sqrt{b^2 - a^2}}$ **(B)** $\frac{b}{a}$ **(C)** $\frac{\sqrt{b^2 - a^2}}{b}$ **(D)** $\frac{a}{\sqrt{b^2 - a^2}}$

14. If α, β are the zeroes of the quadratic polynomial $p(x) = x^2 - (k + 6)x + 2(2k - 1)$, then the value of k, if $\alpha + \beta = \frac{1}{2}\alpha\beta$, is:

(A) –7 **(B)** 7 **(C)** –3 **(D)** 3

15. In the figure given below, $\angle BAC = 90°$ and $AD \perp BC$. Then:



(A) $BD \times CD = BC^2$ **(B)** $AB \times AC = BC^2$ **(C)** $BD \times CD = AD^2$ **(D)** $AB \times AC = AD^2$

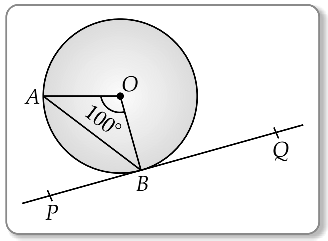
16. Which of the following cannot be the probability of an event?

(A) 0.01 **(B)** 3% **(C)** $\frac{16}{17}$ **(D)** $\frac{17}{16}$

17. **An event is very unlikely to happen. Its probability is closest to:**

(A) 0.0001 **(B)** 0.001 **(C)** 0.01 **(D)** 0.1

18. In the given figure, PQ is tangent to the circle with centre at O, at the point B. If $\angle AOB = 100°$, then $\angle ABP$ is equal:



(A) 50° **(B)** 40° **(C)** 60° **(D)** 80°

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(C) Assertion (A) is true but reason (R) is false.
(D) Assertion (A) is false but reason (R) is true.

19. **Assertion:** If a and b are two coprime numbers, then a^3 and b^3 will be coprime numbers.
Reason: Two natural numbers are always coprime numbers then their cube are also coprime numbers.

20. **Assertion:** The coordinates of P(2, 0) an x-axis is equidistance from point A(–2, 0) and B(6, 0).
Reason: Since, P(2, 0) in the midpoint of A(–2, 0) and B(6, 0).

Section - B

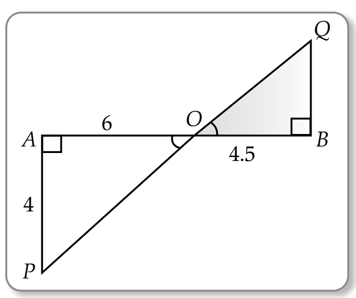
Section B consists of 5 questions of 2 marks each.

21. If one of the zeroes of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

22. The larger of two supplementary angles exceeds the smaller by 18°. Find the angles.

OR

In the given figure, if $\angle A = 90°$, $\angle B = 90°$, $OB = 4.5$ cm, $OA = 6$ cm and $AP = 4$ cm, then find QB.



23. Out of the two concentric circles, the radius of the outer circle is 5 cm and the chord AC of length 8 cm is a tangent to the inner circle. Find the radius of the inner circles.

24. If $\sin(A + B) = 1$ and $\sin(A - B) = \frac{1}{2}$, $0 \leq A + B = 90°$ and A > B, then find A and B.

25. Find the area (in cm^2) of the circle that can be inscribed in a square of side 8 cm.

OR

The curved surface area of a cylinder is 264 m^2 and its volume is 924 m^3. Find the ratio of its height to its diameter.

Section - C

Section C consists of 6 questions of 3 marks each.

26. Prove that $7\sqrt{5}$ is an irrational number.

27. Solve for x: $\frac{x-1}{2x+1}+\frac{2x+1}{x-1}=2$, where $x\neq\frac{-1}{2}, 1$

28. A dealer sells a toy for ₹ 24 and gains as much percent as the cost price of the toy.

(i) Find the cost price of the toy.

(ii) Which mathematical concept is used in the above problem?

OR

In forest, trees are cut down every year. So, animals do not find the shadowy area to sleep or rest. Animals of forest wander whole day to find the area full of trees. A vertical row of trees 12 m long casts a shadow 8 m long on the ground. At the same time, a bamboo tree casts the shadow 40 m long on the ground.

(i) Determine the height of the bamboo tree.

(ii) Which mathematical concept is used in this problem?

29. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

30. Prove that: $\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta} = 1 + \tan\theta + \cot\theta$.

OR

Prove that: $1+\frac{\cot^2\alpha}{1+\operatorname{cosec}\alpha}=\operatorname{cosec}\alpha$

31. A bag contains 18 balls out of which x balls are red.

(i) If one ball is drawn at random from the bag, what is the probability that it is not red?

(ii) If 2 more red balls are put in the bag, the probability of drawing a red ball will be $\frac{9}{8}$ times the probability of drawing a red ball in the first case. Find the value of x.

Section - D

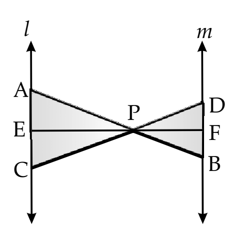
Section D consists of 4 questions of 5 marks each.

32. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum ₹ 1,008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got ₹ 1,028. Find the cost price of the saree and the list price (price before discount) of the sweater.

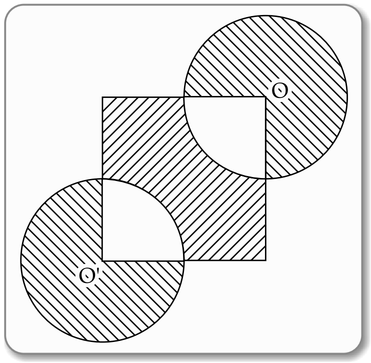
OR

A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs ₹ 2,530. Also, one reserved first class ticket and one reserved first class half ticket from A to B costs ₹ 3,810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.

33. In the figure given below, $l \parallel m$ and line segments AB, CD and EF are concurrent at point P. Prove that $\frac{AE}{BF}=\frac{AC}{BD}=\frac{CE}{FD}$.



34. In the given figure, the side of square is 28 cm and radius of each circle is half of the length of the side of the square, where O and O′ are centres of the circle. Find the area of the shaded region.



OR

A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid (use $\pi = \frac{22}{7}$)

35. On annual day of a school, 400 students participated in the function. Frequency distribution showing their ages is as shown in the following table:

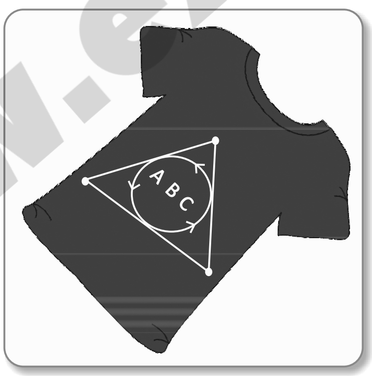
Age (in years)	05 – 07	07 – 09	09 – 11	11 – 13	13 – 15	15 – 17	17 – 19
Number of students	70	120	32	100	45	28	5

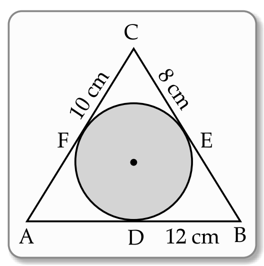
Find mean and median of the above data.

Section - E

Case study based questions are compulsory.

36. Varun has been selected by his school to design logo for sports day T-shirts for students and staff. The logo is designed in different geometry and different colours according to the theme. In the given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F, respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.





(i) Find the length of AD and BE. **1**

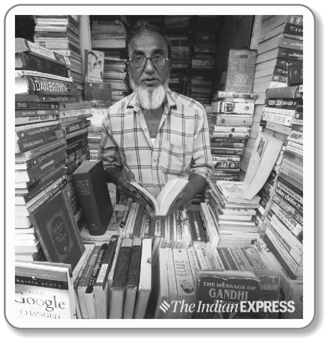
(ii) If the radius of the circle is 4 cm, find the area of ΔOAB. **1**

(iii) Calculate the perimeter of ΔABC. **2**

OR

If ΔABC in stitched by Gold wire then calculate the amount of used role x-wise in it. The rate of gold-wise in ₹ 1500 per m.

37. A book store shopkeeper gives books on rent for reading. He has variety of books in his store related to fiction, stories and quizzes, etc. He takes a fixed charge for the first two days and an additional charge for subsequent day. Amruta paid ₹ 22 for a book kept for 6 days; while Radhika paid ₹ 16 for keeping the book for 4 days.



Assume that the fixed charge be ₹ x and additional charge (per day) be ₹ y.

Based on the above information, answer the following questions:

(i) Frame the algebraic equation for Radhika. **1**

(ii) Frame the algebraic equation for Amruta. **1**

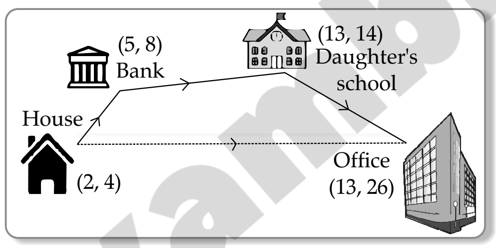
(iii) What are the additional charges for each subsequent day for a book? **2**

OR

Which is the total amount paid by both, if both of them have kept the book for 2 more days?

38. Ayush Starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office.

(Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.



(i) What is the distance between house and bank? **1**

(ii) What is the distance between Daughter's School and bank? **1**

(iii) What is the distance between house and office? **2**

OR

What is the total distance travelled by Ayush to reach the office?

■■■

Sample Question Paper-3

Mathematics Standard (041)

Class-X

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. The LCM of two numbers is 2400. Which of the following cannot be their HCF?

(A) 300 **(B)** 400 **(C)** 500 **(D)** 600

2. The product of HCF and LCM of two numbers is 50 and if one of the numbers is 10 then the other number is

(A) 10 **(B)** 20 **(C)** 5 **(D)** 15

3. If the zeroes of quadratic polynomial are 1,1 then the polynomial can be

(A) $x^2 + x + 1$ **(B)** $x^2 - 2x + 1$ **(C)** $x^2 + 3x + 2$ **(D)** $x^2 + 2x + 2$

4. It is given that, $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, the true statement is:

(A) $DE = 12$ cm, $\angle F = 50°$ **(B)** $DE = 12$ cm, $\angle F = 100°$

(C) $EF = 12$ cm, $\angle D = 100°$ **(D)** $EF = 12$ cm, $\angle D = 30°$

5. A line intersects the y-axis and x-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively

(A) (0, –5) and (2, 0) **(B)** (0, 10) and (–4, 0) **(C)** (0, 4) and (–10, 0) **(D)** (0, –10) and (4, 0)

6. How many zero(es) does the polynomial $293x^2 - 293x$ have?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

7. If $\cot\theta = \frac{1}{\sqrt{3}}$, the value of $\sec^2\theta + \operatorname{cosec}^2\theta$ is

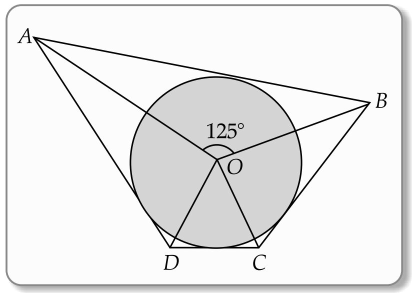
(A) 1 **(B)** $\frac{40}{9}$ **(C)** $\frac{38}{9}$ **(D)** $5\frac{1}{3}$

8. A pole 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then the sun's elevation is:

(A) 60° **(B)** 45° **(C)** 30° **(D)** 90°

9. In the given figure, if $\angle AOB$ = 125°, then $\angle COD$ is equal to:

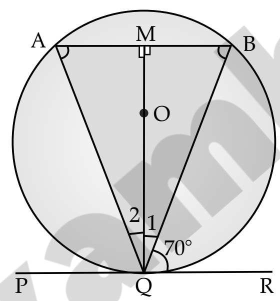
(A) 62.5° **(B)** 45° **(C)** 35° **(D)** 55°



10. It is given that $\Delta ABC \sim \Delta DFE$, $\angle A = 30°$, $\angle C = 50°$, AB = 5 cm, AC = 8 cm and DF = 7.5 cm. Then, the following is true:

(A) DE = 12 cm, $\angle F = 50°$ **(B)** DE = 12 cm, $\angle F = 100°$ **(C)** EF = 12 cm, $\angle D = 100°$ **(D)** EF = 12 cm, $\angle D = 30°$

11. In the given figure, if PQR is the tangent to a circle at Q, whose centre is O, AB is a chord parallel to PR and $\angle BQR$ = 70°, then $\angle AQB$ is equal to



(A) 20° **(B)** 40° **(C)** 35° **(D)** 45°

12. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm is:

(A) 31 cm **(B)** 25 cm **(C)** 62 cm **(D)** 50 cm

13. If two solid hemispheres of same base radius 'r' are joined together along their bases, then curved surface area of this new solid is:

(A) $4\pi r^2$ **(B)** $6\pi r^2$ **(C)** $3\pi r^2$ **(D)** $8\pi r^2$

14. While computing mean of grouped data, we assume that the frequencies are:

(A) evenly distributed over all the classes **(B)** centred at the class marks of the classes
(C) centred at the upper limits of the classes **(D)** centred at the lower limits of the classes

15. The area of a quadrant of a circle where the circumference of circle is 176 m, is:

(A) 2464 m^2 **(B)** 1232 m^2 **(C)** 616 m^2 **(D)** 308 m^2

16. Two identical fair dice have numbers 1 to 6 written on their faces. Both are tossed simultaneously. What is the probability that the product of the numbers that turn up is 12?

(A) $\frac{1}{36}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{3}$

17. If an event that cannot occur, then its probability is:

(A) 1 **(B)** $\frac{3}{4}$ **(C)** $\frac{1}{2}$ **(D)** 0

18. Given that $\sin\alpha = \frac{1}{2}$ and $\cos\beta = \frac{1}{2}$, then the value of $(\alpha + \beta)$ is

(A) 0° **(B)** 30° **(C)** 60° **(D)** 90°

DIRECTION: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

19. **Statement A(Assertion):** If product of two numbers is 2890 and their HCF is 17, then their LCM is 450.
Statement R(Reason): LCM is always greater than HCF.

20. **Statement A(Assertion):** The distance between two points (x_1, y_1) and (x_2, y_2) is $\left|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right|$ unit.

Statement R(Reason): The distance between Y(4, 0) and Z(0, 3) is 5 units.

Section - B

Section B consists of 5 questions of 2 marks each.

21. Determine the values of m and n so that the following system of linear equations have infinite number of solutions:

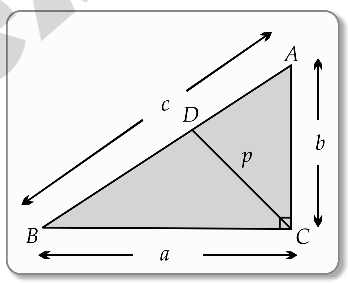
$(2m-1)x + 3y - 5 = 0$

and $3x + (n-1)y - 2 = 0$

OR

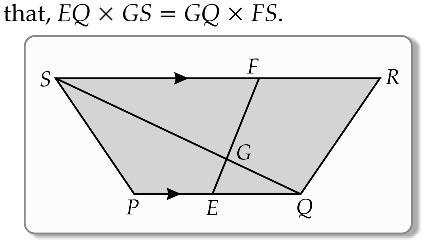
Write all the values of p for which the quadratic equation $x^2 + px + 16 = 0$ has equal roots. Find the roots of the equation so obtained.

22. ABC is a right triangle, right angled at C. Let $BC = a$, $CA = b$, $AB = c$ and p be the length of perpendicular from C to AB. Prove that, $cp = ab$.



23. If the circumferences of two concentric circles forming a ring are 88 cm and 66 cm, respectively. Find the width of the ring.

24. In the figure, PQRS is a trapezium in which PQ || RS. On PQ and RS, there are points E and F, respectively such that EF intersects SQ at G. Prove that, $EQ \times GS = GQ \times FS$.



25. Prove that:

$$\frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta} = 1$$

OR

One day, three boys were playing cricket in playground. One boy hits sixer, so the ball flew and entered in the light house which is at the backside of the play ground. So they decided to go in the light house to bring the ball back. They have seen a boat from the top of light house in the nearby river. From the top of light house, 40 m above the water level, the angle of depression of the boat is 60°. Find how far the boat is from the base of the light house.

Section - C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{5}$ is an irrational number.

27. If the ratio of the sums of first n terms of two A.P.'s is $(7n + 1) : (4n + 27)$, find the ratio of their 9th term.

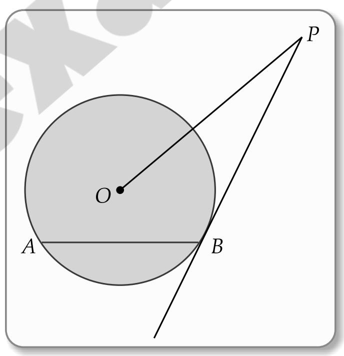
28. Solve the equation: $1 + 4 + 7 + 10 + ... + x = 287$.

29. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45°, respectively. If the height of the tower is 15 m, then find the distance between these points.

OR

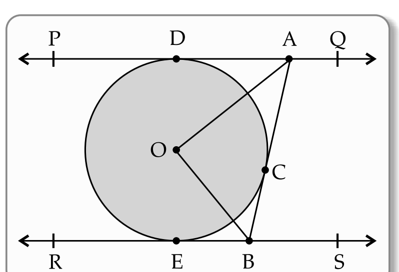
An aeroplane is flying at a height of 300 m above the ground. Flying at this height, the angles of depression from the aeroplane of two points on both banks of a river in opposite directions are 45° and 60°, respectively. Find the width of the river [use $\sqrt{3}$ = 1.732].

30. AB is a chord of circle with centre O. At B, a tangent PB is drawn such that its length is 24 cm. The distance of P from the centre is 26 cm. If the chord AB is 16 cm, find the distance of the chord from the centre of circle.



OR

In figure, PQ and RS are two parallel tangents to a circle with center O and another tangent AB will have point of contact C intersecting PQ at A and RS at B. Prove that, ∠AOB = 90°.



31. Two different dice are thrown together. Find the probability that the numbers obtained.

(i) have a sum less than 7.

(ii) have a product less than 16.

(iii) is a doublet of odd numbers. AI

Section - D

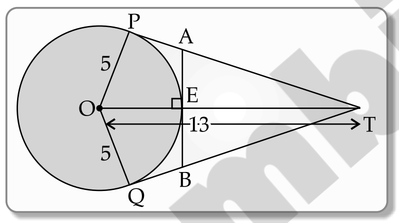
Section D consists of 4 questions of 5 marks each.

32. Two water taps together can fill a tank in $1\frac{7}{8}$ h. The tap with larger diameter takes 2 h. less than the tap with smaller one to fill the tank separately. Find the time in which both the taps can fill the tank separately. AI

OR

A motorboat covers a distance of 16 km upstream and 24 km downstream in 6 h. In the same time it covers a distance of 12 km upstream and 36 km downstream. Find the speed of the boat in still water and that of the stream. AI

33. In the given Fig. O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.



34. A vessel full of water is in the form of an inverted cone of height 8 cm and the radius of its top, which is open, is 5 cm. When 100 spherical lead balls are dropped into vessel. then One-fourth of the water flows out of the vessel. Find the radius of a spherical ball.

35. In a higher secondary school, an inter-school quiz competition was held among 110 students. The marks obtained by 110 students in an examination are given below:

Marks	30–35	35–40	40–45	45–50	50–55	55–60	60–65
Number of students	14	16	28	23	18	8	3

Find the mean.

OR

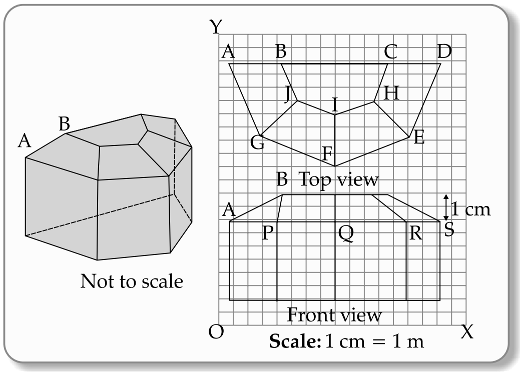
One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting (a) non-face card, (b) black king or a red queen, (c) spade card.

Section - E

Case study based questions are compulsory.

36. The diagram show the plans for a sun room. It will be built onto the wall of a house. The four walls of the sun room are square clear glass panels. The roof is made using:

- Four clear glass panels, trapezium in shape, all of the same size.
- One tinted glass panel, half a regular octagon in shape.



(i) Refer to top view, find the mid-point of the segment joining the points J(6, 17) and I(9, 16): **1**

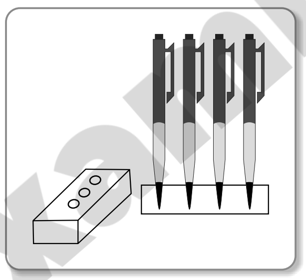
(ii) Refer to front view, Find the distance of the point P from the Y-axis is: **1**

(iii) Refer to front view, find the co-ordinates of the point which divides the line segment joining the points A and B in the ratio 1 : 3 internally: **2**

OR

Refer to front view, if a point (x, y) is equidistant from the Q(9, 8) and S(17, 8), then find the value of x.

37. A carpenter made a wooden pen stand. It is in the shape of cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. (See Figure).



(i) What is the volume of cuboid? **1**

(ii) What is the volume of a conical depression ? **1**

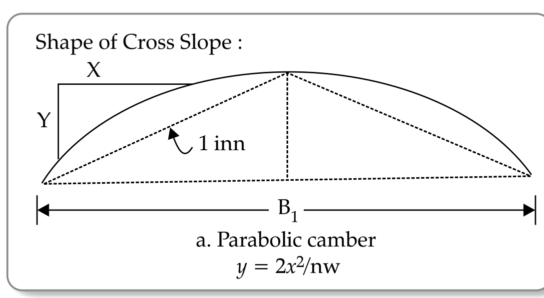
(iii) What is the total volume of conical depressions? **2**

OR

(iv) What is the volume of wood in the entire stand?

38. **Applications of Parabolas: Highway Overpasses/Underpasses**

A highway underpass is parabolic in shape.



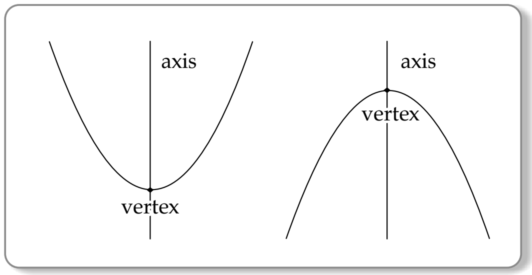
Parabola

A parabola is the graph that results from

$$p(x) = ax^2 + bx + c.$$

Parabolas are symmetric about a vertical line known as the Axis of Symmetry.

The Axis of Symmetry runs through the maximum or minimum point of the parabola which is called the vertex.



(i) What is zeroes of x, f the highway overpass is represented by $x^2 - 2x - 8$. **1**

(ii) The highway overpass is represented graphically. **1**

Zeroes of a polynomial can be expressed graphically. How to find the number of zeroes of polynomial which is equal to number of points in the given graph of polynomial: **2**

(iii) What is the equation of Highway Underpass whose one zero is 6 and sum of the zeroes is 0, is:

OR

Find The number of zeroes that polynomial $f(x) = (x - 2)^2 + 4$ can have is:

■■■

Sample Question Paper-4

Mathematics Standard (041)

Class-X

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Questions of 5 marks, 2 Questions of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.*
(viii) *Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. $n^2 - 1$ is divisible by 8, if n is:
(A) an integer **(B)** a natural number **(C)** an odd integer **(D)** an even integer

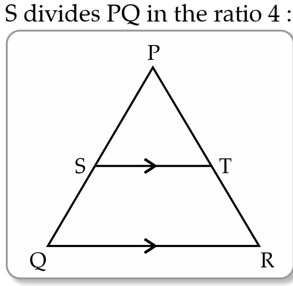
2. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is:
(A) xy **(B)** xy^2 **(C)** x^3y^3 **(D)** x^2y^2

3. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $a \neq 0$ are equal, then:
(a) c and a have opposite signs **(b)** c and b have opposite signs
(c) c and a have the same sign **(d)** c and b have the same sign

4. Which of the following equations has the sum of its roots as 3?
(A) $2x^2 - 3x + 6 = 0$ **(B)** $-x^2 + 3x - 3 = 0$ **(C)** $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ **(D)** $3x^2 - 3x + 3 = 0$

5. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1:2 internally lies in the:
(A) I quadrant **(B)** II quadrant **(C)** III quadrant **(D)** IV quadrant

6. In the following figure, ST || QR, point S divides PQ in the ratio 4 : 5. If $ST = 1.6$ cm, what is the length of QR?



(Note: the figure is not to scale)
(A) 0.71 cm **(B)** 2 cm
(C) 3.6 cm **(D)** cannot be calculated from the given data

7. The angle of depression of a car parked on the road from the top of 150 m high tower is 30°. The distance of the car from the tower (in metres) is:

(A) $50\sqrt{3}$ **(B)** $150\sqrt{3}$ **(C)** $150\sqrt{2}$ **(D)** 75

8. If ΔABC is right angled at C, then the value of cos (A+B) is:

(A) 0 **(B)** 1 **(C)** $\frac{1}{2}$ **(D)** $\frac{\sqrt{3}}{2}$

9. If in two triangles *DEF* and *PQR*, ∠D = ∠Q and ∠R = ∠E, then which of the following is not true?

(A) $\frac{EF}{PR}=\frac{DF}{PQ}$ **(B)** $\frac{DE}{DF}=\frac{FE}{RP}$ **(C)** $\frac{DE}{QR}=\frac{DF}{PQ}$ **(D)** $\frac{EF}{RP}=\frac{DE}{QR}$

10. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. The value of ∠APB is:

(A) 30° **(B)** 45° **(C)** 60° **(D)** 90° AI

11. Two concentric circles are of radii 5 cm and 3 cm. The length of the chord of larger circle (in cm) which touches the smaller circle is:

(A) 6 cm **(B)** 8 cm **(C)** 10 cm **(D)** 2 cm

12. Area of a sector of angle *p* (in degrees) of a circle with radius R is:

(A) $\frac{p}{180^{\circ}}\times 2\pi R$ **(B)** $\frac{p}{90^{\circ}}\times 2\pi R$ **(C)** $\frac{p}{360^{\circ}}\times 2\pi R$ **(D)** $\frac{p}{720^{\circ}}\times 2\pi R^2$

13. The radius of a sphere (in cm) whose volume is 12π cm^3, is:

(A) 3 **(B)** $3\sqrt{3}$ **(C)** $3^{2/3}$ **(D)** $3^{1/3}$

14. In the formula $\bar{x}=a+\frac{\sum x_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are the deviations from *a* of:

(A) lower limits of the classes **(B)** upper limits of the classes

(C) mid-points of the classes **(D)** frequencies of the class marks

15. Savita has a lamp placed at the centre of her square yard, each side measuring 20 m. The light of lamp covers a circle of radius 10 m on yard. The area of the yard is not lit by the lamp; is:

(A) 400π sq. m **(B)** 100π sq. m **(C)** (40 – 10π) sq. m **(D)** (400 – 100π) sq. m AI

16. If the probability of an event is *p*, then the probability of its complementary event will be:

(A) $p-1$ **(B)** p **(C)** $1-p$ **(D)** $1-\frac{1}{p}$

17. The probability expressed as a percentage of a particular occurrence can never be:

(A) less than 100 **(B)** less than 0

(C) greater than 1 **(D)** anything but a whole number

18. The value of $9\sec^2 A - 9\tan^2 A$ is:

(A) 1 **(B)** 9 **(C)** 8 **(D)** 0

DIRECTION: In the question numbers 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option:

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

19. **Assertion:** The smallest number which is divisible by both 306 and 657; is 22338.

Reason: The HCF of 306 and 657 is 9.

20. **Assertion:** The line segment joining the points (x_1, y_1) and (x_2, y_2) is divided by the *y*-axis in the ratio *k* : 1; is called section formula.

Reason: The line segment joining the points P(–3, 2) and Q(5, 7) is divided by the *y*-axis in the ratio is 3 : 5.

Section - B

Section B consists of 5 questions of 2 marks each.

21. If α and β are the zeroes of the polynomial $x^2 + 6x + k$ such that $\alpha - \beta = 2$, find the value of k.

OR

Find the zeroes of the quadratic polynomial $7y^2 - \frac{11}{3}y - \frac{2}{3}$ and verify the relationship between the zeroes and the coefficients.

22. Evaluate: $\frac{\sin 90°}{\cos 45°} + \frac{1}{\text{cosec } 30°}$.

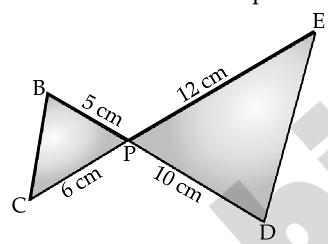
23. A cylinder, a cone and a hemisphere have same base and same height. Find the ratio of their volumes.

OR

A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, then find the circumference of the circle.

24. Prove that the lengths of two tangents drawn from an external point to a circle are equal.

25. In the figure given below, BD and CE intersect each other at the point P. Is ΔPBC ~ ΔPDE? Why?



Section - C

Section C consists of 6 questions of 3 marks each.

26. Given that, $\sqrt{5}$ is irrational, prove that $2\sqrt{5} - 3$ is an irrational number.

27. Represent the following pair of linear equations graphically and hence comment on the condition of consistency of this pair.

$$x - 5y = 6, 2x - 10y = 12.$$

OR

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

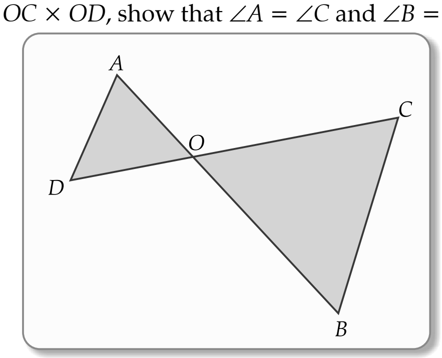
28. If $ad \neq bc$, then prove that the equation:
$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

29. A man in a boat moving away from a light house 100 m high takes 2 min to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute [use $\sqrt{3}$ = 1.732].

OR

Amit, standing on a horizontal plane, finds a bird flying at a distance of 200 m from him at an elevation of 30°. Deepak standing on the roof of a 50 m high building, the angle of elevation of the same bird to be 45°. Amit and Deepak are on opposite sides of the bird. Find the distance of the bird from Deepak.

30. In the given figure, $OA \times OB = OC \times OD$, show that $\angle A = \angle C$ and $\angle B = \angle D$.

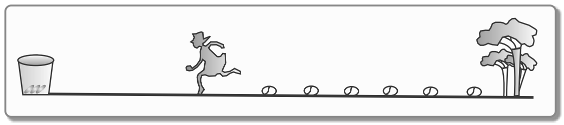


31. A game consists of tossing a coin three times and noting the outcome each time, if getting the same result in all the tosses is a success, find the probability of losing the game. AI

Section - D

Section D consists of 4 questions of 5 marks each.

32. In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and the other potatoes are placed 3 m apart in a straight line. There are 10 potatoes in the line (see figure).



Each competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket.

(i) What is the total distance the competitor has to run?

(ii) Which mathematical concept is used in the above problem?

OR

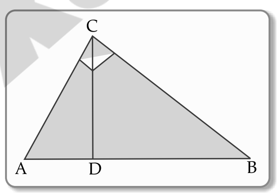
In a class test, the sum of Arun's marks in Hindi and English is 30. He had got 2 marks more in Hindi and 3 marks less in English, the product of the marks would have been 210. Find his marks in the two subjects.

33. Diagonals of a trapezium PQRS intersect each other at the point O, PQ || RS and $PQ = 3RS$.

Prove that $\Delta POS \sim \Delta QOR$. AI

OR

In the given figure, $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



34. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 g mass (use $\pi = 3.14$).

35. Find the mode of the following frequency distribution:

Class interval	f
25 – 35	7
35 – 45	31
45 – 55	33
55 – 65	17
65 – 75	11
75 – 85	1

Section - E

Case study based questions are compulsory.

36. Read the following text and answer the following questions:

Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 s and with each day of practice it takes him 2 s less. He wants to do in 31 s.

(i) Write the given situation in A.P. **1**

(ii) What is the minimum number of days, he needs to practice till his goal is achieved? **1**

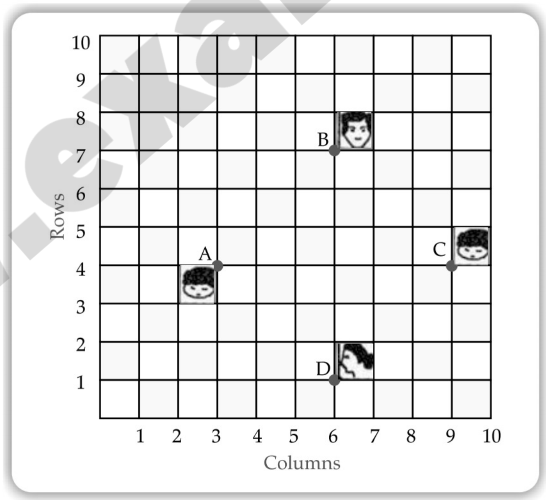
(iii) If nth term of an A.P. is given by
$a_n = 2n + 3$, then find the common difference of A.P. **2**

OR

Find the value of x, for which $2x$, $x + 10$, $3x + 2$ are three consecutive terms of an A.P.

37. Read the following text and answer the below questions:

In a room, 4 friends are seated at the points A, B, C and D as shown in figure. Reeta and Meeta walk into the room and after observing for a few minutes, Reeta asks Meeta.



(i) What is the position of A and D ? **1**

(ii) What is the distance between A and B ? **1**

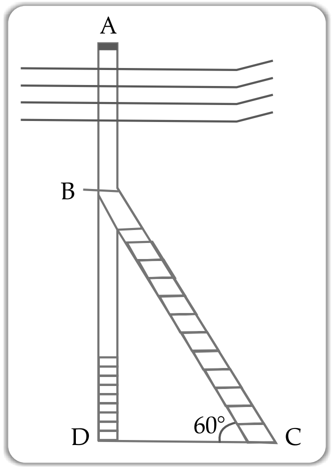
(iii) What is the equation of line CD ? **2**

OR

What is the middle position of B and C ?

38. Read the following and answer the following questions:

An electrician has to repair an electric fault on the pole of height 5 m. He needs to reach a point 1.3 m below the top of the pole to undertake the repair work (see figure).



(i) What is the length of BD ? **1**

(ii) What should be the length of ladder, when inclined at an angle of 60° to the horizontal ? **1**

(iii) How far from the foot of the pole should he place the foot of the ladder ? **2**

OR

If the horizontal angle is changed to 30°, then what should be the length of the ladder ?

■■■

Sample Question Paper-5

Mathematics Standard (041)

Class-X

SOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5, then least prime factor of $(a + b)$ is:

(A) 3 **(B)** 5 **(C)** 2 **(D)** 1

2. The smallest natural number by which 1200 should be multiplied, so that the square root of the product is a rational number is:

(A) 3 **(B)** 5 **(C)** 2 **(D)** 1

3. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are:

(A) both positive **(B)** both negative

(C) one positive and one negative **(D)** both equal

4. If the nth term of an A.P. –1, 4, 9, 14.... is 129. Find the value of n.

(A) 25 **(B)** 27 **(C)** 22 **(D)** 27

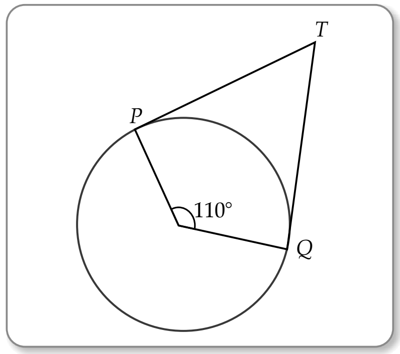
5. If the distance between the points $(4, p)$ and $(1, 0)$ is 5, then the value of p is:

(A) 4 only **(B)** ±4 **(C)** –4 only **(D)** 0

6. The distance between the points $(m, -n)$ and $(-m, n)$ is:

(A) $\sqrt{m^2+n^2}$ units **(B)** $m + n$ units **(C)** $2\sqrt{m^2+n^2}$ units **(D)** $\sqrt{2m^2+2n^2}$ units

7. In the given figure, if TP and TQ are the two tangents to a circle with centre O such that $\angle$ POQ = 110°, then $\angle$ PTQ is equal to:



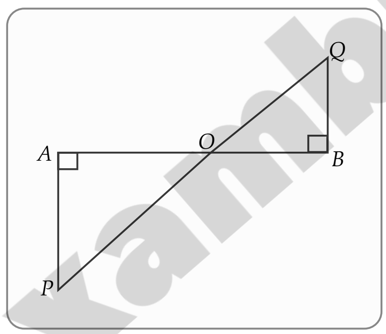
(A) 60° **(B)** 70° **(C)** 80° **(D)** 90°

8. If A(4, –2), B(7, –2) and C(7, 9) are the vertices of a ΔABC, then ΔABC is:

(A) equilateral triangle (B) isosceles triangle

(C) right angled triangle (D) isosceles right angled triangle

9. In the given figure, if $\angle A = 90°$, $\angle B = 90°$, $OB = 4.5$ cm, $OA = 6$ cm and $AP = 4$ cm, then QB is:



(A) 4 cm **(B)** 3 cm **(C)** 5 cm **(C)** 4.5 cm

10. The volume of a right circular cylinder with base radius 7 cm and height 10 cm is: $\left(\text{use } \pi = \frac{22}{7}\right)$

(A) 1540 cm^3 **(B)** 1500 cm^3 **(C)** 1478 cm^3 **(C)** 700 cm^3

11. If the sum of the circumferences of two circles with radii R_1 and R_2 is equal to the circumference of a circle of radius R, then:

(A) $R_1 + R_2 = R$

(B) $R_1 + R_2 > R$

(C) $R_1 + R_2 < R$

(D) Nothing definite can be said about the relation among R_1, R_2 and R

12. The diameter of a car wheel is 42 cm. The number of complete revolutions it will make in moving 132 km is:

(A) 10^4 (B) 10^5 **(C)** 10^6 **(D)** 10^3

13. Consider the following frequency distribution:

Class	0–5	6–11	12–17	18–23	24–29
Frequency	13	10	15	8	11

the upper limit of the median class is:

(A) 7 **(B)** 17.5 **(C)** 18 **(D)** 18.5

14. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the ground level such that $\tan \theta = \frac{15}{8}$, then what is the height of a kite from the ground?

(A) 75 m **(B)** 79.41 m **(C)** 80 m **(D)** 72.5 m

15. A bag contains 12 red and 8 blue marbles. A marble is drawn at random. The probability of drawing a blue marble is:

(A) $\frac{3}{5}$ **(B)** $\frac{2}{5}$ **(C)** $\frac{1}{5}$ **(D)** 1

16. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has:

(A) a unique solution **(B)** exactly two solutions

(C) infinitely many solutions **(D)** no solution

17. If $a \cot \theta + b \operatorname{cosec} \theta = p$ and $b \cot \theta + a \operatorname{cosec} \theta = q$, then $p^2 - q^2 =$

(A) $a^2 - b^2$ **(B)** $b^2 - a^2$ **(C)** $a^2 + b^2$ **(D)** $b - a$

18. The probability that a non-leap year selected at random will contain 53 Sundays is:

(A) $\frac{1}{7}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{3}{7}$ **(D)** $\frac{5}{7}$

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

19. **Assertion:** If a chord AB subtends an angle of 60° at the centre of a circle, then the angle between the tangents at A and B is also 60°.

Reason: The length of the tangent from an external point P on a circle with centre O is always less than OP.

20. **Assertion:** The positive root of $\sqrt{3x^2 + 6} = 9$ is 5.

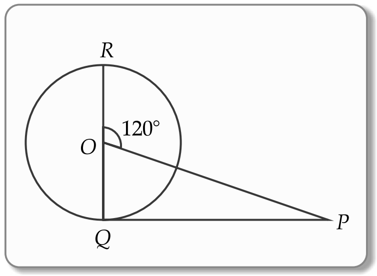
Reason: If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, then the value of a and b are 3 and -6.

Section - B

Section B consists of 5 questions of 2 marks each.

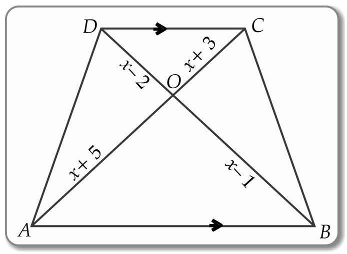
21. If α and β are two roots of the quadratic polynomial $2x^2 + 2x + 1$ then find the product and sum of the roots of the polynomial.

22. PQ is a tangent drawn from an external point P to a circle with centre O and QOR is the diameter of the circle. If $\angle POR = 120°$, then find the measure of $\angle OPQ$.

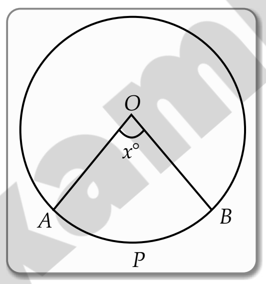


OR

In the given figure, if $AB \parallel DC$, find the value of x.



23. In given figure, O is the centre of a circle. If the area of the sector $OAPB$ is $\frac{5}{36}$ times the area of the circle, then find the value of x.



24. Find the area of the sector of a circle of radius 6 cm whose central angle is 30° (take $\pi = 3.14$).

OR

If the areas of three adjacent faces of a cuboid are X, Y, and Z respectively, then find the volume of cuboid.

25. Prove that:

$$\frac{\tan\theta}{1-\cot\theta}+\frac{\cot\theta}{1-\tan\theta}=1+\sec\theta\ \operatorname{cosec}\theta$$

OR

Prove that: $\dfrac{\cot\theta+\operatorname{cosec}\theta-1}{\cot\theta-\operatorname{cosec}\theta+1}=\dfrac{1+\cos\theta}{\sin\theta}$.

Section - C

Section C consists of 6 questions of 3 marks each.

26. Three sets of English, Hindi and Sociology books dealing with cleanliness have to be stacked in such a way that all the books are stored topicwise and the height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and the number of Sociology books is 336.

(i) Assuming that the books are of the same thickness, determine the number of stacks of English, Hindi and Sociology books.

(ii) Which mathematical concept is used in the problem?

27. Find c if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions?

28. Rinku and his family were going for summer vacation to a hill station. They decide to travel in a train. The train was running late by its expected time. The train covers a distance of 300 km at a uniform speed. If the speed of the train is increased by 5 km/h, it takes 2 h less in journey. Find the original speed of the train.

OR

If the sum of first four terms of an A.P. is 40 and that of first 14 terms is 280. Find the sum of its first n terms.

29. Prove that: $(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

30. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

OR

Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that $AP \times PC = BP \times DP$.

31. Peter and Rina were playing the game. Peter throws two different dice together and finds the product of the two numbers obtained. Rina throws a die and find a square of a number. Who has the better chance to get the number 25.

Section - D

Section D consists of 4 questions of 5 marks each.

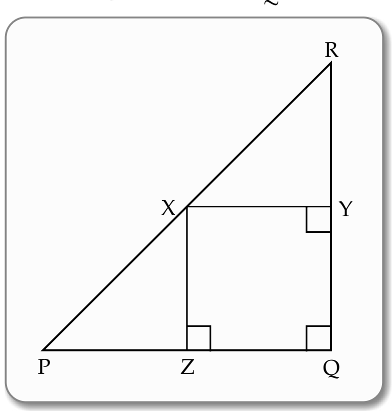
32. A boat goes 30 km upstream and 44 km downstream in 10 h. In 13 h, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

OR

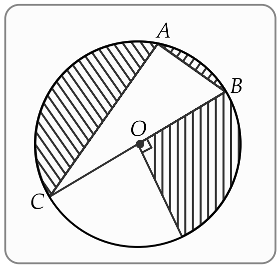
Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find the height of the poles and the distances of the point from the poles.

33. ΔPQR is right angled at Q. $QX \perp PR$, $XY \perp RQ$ and $XZ \perp PQ$ are drawn. Prove that:

$$XZ^2 = PZ \times ZQ.$$



34. In the given figure, O is the centre of the circle with $AC = 24$ cm, $AB = 7$ cm and $\angle BOD = 90°$. Find the area of the shaded region.



OR

A right cylindrical container of radius 6 cm and height 15 cm is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.

35. Find the mode of the following frequency distribution:

Class interval	25 - 30	30 - 35	35 - 40	40 - 45	45 - 50	50 - 55
Frequency	25	34	50	42	38	14

Section - E

Case study based questions are compulsory.

36. **A** garden consists of 135 rose plants planted in certain number of columns. There are another set of 225 marigold plants, which is to be planted in the same number of columns.

Read carefully the above paragraph and answer the following questions:

(i) What is the maximum number of columns in which they can be planted ? **2**

OR

Find the total number of plants

(ii) Find the sum of exponents of the prime factors of the maximum number of columns in which they can be planted. **1**

(iii) What is total numbers of row in which they can be planted **1**

37. Places A and B are 100 km apart on a highway. One car starts from A and another car starts from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 h. If they travel towards each other, they meet in 1 h.

(i) Assuming that the speed of first car and second car be u km/h and v km/h, respectively. **2**
What is the relative speed of both cars while they are travelling in the same direction ?

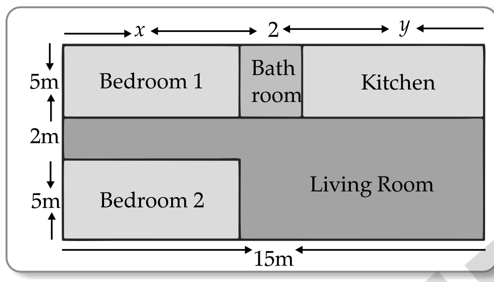
OR

What is the relative speed of both cars while they are travelling towards each other ?

(ii) What is the actual speed of first car? **1**

(iii) What is the actual speed of other car? **1**

38. Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95 sq.m.



Based on the above information, answer the following questions:

(i) Form the pair of linear equations in two variables from this situation. **2**

OR

Find the length of the outer boundary of the layout.

(ii) Find the area of each bedroom and kitchen in the layout. **1**

(iii) Find the area of living room in the layout. **1**

■■■

Self Assessment Paper-1

Mathematics Standard (041)

Class-X

UNSOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. What is the HCF of 3000 and 525?

(A) 75 **(B)** 25 **(C)** 55 **(D)** 35

2. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. The minimum distance each should walk, so that each can cover the same distance in complete steps is:

(A) 2250 cm **(B)** 2520 cm **(C)** 2550 cm **(D)** 2050 cm

3. For what value of k, the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other ?

(A) $k = 3$ **(B)** $k = 10$ **(C)** $k = \frac{10}{3}$ **(D)** $k = \frac{1}{3}$

4. If the nth term of A.P. 12, 15, 18....... 99 is 99. Then value of n is equal to

(A) 20 **(B)** 40 **(C)** 30 **(D)** 35

5. The co-ordinates of a point A, where AB is the diameter of the circle with centre (–2, 2) and B is the point with coordinates (3, 4) is:

(A) (7, 0) **(B)** (– 7, 0) **(C)** (5, 0) **(D)** (– 5, 0)

6. The centre of a circle whose end points of a diameter are (– 6, 3) and (6, 4) is:

(A) (8, –1) **(B)** (4, 7) **(C)** $\left(0, \frac{7}{2}\right)$ **(D)** $\left(4, \frac{7}{2}\right)$

7. The co-ordinates of the point which is reflection of point (–3, 5) in x-axis are:

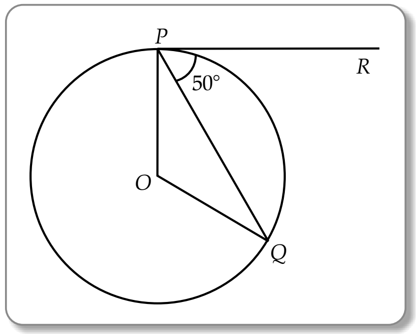
(A) (3, 5) **(B)** (3, –5) **(C)** (–3, –5) **(D)** (–3, 5)

8. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3DE. Then, the two triangles are:

(A) congruent but not similar **(B)** similar but not congruent

(C) neither congruent nor similar **(D)** congruent as well as similar

9. In the given figure, 'O' is the centre of circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is equal to:



(A) 100° **(B)** 80° **(C)** 90° **(D)** 75°

10. $(\sec A + \tan A)(1 - \sin A) =$

(A) $\sec A$ **(B)** $\sin A$ **(C)** $\operatorname{cosec} A$ **(D)** $\cos A$

11. Area of the largest triangle that can be inscribed in a semi-circle of radius 'r' units is:

(A) r^2 sq. units **(B)** $\frac{1}{2}r^2$ sq. units **(C)** $2r^2$ sq. units **(D)** $\sqrt{2}\,r^2$ sq. units

12. In a right circular cone, the cross-section made by a plane parallel to the base is a:

(A) circle **(B)** frustum of a cone **(C)** sphere **(D)** hemisphere

13. If P(A) denotes the probability of an event A, then:

(A) $P(A) < 0$ **(B)** $P(A) > 1$ **(C)** $0 \le P(A) \le 1$ **(D)** $-1 \le P(A) \le 1$

14. If the perimeter of a circle is half to that of a square, then the ratio of the area of the circle to the area of the square is:

(A) 22 : 7 **(B)** 11 : 7 **(C)** 7 : 11 **(D)** 7 : 22

15. For the following distribution:

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

the sum of lower limits of median class and modal class is:

(A) 15 **(B)** 25 **(C)** 30 **(D)** 35

16. The quadratic polynomial whose sum and product of the zeroes are $\frac{21}{8}$ and $\frac{5}{16}$ respectively is:

(A) $\frac{1}{4}(4x^2 - 12x + 5)$ **(B)** $\frac{1}{16}(16x^2 - 42x + 5)$ **(C)** $\frac{1}{8}(x^2 - 42x + 5)$ **(D)** None of these

17. The exponent of 2 in the prime factorisation of 484 is:

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4

18. A vessel having 30 m^3 of water is emptied through two openings, one small and the other large. Water flows out through the smaller opening at the rate of U m^3/h and through the larger one at the rate V m^3/h. Given that, 3U + 2V = 70 and that the vessel gets fully emptied in 1 hour, what is V?

(A) 10 m^3/h **(B)** 20 m^3/h **(C)** 30 m^3/h **(D)** 50 m^3/h

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(C) Assertion (A) is true but reason (R) is false.
(D) Assertion (A) is false but reason (R) is true.

19. **Assertion (A):** If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.

Reason (R): If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

20. **Assertion (A):** If n^{th} term of an A.P. is $(2n + 1)$, then the sum of its first three terms is 15.

Reason (R): The sum of first 16 terms of the A.P. 10, 6, 2, ... is – 320.

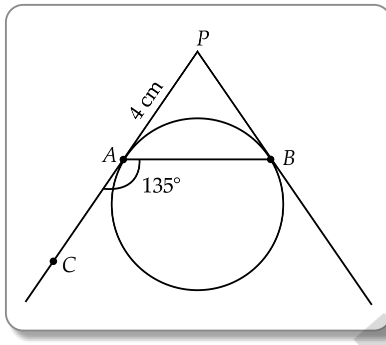
Section - B

Section B consists of 5 questions of 2 marks each.

21. Sumit is 3 times as old as his son. Five years later, he shall be two and a half times as old as his son. How old is Sumit at present?

22. Point A lies on the line segment XY joining $X(6, -6)$ and $Y(-4, -1)$ in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y + 1) = 0$, find the value of k.

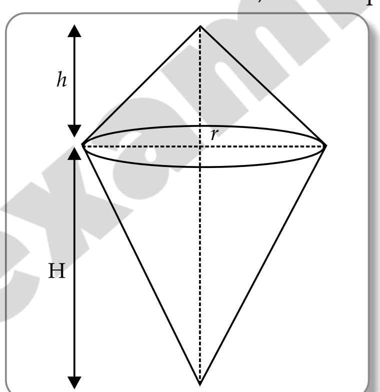
23. In the given figure, PA and PB are tangents to a circle from an external point P such that $PA = 4$ cm and $\angle BAC = 135°$. Find the length of chord AB.



24. Two cubes each of volume 8 cm^3 are joined end to end, then what is the surface area of resulting cuboid.

OR

A solid metallic object is shaped like a double cone as shown in figure. Radius of base of both cones is same but their heights are different. If this cone is immersed in water, find the quantity of water it will displace.



25. Find the value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$.

OR

If $\sin A + \cos B = 1$, $A = 30°$ and B is an acute angle, then find the value of B.

Section - C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{2}$ is an irrational number.

27. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

28. If m^{th} term of A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, find the sum of first mn terms.

OR

How many terms of an A.P. 9, 17, 25, ... must be taken to give a sum of 636 ?

29. Find A and B if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$ and $\cos(A + 4B) = 0$, where A and B are acute angles.

30. Prove that the parallelogram circumscribing a circle is a rhombus.

OR

If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(BC + CA + AB)$.

31. A child has a die whose six faces show the letters as shown below:

A	A	B	C	C	C

The die is thrown once. What is the probability of getting (i) A, (ii) C ?

Section - D

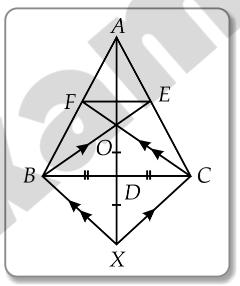
Section D consists of 4 questions of 5 marks each.

32. A journey of 192 km from a town A to town B takes 2 hours more by an ordinary passenger train than a super fast train. If the speed of the faster train is 16 km/h more than the passenger train, find the speed of the faster and the passenger train.

OR

The total cost of a certain length of a piece of cloth is ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less, the cost of the piece would have remained unchanged. How long is the piece and what is its original rate per metre ?

33. In ΔABC, AD is a median and O is any point on AD. BO and CO on producing meet AC and AB at E and F respectively. Now AD is produced to X such that $OD = DX$ as shown in figure.



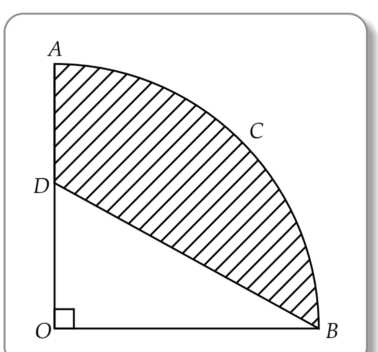
Prove that:

(i) $EF \parallel BC$ **(ii)** $AO : AX = AF : AB$

34. The dimensions of a solid iron cuboid are 4.4 m × 2.6 m × 1.0 m. It is melted and recast into a hollow cylindrical pipe of 30 cm inner radius and thickness 5 cm. Find the length of the pipe.

OR

In the given figure, OACB is a quadrant of a circle with centre O and radius 3·5 cm. If OD = 2 cm, find the area of the shaded region.



35. Find the unknown values in the following table:

Class Interval	Frequency	Cumulative Frequency
0 – 10	5	5
10 – 20	7	x_1
20 – 30	x_2	18
30 – 40	5	x_3
40 – 50	x_4	30

Section - E

Case study based questions are compulsory.

36. Read the following text and answer the following questions on the basis of the same:

Raj and Ajay are very close friends. Both the families decide to go to Ranikhet by their own cars. Raj's car travels at a speed of x km/h while Ajay's car travels 5 km/h faster than Raj's car. Raj took 4 hours more than Ajay to complete the journey of 400 km.

(i) What will be the distance covered by Ajay's car in two hours? **1**

(ii) What is the quadratic equation for the speed of Raj's car? **1**

(iii) How much time taken by Ajay to travel 400 km? **2**

OR

What is the speed of Ajay's car .

37. The weights (in kg) of 50 wrestlers are recorded in the following table:

Weight (in kg)	100 – 110	110 – 120	120 – 130	130 – 140	140 – 150
No. of Wrestlers	4	14	21	8	3

(i) What is the upper limit of modal class. **1**

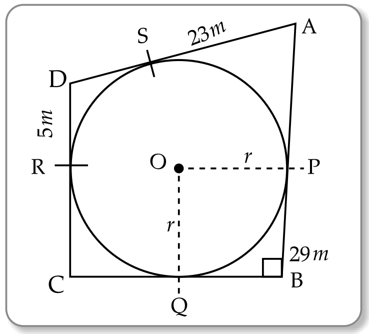
(ii) What is the mode class frequency of the given data. **1**

(iii) How many wrestlers weights have more than 120 kg weight ? **2**

OR

What is the class mark for class 130 – 140 ?

38. ABCD is a playground. Inside the playground a circular track is present such that it touches AB at point P, BC at Q, CD at R and DA at S.



See the above figure and give answer of the following questions:

(i) Calculate the value of DS If $DR = 5$ m. **1**

(ii) Find the length of AS. **1**

(iii) Find the length of PB is. **2**

OR

What is the diameter of given circle?

Self Assessment Paper-2

Mathematics Standard (041)

Class-X

UNSOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*

(ii) *Section A has 20 MCQs carrying 1 mark each*

(iii) *Section B has 5 questions carrying 02 marks each.*

(iv) *Section C has 6 questions carrying 03 marks each.*

(v) *Section D has 4 questions carrying 05 marks each.*

(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*

(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*

(viii) *Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. The LCM of 6, 72 and 120 is:

(A) 360 **(B)** 6 **(C)** 51840 **(D)** 720

2. If one of the zeroes of a quadratic polynomial of the form $x^2 + ax + b$ is the negative of the other, then it

(a) has no linear term and the constant term is negative.

(b) has no linear term and the constant term is positive.

(c) can have a linear term but the constant term is negative.

(d) can have a linear term but the constant term is positive.

3. If n is an even prime number, then $2(7n + 8n)$ ends with:

(A) 1 **(B)** 4 **(C)** 2 **(D)** 6

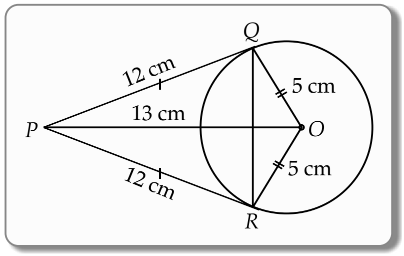
4. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is:

(A) 2 **(B)** -2 **(C)** $\frac{1}{2}$ **(D)** $-\frac{1}{2}$

5. The distance of the point $(-4, -7)$ from the y-axis is:

(A) 14 units **(B)** 7 units **(C)** 4 units **(D)** 8 units

6. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is:



(A) 60 cm^2 **(B)** 65 cm^2 **(C)** 30 cm^2 **(D)** 32.5 cm^2

7. The area of the square that can be inscribed in a circle of radius 8 cm is:

(A) 256 cm^2 **(B)** 128 cm^2 **(C)** $64\sqrt{2}$ cm^2 **(D)** 64 cm^2

8. Volumes of two spheres are in the ratio 64 :27. The ratio of their surface areas is:

(A) 3 : 4 **(B)** 4 : 3 **(C)** 9 : 16 **(D)** 16 : 9

9. When a die is thrown, the probability of getting an odd number less than 3 is:

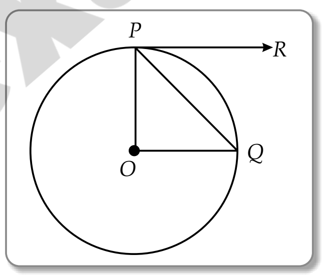
(A) $\frac{1}{6}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{1}{2}$ **(D)** 0

10. The base BC of an equilateral ΔABC lies on the Y-axis. The co-ordinates of C are (0, –3). If the origin is the mid-point of the base BC, what are the co-ordinates of A and B?

(A) $A\left(\sqrt{3}, 0\right)$, $B(0, 3)$ **(B)** $A(\pm 3\sqrt{3}, 0)$, $B(3, 0)$

(C) $A(\pm 3\sqrt{3}, 0)$, $B(0, 3)$ **(D)** $A(-\sqrt{3}, 0)$, $B(3, 0)$

11. If O is centre of a circle, PQ is a chord and the tangent PR at P makes an angle of 50° with PQ, then $\angle POQ$ is:



(A) 80° **(B)** 50° **(C)** 100° **(D)** 40°

12. The distance between the points ($a \cos \theta + b \sin \theta$, 0) and (0, $a \sin \theta - b \cos \theta$), is:

(A) $a^2 + b^2$ units **(B)** $a^2 - b^2$ units **(C)** $\sqrt{a^2 + b^2}$ units **(D)** $\sqrt{a^2 - b^2}$ units

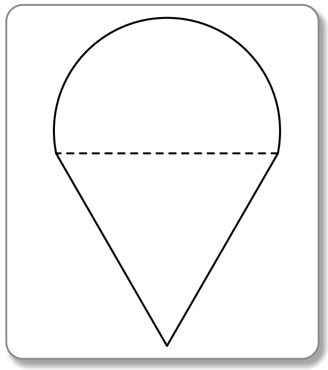
13. The value of $\frac{1}{\tan\theta + \cot\theta}$ =

(A) $\cos \theta \sin \theta$ **(B)** $\sec \theta \sin \theta$ **(C)** $\tan \theta \cot \theta$ **(D)** $\sec \theta \operatorname{cosec} \theta$

14. A plumbline (Sahul) is the combination of:

(A) a cone and a cylinder **(B)** a hemisphere and a cone

(C) frustum of a cone and a cylinder **(D)** sphere and cylinder



15. For the following distribution:

Marks	Number of students
Below 10	3
Below 20	12
Below 30	27
Below 40	57
Below 50	75
Below 60	80

the modal class is:

(A) 10 – 20 **(B)** 20 – 30 **(C)** 30 – 40 **(D)** 50 – 60

16. The common difference of the A.P. $\frac{1}{3q}, \frac{1-6q}{3q}, \frac{1-12q}{3q}$,...... is:

(A) – 1 **(B)** – 2 **(C)** 1 **(D)** 2

17. If the height of a vertical pole is $\sqrt{3}$ times the length of its shadow on the ground, then the angle of elevation of the Sun at that time is:

(A) 30° **(B)** 60° **(C)** 45° **(D)** 75°

18. A right circular cylinder of radius r cm and height h cm (where $h > 2r$) just encloses of sphere of diameter:

(A) r cm **(B)** $2r$ cm **(C)** h cm **(D)** $2h$ cm

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

19. **Assertion (A):** The ordinate of a point A on y-axis is 5 and B has coordinates (–3, 1). Then the length of AB is 5 units.

Reason (R): The point A(2, 7) lies on the perpendicular bisector of line segment joining the points P(6, 5) and Q(0, –4).

20. **Assertion (A):** If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, then triangles will be similar.

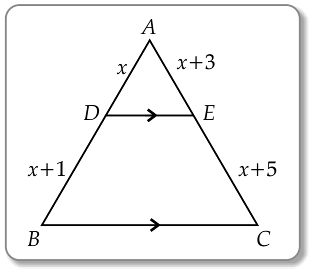
Reason (R): If the ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$, then the ratio of their areas is $\frac{6}{5}$.

Section - B

Section B consists of 5 questions of 2 marks each.

21. A takes 6 days less than B to do a work. If both A and B working together can do it in 4 days, how many days will B take to finish it?

22. In ΔABC, $DE \parallel BC$, find the value of x.



23. The diameter of a wheel is 1.26 m. What is the distance covered in 500 revolutions?

24. In what ratio does the point $\left(\frac{24}{11}, y\right)$ divide the line segment joining the points P (2, – 2) and Q (3, 7)? Also find the value of y.

OR

Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points (8, – 9) and (2, 1). Also, find the co-ordinates of the point of division.

25. If $4 \cos \theta = 11 \sin \theta$, find the value of $\frac{11 \cos \theta - 7 \sin \theta}{11 \cos \theta + 7 \sin \theta}$.

OR

The angle of elevation of the top of a chimney from the foot of a tower is 60° and the angle of depression of the foot of the chimney from the top of the tower is 30°. If the height of tower is 40 m, find the height of smoke emitting chimney. According to pollution control norms, the minimum height of a smoke emitting chimney should be 100 m.

Section - C

Section C consists of 6 questions of 3 marks each.

26. Prove that $2 + 5\sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

27. Solve the following pair of equations:

$$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \text{ and } \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1.$$

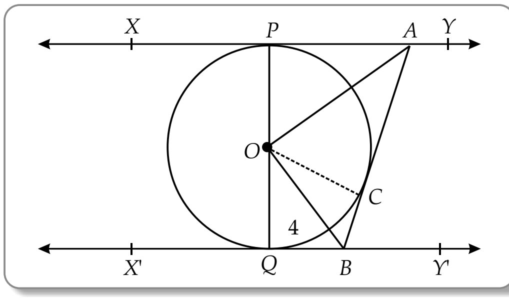
28. A part of monthly hostel charge is fixed and the remaining depends on the number of days one has taken food in the mess. When Swati takes food for 20 days, she has to pay ₹ 3,000 as hostel charges whereas Mansi who takes food for 25 days has to pay ₹ 3,500 as hostel charges. Find the fixed charges and the cost of food per day.

OR

If the pth term of an A.P. is q and q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.

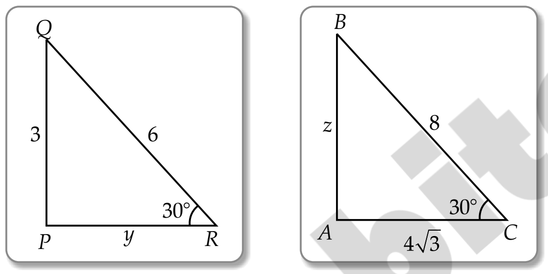
29. If $\sec \theta = x + \frac{1}{4x}$, $x \neq 0$, find $(\sec \theta + \tan \theta)$.

30. In the given figure, XY and $X'Y'$ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C, is intersecting XY at A and $X'Y'$ at B. Prove that $\angle AOB = 90°$.



OR

In the given figure, $\Delta ABC \sim \Delta PQR$. Find the value of $y + z$.



31. Ruhi's father organised a magic show in her birthday party. Various magics by magician are shown using coloured balls. Magician has a bag. That bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag.

Section - D

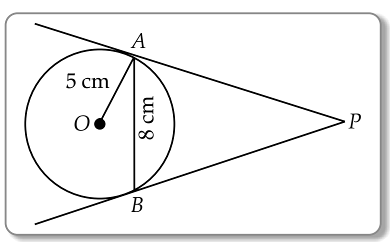
Section D consists of 4 questions of 5 marks each.

32. The angle of depression of two ships from an aeroplane flying at the height of 7500 m are 30° and 45°. If both the ships are in the same line and one ship is exactly behind the other, find the distance between the ships.

OR

From the top of a tower, 100 m high, a man observes two cars on the opposite sides of the tower and in same straight line with its base, with angles of depression 30° and 45°. Find the distance between the cars. [Take $\sqrt{3} = 1.732$]

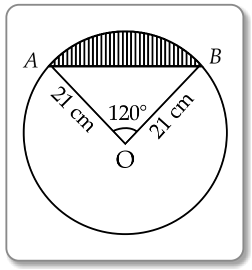
33. In figure AB is a chord of length 8 cm of a circle of radius 5 cm. The tangents to the circle at A and B intersect at P. Find the length of AP.



34. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius on its circular face. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

OR

Find the area of the segment shown in figure, if radius of the circle is 21 cm and $\angle AOB = 120°$ $\left(\text{Use } \pi = \frac{22}{7}\right)$



35. The median of the following data is 525. Find the values of x and y if the total frequency is 100.

Class Interval	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 900	900 – 1000
Frequency	2	5	x	12	17	20	y	9	7	4

Section - E

Case study based questions are compulsory.

36. Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of ₹ 1,18,000 by paying every month starting with the first instalment of ₹ 1000. If he increases the instalment by ₹ 100 every month, answer the following:

(i) What is the amount paid by him in 30th instalment? **1**

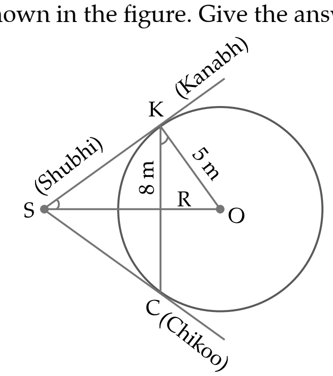
(ii) The amount paid by him in 30 instalments is: **1**

(iiii) What is the amount paid in last instalment, if total instalments are 40 **2**

OR

What is the ratio of the 1st instalment to the last instalment.

37. There is a circular filed of radius 5 m. Kanabh, Chikoo and Shubhi are playing with ball, in which Kanabh and Chikoo are standing on the boundary of the circle. The distance between Kanabh and Chikoo is 8 m. From Shubhi point S, two tangents are drawn as shown in the figure. Give the answer of the following questions.



(i) What is the relation between the lengths of SK and SC ? **1**

(ii) What is the length (distance) of OR? **2**

OR

The sum of angles SKR and OKR.

(iii) Find the distance between Kanabh and Shubhi. **1**

38. To enhance the reading skills of grade X students, the school nominates you and two of your friends to set up a class library. There are two sections- section A and section B of grade X. There are 32 students in section A and 36 students in section B.

(i) What is the minimum number of books you will acquire for the class library, so that they can be distributed equally among students of Section A or Section B? **1**

(ii) If the product of two positive integers is equal to the product of their HCF and LCM is true then, find HCF (32, 36). **2**

OR

Expressed 36 as a product of its primes.

(iii) If p and q are positive integers such that $p = ab^2$ and $q = a^2b$, where a, b are prime numbers, then the LCM (p, q). **1**

Self Assessment Paper-3

Mathematics Standard (041)

Class- X

UNSOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take π = 22/7 wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. HCF of 96 and 404 is:
(A) 4 **(B)** 32 **(C)** 96 **(D)** 404

2. Product of two numbers is equal to
(A) $\frac{\text{LCM}}{\text{HCF}}$ **(B)** $\frac{\text{HCF}}{\text{LCM}}$ **(C)** LCM × HCF **(D)** $(\text{LCM} \times \text{HCF})^2$

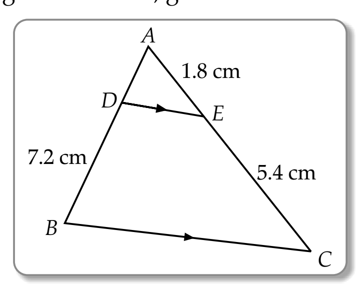
3. If a pair of linear equations is consistent, then the lines will be:
(A) parallel **(B)** always coincident
(C) intersecting or coincident **(D)** never intersecting

4. The sum of first 8 multiplies of 3 is:
(A) 110 **(B)** 108 **(C)** 112 **(D)** 120

5. If the point C (k, 4) divides the join of points A(2, 6) and B (5, 1) in the ratio 2 : 3, then the value of k is:
(A) 16 **(B)** $\frac{28}{5}$ **(C)** $\frac{16}{5}$ **(D)** $\frac{8}{5}$

6. If the angle between two tangents drawn from an external point P to a circle of radius 'a' and centre O, is 60°, then length of OP is:
(A) a **(B)** $2a$ **(C)** $3a$ **(D)** $4a$

7. In the given figure, DE || BC. The length of side AD, given that AE = 1.8 cm, BD = 7.2 cm and CE = 5.4 cm is:



(A) 3.6 cm **(B)** 2.4 cm **(C)** 4.4 cm **(D)** 6.8 cm

8. The co-ordinates of the point which is reflection of point (– 4, 7) is x-axis; is:

(A) (4, – 7) **(B)** (4, 7) **(C)** (– 4, – 7) **(D)** (– 4, 7)

9. $5\sec^2 A - 5\tan^2 A$ is equal to

(A) 1 **(B)** 5 **(C)** 8 **(D)** 0

10. The area of the circle that can be inscribed in a square of side 6 cm is:

(A) 36π cm^2 **(B)** 18π cm^2 **(C)** 12π cm^2 **(D)** 9π cm^2

11. The radius of solid sphere is r cm. It is divided into two equal parts. Find the whole surface of two parts.

(A) $2\pi r^2$ **(B)** $3\pi r^2$ **(C)** $6\pi r^2$ **(D)** πr^2

12. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is:

(A) 5 units **(B)** 3 units **(C)** $\sqrt{34}$ units **(D)** 4 units

13. In the formula $\bar{x} = a + \frac{\sum x_i d_i}{\sum f_i}$ for finding the mean of grouped data d_i's are the deviations from a of:

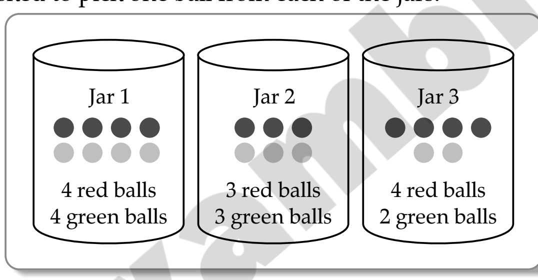
(A) lower limits of the classes **(B)** upper limits of the classes

(C) mid-points of the classes **(D)** frequencies of the class marks

14. Two cubes have their volumes in the ratio 1 : 64. The ratio of their surface areas is:

(A) 1 : 9 **(B)** 1 : 4 **(C)** 1 : 8 **(D)** 1 : 16

15. Romy is blind folded and asked to pick one ball from each of the jars.



The chance of Romy picking a red ball is same in:

(A) jars 2 and 3 **(B)** jars 1 and 3 **(C)** jars 1 and 2 **(D)** all the three jars

16. Which of the following equations has not 2 as a root?

(A) $x^2 - 4x + 5 = 0$ **(B)** $x^2 + 3x - 12 = 0$ **(C)** $2x^2 - 7x + 6 = 0$ **(D)** $3x^2 - 6x - 2 = 0$

17. The 21st term of the A.P. $-4\frac{1}{2}, -3, -1\frac{1}{2}, ...$ is:

(A) $\frac{51}{2}$ **(B)** $\frac{51}{4}$ **(C)** $\frac{51}{5}$ **(D)** $\frac{51}{8}$

18. $\sin^2 30° + \tan^2 45°$ is equal to:

(A) $\frac{1}{4}$ **(B)** $\frac{5}{4}$ **(C)** 1 **(D)** $\frac{3}{4}$

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

19. **Assertion (A):** The length of the tangent drawn from a point 8 cm away from the centre of circle of radius 6 cm is $2\sqrt{7}$ cm.

Reason (R): If the angle between two radii of a circle is 130°, then the angle between the tangents at the end points of radii at their point of intersection is 50°.

20. **Assertion (A):** Cot A is the product of Cot and A.

Reason (R): The value of sinθ increases as θ increases.

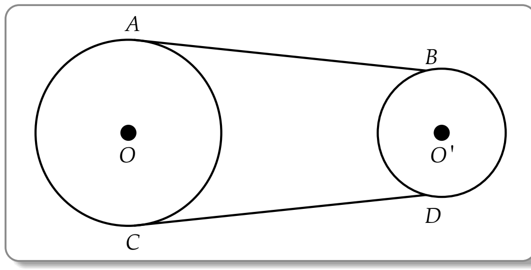
Section - B

Section B consists of 5 questions of 2 marks each.

21. Ashima plays a game with her friend. She asked to assume any two digits number then follow this instruction as seven times a two digit number is equal to four times the number obtained by reversing the order of its digits. If the difference of the digits is 3, determine the number that she has assumed.

22. Find the ratio in which the y-axis divides the line segment joining the points (– 1, – 4) and (5, – 6). Also, find the co-ordinates of the points of intersection.

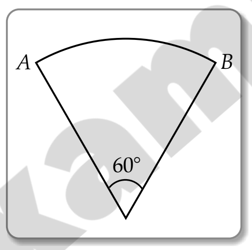
23. In the figure, AB and CD are common tangents to two circles of unequal radii. Prove that $AB = CD$.



24. A solid metallic cuboid of dimensions 9 m × 8 m × 2 m is melted and recast into solid cubes of edge 2 m. Find the number of cubes so formed.

OR

In the given figure, find the perimeter of the sector of a circle with radius 10.5 cm and of angle 60°. $\left(\text{Take } \pi = \dfrac{22}{7}\right)$.



25. If $\tan\theta = \dfrac{1}{\sqrt{5}}$,

(i) Evaluate: $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$ **(ii)** Verify the identity: $\sin^2\theta + \cos^2\theta = 1$.

OR

A vertical tower stands on horizontal plane and is surmounted by a vertical flag-staff of height 6 m. The angles at a point on the bottom and top of the flag-staff with the ground are 30° and 45° respectively. Find the height of the tower.

(Take $\sqrt{3}$ = 1.73)

Section - C

Section C consists of 6 questions of 3 marks each.

26. In which quadrant lies the point which divides the line segment joining the points (8, – 9) and (2, 3) in ratio 1 : 2 internally ?

27. If α and β are the zeroes of polynomial $P(x) = 3x^2 + 2x + 1$, find the polynomial whose zeroes are $\dfrac{1-\alpha}{1+\alpha}$ and $\dfrac{1-\beta}{1+\beta}$.

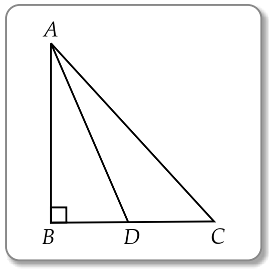
28. The ratio of the length of a vertical rod and the length of its shadow is 1 : $\sqrt{3}$. Find the angle of elevation of the sun at that moment ?

OR

During the Sunset, shadow of the tower increases. The shadow of a tower at is three times as long as its shadow when the angle of elevation of the Sun is 60°. Find the angle of elevation of the Sun at the time of the longer shadow.

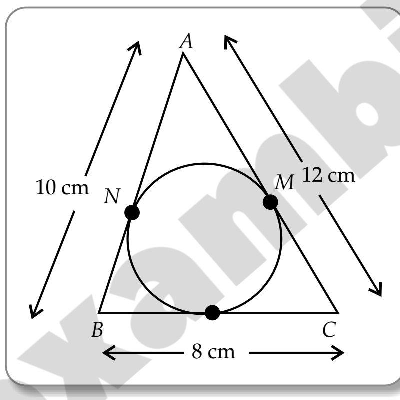
29. Prove that $\dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\operatorname{cosec}^2 A}{\sec^2 A - \operatorname{cosec}^2 A} = \dfrac{1}{1 - 2\cos^2 A}$

30. In the given figure, ABC is a right angled triangle at $\angle B = 90°$. D is the mid-point of BC. Show that $AC^2 = AD^2 + 3CD^2$.



OR

In the given figure, a circle is inscribed in a ΔABC having sides BC = 8 cm, AB = 10 cm and AC = 12 cm. Find the length BL, CM and AN.



31. A die is thrown once. Find the probability of getting a number which **(i)** is a prime number, **(ii)** lies between 2 and 6.

Section - D

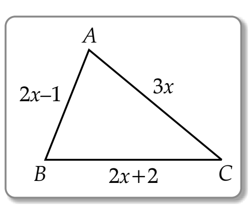
Section D consists of 4 questions of 5 marks each.

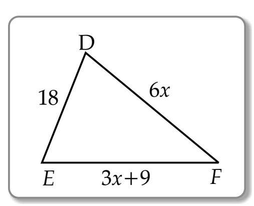
32. Speed of a boat in still water is 15 km/hr. It goes 30 km upstream and returns back at the same point in 4 hours 30 minutes. Find the speed of the stream.

OR

Two taps running together can fill a tank in $3\frac{1}{13}$ hours. If one tap takes 3 hours more than the other to fill the tank, than how much time will each tap take to fill the tank?

33. In the given figure, if $\Delta ABC \sim DEF$ and their sides of the given figure lengths (in cm) are marked along them, then find the lengths of sides of each triangle.





34. In a rain water harvesting system, the rain water from a roof 22 m $\times$ 20 m drains into a cylindrical tank having diameter of base 2 m and height 3.5 m. If the tank is full, find the rainfall in cm. Write your views on water conservation.

OR

Rohan wants to renovate his room. He calls an architect for this work to measure the room. The length, breadth and height of a room are 8m 50 cm; 6m 25 cm and 4 m 75 cm respectively. He wants to put the longest rod that can measure the dimensions of the room exactly.

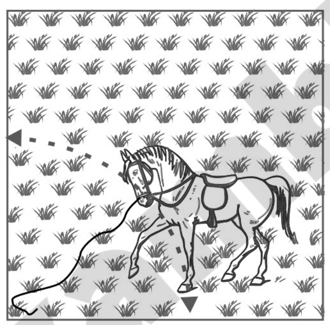
35. The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

Daily expenditure (in ₹)	100 – 150	150 – 200	200 – 250	250 – 300	300 – 350
Number of households	4	5	12	2	2

Section - E

Case study based questions are compulsory.

36. A horse is tied to a peg at one corner of a square shaped grass field of sides 15 m by means of a 5 m long rope (see the given figure).



(i) What is the area of the grass field? 1

(ii) What would be the area of part of the field in which the horse can graze. 1

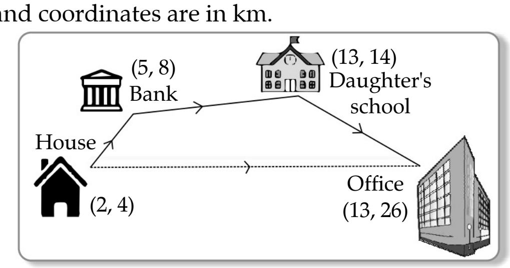
(iii) What would be the grazing area if the rope were 10 m long instead of 5 m. 2

OR

Calculate the increase area in grazing if the rope were 10m long instead of 5m.

37. Ayush Starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office.

(Assume that all distances covered are in straight lines). If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.



(i) What is the distance between house and bank? 1

(ii) What is the distance between Daughter's School and bank? 2

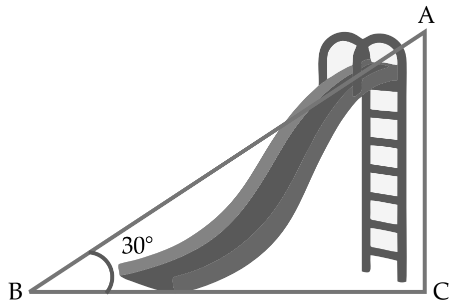
OR

What is the distance between house and office?

(iii) What is the total distance travelled by Ayush to reach the office? 1

38. Authority wants to construct a slide in a city park for children. The slide was to be constructed for children below the age of 12 years. Authority prefers the top of the slide at a height of 4 m above the ground and inclined at an angle of 30° to the ground.

Based on the following figure related to the slide answer the questions:



(i) The distance of AB is: **1**

(ii) The value of $\sin^2 30° + \cos^2 60°$ is: **2**

OR

If $\cos A = \frac{1}{2}$, then the value of $12 \cot^2 A - 2$ is:

(iii) In the given figure, the value of $(\sin C \times \cos A)$ is: **1**

■■■

Self Assessment Paper-4

Mathematics Standard (041)

Class-X

UNSOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. If $p = 2^4 \times 3^3 \times 5^4 \times 9$, then the number of consecutive zeroes in p, where p is a natural number; is:

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

2. $5 - \sqrt{3}$ is a

(A) composite number **(B)** rational number **(C)** irrational number **(D)** none of these

3. If the product of zeroes of the polynomial $f(x) = x^2 - 8x + k$ is 20, then the value of k is:

(A) 20 **(B)** 30 **(C)** 40 **(D)** 60

4. The pair of equations $y = 0$ and $y = -7$ has:

(A) one solution **(B)** two solutions **(C)** infinitely many solutions **(D)** no solution

5. A line intersects the y-axis and x-axis at the points P and Q respectively. If (2, – 5) is the mid-point of PQ, then the coordinates of P and Q are:

(A) P (0, – 10), Q(4, 0) **(B)** P (– 10, 0), Q (0, 4) **(C)** P (0, – 10), Q (0, 4) **(D)** P (– 10, 0), Q (4, 0)

6. If the points A(4, 3) and B(x, 5) are on the circle with centre O(2, 3), then the value of x is:

(A) 3 **(B)** 4 **(C)** 1 **(D)** 2

7. A circle touches all the four sides of a quadrilateral ABCD. Then AB + CD is equal to

(A) AD – BC **(B)** BC – AD **(C)** AD + BC **(D)** $(AD + BC)^2$

8. ABC is an isosceles triangle right angled at C with AC = 4 cm. The length of AB is:

(A) $4\sqrt{2}$ cm **(B)** $3\sqrt{2}$ cm **(C)** 5 cm **(D)** $5\sqrt{2}$ cm

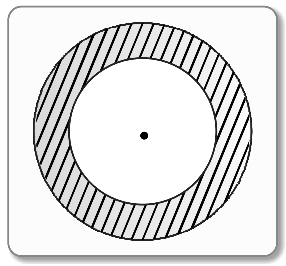
9. If a tower 30 m high, casts a shadow $10\sqrt{3}$m long on the ground, then angle of elevation of Sun is:

(A) 60° **(B)** 30° **(C)** 90° **(D)** 45°

10. Points $A(-1, y)$ and $B(5, 7)$ lie on a circle with centre $O(2, -3y)$. The values of y are:

(A) 1, –7 **(B)** –1, 7 **(C)** 2, 7 **(D)** –2, –7

11. Two coins of diameter 2 cm and 4 cm respectively are kept one over the other as shown in the figure, then the area of the shaded ring shaped region in square cm is



(A) 3π sq cm **(B)** 4π sq cm **(C)** 2π sq cm **(D)** 5π sq cm

12. A bag contains 3 red and 5 black ball. A ball is drawn at random from the bag. The probability that the drawn ball is not red is:

(A) $\frac{1}{8}$ **(B)** $\frac{3}{8}$ **(C)** $\frac{5}{8}$ **(D)** 1

13. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be:

(A) 10 m **(B)** 15 m **(C)** 20 m **(D)** 24 m

14. A dice is rolled twice. The probability that 5 will not come up either time is:

(A) $\frac{11}{36}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{13}{36}$ **(D)** $\frac{25}{36}$

15. A medicine-capsule is in the shape of a cylinder of diameter 0.5 cm with two hemispheres stuck to each to its ends. The length of entire capsule is 2 cm. The capacity of the capsule is:

(A) 0.36 cm^3 **(B)** 0.35 cm^3 **(C)** 0.34 cm^3 **(D)** 0.33 cm^3

16. The angle of depression of a car, standing on the ground, from the top of a 75 m high tower, is 30°. The distance of the car from the base of the tower (in m.) is:

(A) $25\sqrt{3}$ (B) $50\sqrt{3}$ **(C)** $75\sqrt{3}$ **(D)** 150

17. Aruna has only ₹ 1 and ₹ 2 coins with her. If the total number of coins that she has is 50 and the amount of money with their is ₹ 75, then the number of ₹ 1 and ₹ 2 coins are, respectively .... and ....

(A) 20, 30 **(B)** 25, 25 **(C)** 10, 40 **(D)** 15, 35

18. Two dice are thrown simultaneously. The probability of getting a total of at least 10 is:

(A) $\frac{1}{2}$ **(B)** $\frac{5}{12}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{12}$

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)
(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)
(C) Assertion (A) is true but reason (R) is false.
(D) Assertion (A) is false but reason (R) is true.

19. **Assertion (A):** In a right circular cone, the cross-section made by a plane parallel to the base is a circle.
Reason (R): If the volume and the surface area of a solid hemisphere are numerically equal, then the diameter of hemisphere is 9 units.

20. **Assertion (A):** HCF of two or more numbers
= Product of the smallest power of each common prime factor, involved in the numbers.
Reason (R): The HCF of 12, 21 and 15 is 3.

Section - B

Section B consists of 5 questions of 2 marks each.

21. If the sum of first n terms of an A.P. is n^2 then find its 10^{th} term.

22. The line segment joining the points A(2, 1) and B (5, – 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k.

23. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord.

24. A solid sphere of radius r is melted and recast into the shape of a solid cone of height r. Find the radius of the base of a cone.

OR

The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3, find the ratio of their volumes.

25. Evaluate: $\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2 \tan 45° \cos^2 0° \sin 90°$

OR

If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b \cos\theta = n$, prove that $m^2 + n^2 = a^2 + b^2$

Section - C

Section C consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{7}$ is an irrational number.

27. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots then show that $c^2 = a^2(1 + m^2)$.

28. How many terms of the Arithmetic progression 45, 39, 33, ... must be taken so that their sum is 180? Explain the double answer.

OR

If the m^{th} term of an A.P. is $\frac{1}{n}$ and n^{th} term is $\frac{1}{m}$, then show that its $(mn)^{th}$ term is 1.

29. Two trees of height a and b are p meters apart.

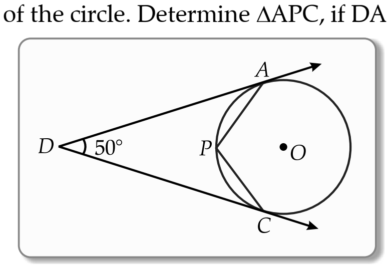
(i) Prove that the height of the point of intersection of the lines joining the top of each tree to the foot of the opposite trees is given by $\frac{ab}{a+b}$ m.

(ii) Which mathematical concept is used in this problem?

30. X is a point on the side BC of ΔABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that $TX^2 = TB \times TC$.

OR

In the given figure, O is the centre of the circle. Determine ΔAPC, if DA and DC are tangents and ΔADC = 50°.



31. Two friends are playing a game with a die. One of them throws a die twice. Find the probability that
(i) 5 will come up at least once.
(ii) 5 will not come up either time.

Section - D

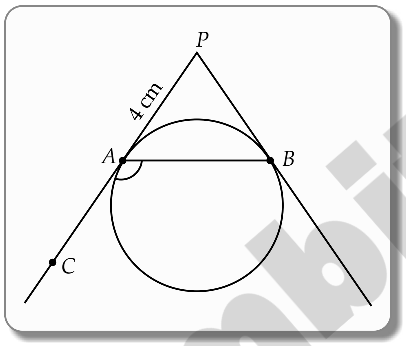
Section D consists of 4 questions of 5 marks each.

32. A man observes a car from the top of a tower, which in moving towards the tower with a uniform speed. If the angle of depression of the car changes from 30º to 45º in 12 min, find the time taken by the car now to reach the tower.

OR

An observer finds the angle of elevation of the top of the tower from a certain point on the ground as 30°. If the observer moves 20 m towards the base of the tower, the angle of elevation of the top increased by 15º, find the height of the tower.

33. In the given figure, PA and PB are tangents to a circle from an external point P such that PA = 4 cm and $\angle BAC$ = 135°. Find the length of chord AB.



34. In a hospital, used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital having length 25 m and breadth 20 m (provided no water is absorbed by land). If tank is filled completely then what will be the height of standing water used for irrigating the park. Write your views on recycling of water.

OR

A chord PQ of a circle of radius 10 cm subtends an angle of 60° at the centre of circle. Find the area of major and minor segments of the circle.

35. Consider the following frequency distribution of the heights of 60 students of a class.

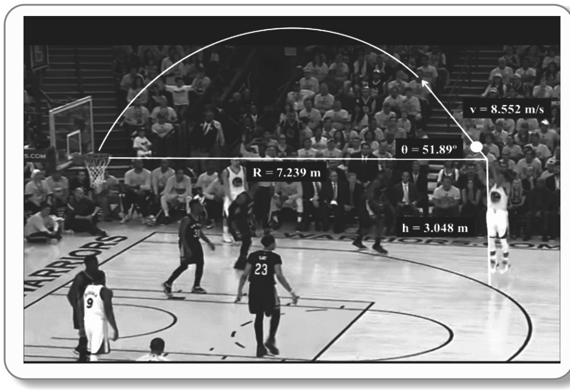
Heights (in cm)	No. of students
150-155	15
155-160	13
160-165	10
165-170	8
170-175	9
175-180	5

Find the upper limit of the median class in the given data.

Section - E

Case study based questions are compulsory.

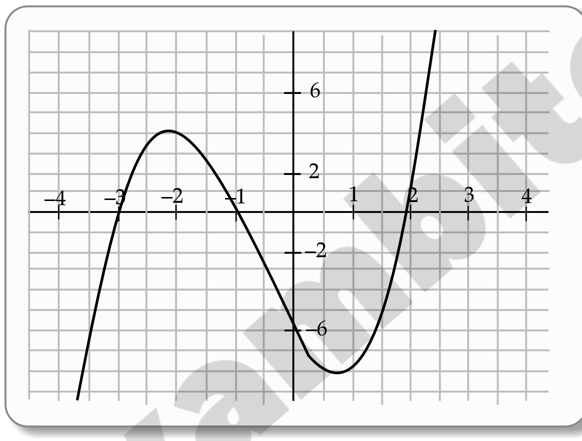
36. Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial.



(i) What is the shape of the path ? **1**

(ii) Write the general solution for $ax^2 + bx + c = 0$ **1**

(iii) Observe the following graph and answer

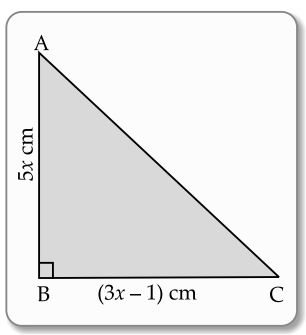


In the above graph, how many zeroes are there for the polynomial? **2**

OR

What will be the expression of the polynomial?

37. There is a triangular playground as shown in the below figure. Many Children and people are playing and walking in the ground.



As we see in the above figure of right angled triangle playground, the length of the sides are $5x$ cm and $(3x – 1)$ cm and area of the triangle is 60 cm^2.

(i) Find the value of x. **1**

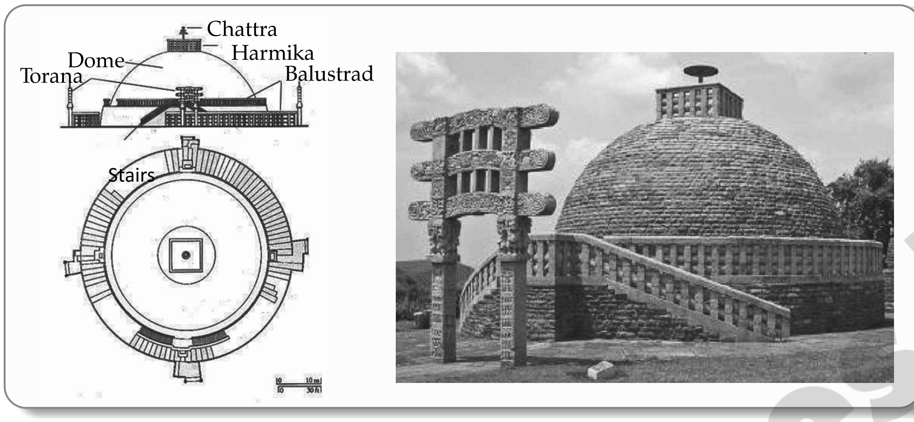
(ii) Find the length of AB. **2**

OR

Find the length of AC.

(iii) Calculate the perimeter of ΔABC. **1**

38. A The Great Stupa at Sanchi is one of the oldest stone structures in India, and an important monument of Indian Architecture. It was originally commissioned by the emperor Ashoka in the 3rd century BCE. Its nucleus was a simple hemispherical brick structure built over the relics of the Buddha. It is a perfect example of combination of solid figures. A big hemispherical dome with a cuboidal structure mounted on it. $\left(\textbf{Take } \pi = \frac{22}{7}\right)$



(i) Calculate the volume of the hemispherical dome if the height of the dome is 21 m. **1**

(ii) Find the Volume of Sphere if radius is 7 meter. **2**

OR

Calculate the total surface area of the combined figure i.e. hemispherical dome with radius 14 m and cuboidal shaped top with dimensions 8 m × 6 m × 4 m is

(iii) Calculate he volume of the cuboidal shaped top is with dimensions mentioned in question ii (OR). **1**

■■■

Self Assessment Paper-5

Mathematics Standard (041)

Class-X

UNSOLVED

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

(i) *This Question Paper has 5 Sections A-E.*
(ii) *Section A has 20 MCQs carrying 1 mark each*
(iii) *Section B has 5 questions carrying 02 marks each.*
(iv) *Section C has 6 questions carrying 03 marks each.*
(v) *Section D has 4 questions carrying 05 marks each.*
(vi) *Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.*
(vii) *All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E*
(viii) *Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.*

Section - A

Section A consists of 20 questions of 1 mark each.

1. The HCF of 18, 21 and 27 is:

(A) 7 **(B)** 6 **(C)** 9 **(D)** 3

2. If $x = 25 \times 7$, $y = 22 \times 32 \times 5$ and z = 3n × 52 and LCM $(x, y, z) = 2^5 \times 3^4 \times 5^2 \times 7$, then n is equal to

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5

3. How many zeroes does $(x - 1)(x + 2)$ have ?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3

4. The pair of linear equations

$\frac{3x}{2} + \frac{5y}{3} = 7$ and $9x + 10y = 14$ is:

(A) consistent **(B)** inconsistent

(C) consistent with one solution **(D)** consistent with many solutions

5. The distance of the point P(3, 5) from the x-axis is:

(A) 1 **(B)** 5 **(C)** 3 **(D)** – 5

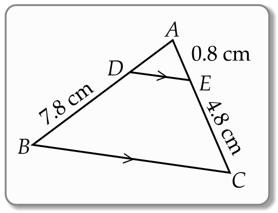
6. If the point P(k, 0) divides the line segment joining the points A(3, – 3) and B(– 5, 2) in the ratio 2 : 3, then the value of k is:

(A) 4 **(B)** 5 **(C)** $-\frac{1}{5}$ **(D)** $\frac{1}{5}$

7. The distance between the points A(0, 4) and B(0, – 3) is:

(A) 7 units **(B)** 5 units **(C)** 4 units **(D)** 3 units

8. If figure, DE || BC. Find the length of side AD, given that AE = 0.8 cm, BD = 7.8 cm and CE = 4.8 cm.



(A) 2.4 cm **(B)** 1.6 cm **(C)** 2.8 cm **(D)** 1.3 cm

9. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 4DE$. Then, the two triangles are:

(A) congruent but not similar **(B)** similar but not congruent
(C) neither congruent nor similar **(D)** congruent as well as similar

10. If $\sin\theta = \frac{1}{\sqrt{2}}$, then the value of $\tan^2\theta + \cot^2\theta$ is:

(A) 3 **(B)** 5 **(C)** 2 **(D)** $\sqrt{5}$

11. If the probability of winning a game is 0.03, then the probability of losing it; is:

(A) 0.93 **(B)** 0.97 **(C)** 0.95 **(D)** 0.96

12. 2.2 dm^3 of copper is to be drawn into a cylindrical wire of diameter 0.50 cm, the length of wire is:

(A) 112 m **(B)** 112 dm **(C)** 112 cm **(D)** 11.2 m

13. The radius of two cylinders are in the ratio of 2 : 3 and their heights are in the ratio of 5 : 3, then the ratio of their volumes is:

(A) 10 : 17 **(B)** 20 : 27 **(C)** 17 : 27 **(D)** 20 : 37

14. Class marks for the class interval 70 – 80 is:

(A) 65 **(B)** 75 **(C)** 85 **(D)** 70

15. If the total surface area of a right circular cone of slant height 13 cm is 90π cm^2, then its radius is:

(A) 5 cm **(B)** 18 cm **(C)** 15 cm **(D)** 4 cm

16. If one zero of quadratic polynomial $x^2 + 5x + k$ is 3, then the value of k is:

(A) 24 **(B)** – 24 **(C)** 6 **(D)** – 6

17. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have:

(A) a unique solution **(B)** exactly two solutions
(C) infinitely many solutions **(D)** no solution

18. The value of $\sin^2 30° + 2\cot 45° - \cos^2 60°$ is:

(A) 2 **(B)** 1 **(C)** 3 **(D)** 5

DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option

(A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(B) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

19. **Assertion (A):** A bag contains slips numbered from 1 to 100. If Fatima chooses a slip at random from the bag, it will either be an odd number or an even number. Since this situation has only two possible outcomes, the probability of each is $\frac{1}{2}$.

Reason (R): When we toss a coin, there are two possible outcomes: head or tail. Therefore, the probability of each outcome is $\frac{1}{2}$.

20. **Assertion (A):** If the equation $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines, then the value of $k = 2$.

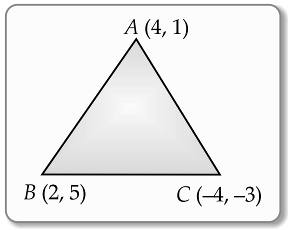
Reason (R): If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is 15.

Section - B

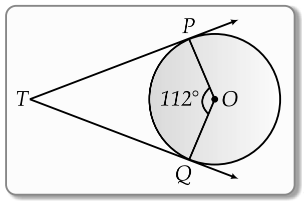
Section B consists of 5 questions of 2 marks each.

21. Find a quadratic polynomial where zeroes are $7-2\sqrt{3}$ and $7+2\sqrt{3}$

22. A(4, 1) B(2, 5) and C(– 3, – 2) are the vertices of ΔABC. Find the length of median AD.



23. In the figure, TP and TQ are two tangents to a circle with centre O. If ∠POQ = 112°, then find ∠PTQ.



24. If the volume of a cube is 125 cm^3, then what is the surface area of a cube ?

OR

From a solid right circular cylinder of height 14 cm and base radius 6 cm, a right circular cone of same height and same base removed. Find the volume of the remaining solid.

25. If tan $(5x + 30°) = 1$, then find the value of x.

OR

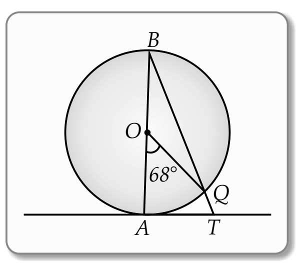
If $\sin(3x + 30°) = \dfrac{\sqrt{3}}{2}$, then find the value of $5x + 10$.

Section - C

Section C consists of 6 questions of 3 marks each.

26. Prove that $3\sqrt{7}$ is an irrational number.

27. In given figure, AB is the diameter of a circle with centre O and AT is a tangent. If $\angle AOQ = 68°$, find $\angle ATQ$.



28. Write the discriminant of the quadratic equation $(x + 4)^2 = 3(7x - 4)$.

OR

In a cricket match, Harbhajan took three wickets less than twice the number of wickets taken by Zahir. The product of the number of wickets taken by these two is 20. Represent the above situation in the form of quadratic equation.

29. If $x = a \sec\alpha \cos\beta$, $y = b \sec\alpha \sin\beta$ and $z = c \tan\alpha$, then evaluate $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}-\dfrac{z^2}{c^2}$.

30. The first term of an A.P. is 4, the last term is 94 and the sum of all its terms is 980. Find the number of terms and the common difference of the A.P..

OR

The 14^{th} term of an A.P. is twice its 8^{th} term. If the 6^{th} term is – 8, then find the sum of its first 20 terms.

31. A bag contains 20 white and some red balls. If the probability of drawing a black ball from the bag is 4 times that of drawing a white ball, find the number of red balls in the bag.

Section - D

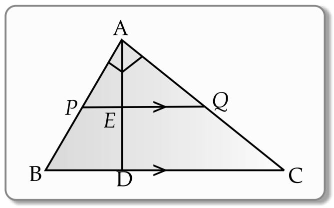
Section D consists of 4 questions of 5 marks each.

32. Two points A and B are on the same side of a tower and in the same straight line with its base. The angle of depression of these points from the top of the tower are 60° and 45° respectively. If the height of the tower is 18 m, then find the distance between these points.

OR

The angle of depression of two ships from an aeroplane flying at the height of 6000 m are 30° and 45°. If both the ships are in the same line that one ship is exactly behind the other, find the distance between the ships.

33. In the figure, PQ || BC. Prove that median AD bisects PQ.



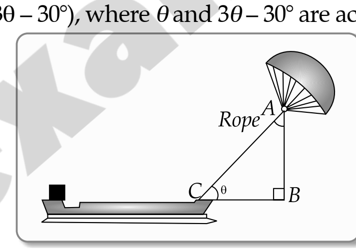
34. Water is flowing at the rate of 15 km/h through a cylindrical pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time the level of water in pond rise by 21 cm?

OR

'Sky sails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Sky sails' technology allows the towing kite to gain a height of anything between 100 metres – 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the questions:

(i) In the given figure, if $\sin \theta = \cos (3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then find the value of θ.



(ii) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200 m?

35. **Find the mode of the following frequency distribution:**

Class	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	3	8	9	12	3	2

Section - E

Case study based questions are compulsory.

36. John and Jivanti are playing with the marbles in the playground. They together have 45 marbles and John has 15 marbles more than Jivanti.

(i) How many marbles Jivanti had ? **1**

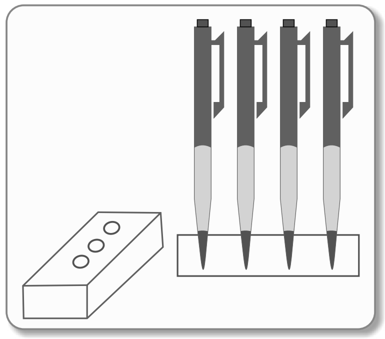
(ii) If 45 is replaced by 55 in the above case discussed in the question, then find the number of marbles Jivanti have. **1**

(iii) From the given passage, find the number of marbles John have. **2**

OR

The given problem is based on which mathematical concept?

37. A carpenter made a wooden pen stand. It is in the shape of cuboid with four conical depressions to hold pens. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm. (See Figure).



(i) What is the volume of cuboid? **1**

(ii) What is the volume of a conical depression ? **2**

OR

What is the total volume of conical depressions?

(iii) What is the volume of wood in the entire stand? **1**

38. On a weekend Rani was playing cards with her family. The deck has 52 cards. If her brother drew one card.

(i) Find the probability of getting a king of red colour. **1**

(ii) Find the probability of getting a jack of hearts. **2**

OR

Find the probability of getting a red face card.

(iii) Find the probability of getting a spade. **1**

SOLUTIONS

Sample Question Paper-2

Mathematics Standard (041)

Section - A

1. Option (A) is correct.

Explanation: First number = 70 – 5 = 65

Second number = 125 – 8 = 117

The largest number = HCF of both numbers 65, 117

$\therefore$ prime factors of 65 = 5 × 13

prime factors of 117 = 3 × 3 × 13

$\Rightarrow$ HCF (65, 117) = 13.

2. Option (D) is correct.

First subtract the remainders from their respective numbers,

$$1251 - 1 = 1250$$
$$9377 - 2 = 9375$$
$$15{,}628 - 3 = 15{,}625$$

According to the prime factorisation,

$$1250 = 2 \times 5 \times 5 \times 5 \times 5$$
$$9375 = 3 \times 5 \times 5 \times 5 \times 5 \times 5$$
$$15{,}625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$$

HCF(1250, 9375, 15,625) = 5 × 5 × 5 × 5 = 625

[CBSE Marking Scheme, 2021]

3. Option (D) is correct.

Explanation: We know that if we divide or multiply a polynomial by any constant (real number), then the zeroes of polynomial remains same.

Here, $\alpha = -2$ and $\beta = +5$

$\therefore\ \alpha + \beta = -2 + 5 = 3$ and $\alpha\beta = -2 \times 5 = -10$

So, required polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - 3x - 10$

If we multiply this polynomial by any real number, let 5 and 2, we get $5x^2 - 15x - 50$ and $2x^2 - 6x - 20$ which are different polynomials but having same zeroes –2 and 5. So, we can obtain so many (infinite polynomials) from two given zeroes. **1**

4. Option (B) is correct.

Explanation: Given,

$$2x + y = 23 \qquad \text{(i)}$$

and $$4x - y = 19 \qquad \text{(ii)}$$

On adding Eq. (i) and (ii), we get

$$6x = 42 \Rightarrow x = 7$$

Putting the value of x in eqn. (i), we get

$$14 + y = 23$$

$$\Rightarrow \quad y = 23 - 14 = 9$$

Hence, $5y - 2x = 5 \times 9 - 2 \times 7 = 45 - 14 = 31.$

and $\dfrac{y}{x} - 2 = \dfrac{9}{7} - 2 = \dfrac{9-14}{7} = \dfrac{-5}{7}.$

5. Option (C) is correct.

Explanation:

$$\left(\sqrt{2}x\right)^2 + \left(\sqrt{3}\right)^2 + 2\times\sqrt{2}x\times\sqrt{3} + x^2 = 3x^2 - 5x$$
$$2x^2 + 3 + 2\sqrt{6}x + x^2 = 3x^2 - 5x$$
$$3x^2 + 2\sqrt{6}x + 3 = 3x^2 - 5x$$
$$x\left(5 + 2\sqrt{6}\right) + 3 = 0$$

It is not of the form of $ax^2 + bx + c = 0$. **1**

6. Option (B) is correct.

Explanation: Here, $d = \dfrac{-3}{4}$

Let the n^{th} term be first negative term

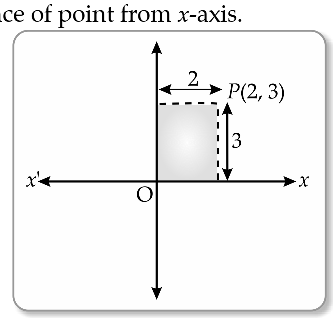
$$\therefore \quad 20 + (n-1)\left(\frac{-3}{4}\right) < 0$$
$$\Rightarrow \quad 3n > 83$$
$$\Rightarrow \quad n > 27\frac{2}{3}$$

Hence, 28^{th} term is first negative term **1**

[CBSE Marking Scheme 2017]

7. Option (B) is correct.

Explanation: Since y-coordinate of a given point is the distance of point from x-axis.

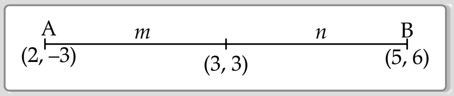


1

then 3 units is distance of p from x coordinate

8. Option (A) is correct.

Explanation: Let $P(x, 0)$ be a point on x-axis which divides the join of A(2, – 3) and B(5, 6) in the ratio $m : n$, then using section formula



$$Y = \frac{my_2 + ny_1}{m+n}$$

$$\Rightarrow \quad 0 = \frac{m \times 6 + n \times (-3)}{m+n}$$

$$\Rightarrow \quad 6m - 3n = 0$$

$$\Rightarrow \quad 2m - n = 0$$

$$\Rightarrow \quad 2m = n$$

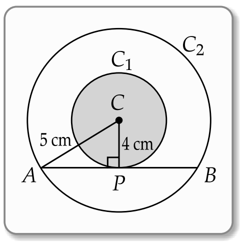
$$\Rightarrow \quad \frac{m}{n} = \frac{1}{2}$$

i.e., $m : n = 1 : 2.$

[CBSE Marking Scheme, 2021-22]

9. **Option (B) is correct.**

Explanation: Let C_1, C_2 be two concentric circles with their centre C.



Chord AB of circle C_2 touches C_1 at P

AB is tangent at P and PC is radius

$\therefore \quad CP \perp AB$

Given, $\angle P = 90°$, $CP = 4$ cm and $CA = 5$ cm

$\therefore$ In right angle ΔPAC,

$$AP^2 = AC^2 - PC^2$$
$$= 5^2 - 4^2$$
$$= 25 - 16$$
$$= 9$$

$\Rightarrow \quad AP = 3$ cm

$\because$ Perpendicular from centre to chord bisects the chord.

$\therefore \quad AB = 2AP$

$= 2 \times 3$

$= 6$ cm. **1**

10. **Option (B) is correct.**

Explanation: In ΔABC,

$DE \parallel AC$ (given)

$\therefore \quad \dfrac{BD}{DA} = \dfrac{BE}{EC}$ (from BPT) (i)

In ΔABE,

$DF \parallel AE$ (given)

$\therefore \quad \dfrac{BD}{DA} = \dfrac{FB}{FE}$ (from BPT) (ii)

From Eq. (i) and (ii), we get

$$\frac{BF}{FE} = \frac{BE}{EC}$$

11. **Option (A) is correct.**

Explanation: OP is radius and PR is tangent at P.

So, $\angle OPR = 90°$

$\Rightarrow \quad \angle OPQ + 50° = 90°$

$\Rightarrow \quad \angle OPQ = 90° - 50°$

$\Rightarrow \quad \angle OPQ = 40°$

In ΔOPQ, $\quad OP = OQ$ [radii of same circle]

$\therefore \quad \angle Q = \angle OPQ = 40°$

[angles opposite to equal sides are equal]

But, $\quad \angle POQ = 180° - \angle P - \angle Q$

$= 180° - 40° - 40°$

$= 180° - 80° = 100°$

$\Rightarrow \quad \angle POQ = 100°$

12. **Option (D) is correct.**

Explanation: Given, $\sin\alpha = \dfrac{1}{2} = \sin 30° \quad \left[\because \sin 30° = \dfrac{1}{2}\right]$

$\Rightarrow \quad \alpha = 30°$

And, $\cos\beta = \dfrac{1}{2} = \cos 60°$

$\Rightarrow \quad \beta = 60° \quad \left[\because \cos 60° = \dfrac{1}{2}\right]$

$\therefore \quad \alpha + \beta = 30° + 60° = 90°$

13. **Option (C) is correct.**

Explanation: Given, $\sin\theta = \dfrac{a}{b}$

$$\left[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos\theta = \sqrt{1 - \sin^2\theta}\right]$$

$$\cos\theta = \sqrt{1 - \left(\frac{a}{b}\right)^2} = \sqrt{1 - \frac{a^2}{b^2}} = \frac{\sqrt{b^2 - a^2}}{b}$$

14. **Option (B) is correct.**

Explanation: $p(x) = x^2 - (k + 6)x + 2(2k - 1)$ **is the given polynomial**

Here, $a = 1$, $b = -(k + 6)$ & $c = 2(2k - 1)$

Sum of zeroes $= \alpha + \beta$

$= \dfrac{-b}{a}$

$= k + 6$

Product of zeroes $= \alpha\beta$

$= \dfrac{c}{a}$

$= \dfrac{2(2k-1)}{1} = 2(2k - 1)$

It is given that,

$$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$$\Rightarrow \quad k + 6 = \frac{1}{2}2(2k - 1)$$

$$\Rightarrow \quad k + 6 = 2k - 1$$

$$\Rightarrow \quad -k = -7$$

or $\quad k = 7$

15. Option (C) is correct.

Explanation: In ΔABD and ΔCAD, $\angle ADB = \angle ADC = 90°$ and $\angle ABD = \angle CAD = \theta$.

By AA Similarity, we get, $\Delta ABD \sim \Delta CAD$

$\Rightarrow \dfrac{AD}{BD} = \dfrac{CD}{AD} \Rightarrow BD \times CD = AD^2$

16. Option (D) is correct.

Explanation: Probability of an event is always a proper fraction.

Also, $0 \le P(E) \le 1$

But, $\dfrac{17}{16} > 1$

Therefore, $\dfrac{17}{16}$ can never be probability of any event.

17. Option (A) is correct.

Explanation: The probability of the event, which is very unlikely to happen, will be very close to zero. So it's probability is 0.0001 which is minimum amongst the given values.

18. Option (A) is correct.

(A) 50°

[Topper Answer, 2020] 3

Detailed solution:

$$OA = OB \quad \text{[radius of circle]}$$

$$\angle OAB = \angle OBA$$

[isosceles triangle property]

Now in ΔOAB, $\angle OAB = \dfrac{180 - 100}{2} = 40$

[sum angle property of ΔOAB]

Hence, $\angle ABP = \angle OBP - \angle ABO$

$= 90° - 40°$

[$\because \angle OBP = 90°$, angles between tangent and radius]

$= 50°$.

19. Option (C) is correct.

Explanation: By the property of coprime numbers their cubes are always coprime numbers. Since, in Reason, all natural numbers are not coprime.

20. Option (B) is correct.

Explanation: Point P(2, 0) need not be necessary the midpoint of A and B. P must lies an the locus of A and B.

Section - B

21. Let α and β be the zeroes of the polynomial.

Given, $\beta = 7\alpha$

$\therefore \quad \alpha + 7\alpha = -\left(-\dfrac{8}{3}\right)$ ½

$\therefore \quad 8\alpha = \dfrac{8}{3}$

So, $\quad \alpha = \dfrac{1}{3}$ ½

and $\quad \alpha \times 7\alpha = \dfrac{2k+1}{3}$

($\because$ Product of zeroes $= \dfrac{c}{a}$)

$\Rightarrow \quad 7\alpha^2 = \dfrac{2k+1}{3}$

$\Rightarrow \quad 7\left(\dfrac{1}{3}\right)^2 = \dfrac{2k+1}{3}$ ½

$\Rightarrow \quad 7 \times \dfrac{1}{9} = \dfrac{2k+1}{3}$

or, $\quad \dfrac{7}{3} - 1 = 2k \quad$ or, $2k = \dfrac{4}{3}$

$\therefore \quad \dfrac{2}{3} = k \Rightarrow k = \dfrac{2}{3}$ ½

22. Let larger angle be $x°$

$\therefore$ Smaller angle $= 180° - x°$

According to question

$x° - (180° - x°) = 18°$ ½

$\Rightarrow \quad 2x° = 180° + 18°$

$\Rightarrow \quad 2x° = 198°$

$\Rightarrow \quad x° = 99°$ ½

$\therefore$ The two angles are 99°, 81° **1**

[CBSE Marking Scheme 2019]

OR

In ΔPAO and ΔQBO,

$\angle A = \angle B = 90°$ (Given)

$\angle POA = \angle QOB$

(vertically opposite angles)

Since, $\Delta PAO \sim \Delta QBO$, (by AA)

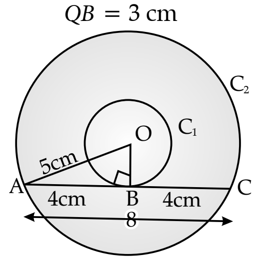
Then, $\quad \dfrac{OA}{OB} = \dfrac{PA}{QB}$

or, $\quad \dfrac{6}{4.5} = \dfrac{4}{QB}$

or, $\quad QB = \dfrac{4 \times 4.5}{6}$

$\therefore \quad QB = 3$ cm **1**

23.



Let us consider two concentric circles C_1 and C_2 with centre O.

Chord AC of circle C_2 is tangent of circle C_1 at B.

We know that tangent AC and radius BO at point B are perpendicular.

$\because$ Perpendicular from centre to chord bisects the chord.

$\therefore AB = CB = \frac{AC}{2} = \frac{8}{2} = 4$ cm **1**

In right ΔABO,

$OB^2 = OA^2 - AB^2$ [By Pythagoras theorem]

$= 5^2 - 4^2 = 25 - 16 = 9$ **1**

$\Rightarrow OB = 3$ cm

Hence, radius of circle C_1 is 3 cm. **1**

24. Since, $\sin(A + B) = 1 = \sin 90°$

or, $A + B = 90°$ (i)

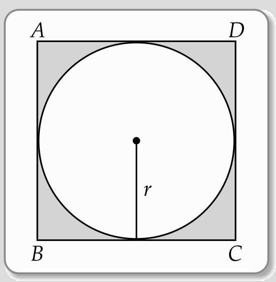
$\sin(A - B) = \frac{1}{2} = \sin 30°$ **1**

or, $A - B = 30°$ (ii)

Solving Eq. (i) and (ii),

$A = 60°$ and $B = 30°$ **1**

25.



1

Side of square = diameter of circle = 8 cm

$\therefore$ Radius of circle, $r = \frac{8}{2} = 4$ cm

Area of circle $= \pi r^2$

$= \pi \times 4 \times 4 = 16\pi$ cm^2 **½**

[CBSE Marking Scheme, 2012]

OR

Curved surface area of cylinder $= 2\pi rh$

Volume of cylinder $= \pi r^2 h$

$$\frac{\pi r^2 h}{2\pi rh} = \frac{924}{264} \Rightarrow \frac{r}{2} = \frac{7}{2}$$ **1**

$\therefore$ $r = 7$ m

$2\pi rh = 264$

or, $2 \times \frac{22}{7} \times 7 \times h = 264$

or, $h = 6$ m

$$\therefore \quad \frac{h}{2r} = \frac{6}{14} = \frac{3}{7}$$ **1**

Hence, the ratio of its height to its diameter is

$h : d = 3 : 7$

Section - C

26. We prove this by using the method of contradiction. Assume that $7\sqrt{5}$ is a rational number.

Then, $7\sqrt{5} = \frac{p}{q}$

where p and q are coprime numbers $(q \neq 0)$

$\Rightarrow \sqrt{5} = \frac{p}{7q}$ **1**

Since $\sqrt{5}$ is an irrational number and $\frac{p}{7q}$ is a rational number.

$\therefore$ which contradicts our assumption.

Therefore, $7\sqrt{5}$ is irrational. **1**

27. $\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 2$

$\Rightarrow (x-1)^2 + (2x+1)^2 = 2(2x+1)(x-1)$ **1**

$\Rightarrow x^2 + 1 - 2x + 4x^2 + 1 + 4x = 4x^2 - 4x + 2x - 2$

$\Rightarrow x^2 + 4x + 4 = 0$ **1**

$\Rightarrow (x+2)^2 = 0$ **1**

$\Rightarrow x = -2$

[CBSE Marking Scheme 2017]

28. (i) Let the cost price of the toy be ₹ x, then gain = x%

or, Gain $= ₹\left(x \times \frac{x}{100}\right)$

$= ₹\left(\frac{x^2}{100}\right)$ **1**

$\therefore$ S.P. = C.P. + Gain

$= x + \frac{x^2}{100}$

But S.P. = ₹ 24

$\therefore$ $x + \frac{x^2}{100} = 24$

or, $100x + x^2 = 2400$

or, $x^2 + 100x - 2400 = 0$

or, $x^2 + 120x - 20x - 2400 = 0$

or, $x(x + 120) - 20(x + 120) = 0$

or, $(x - 20)(x + 120) = 0$

or, $x = 20$ or $x = -120$ **1**

or, $x = 20$,

[$\because x = -120$ is not possible]

Hence the cost price of the toy is ₹ 20.

(ii) Quadratic equations. **1**

Commonly Made Error

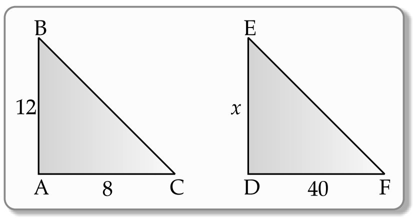
- Some students do not know how to frame the equation. Some frame it correctly but fail to solve it.

Answering Tip

- Solve more application based problems on Quadratic Equations.

OR

(i) Let AB be the height of vertical row of trees and AC be the length of its shadow. Also, let DE be the height of bamboo tree and DF be the length of its shadow. Join BC and EF. Let $DE = x$.



We have $AB = 12$ m

$AC = 8$ m

and $DF = 40$ m **1**

In ΔABC and ΔDEF,

$\angle A = \angle D = 90°$

Since, the Sun casts equal angle at the same time

$\angle C = \angle F$ **1**

Therefore, by AA criterion of similarity, we have

$\Delta ABC \sim \Delta DEF$ (*AA* Criterion)

or, $\dfrac{AB}{DE} = \dfrac{AC}{DF}$

or, $\dfrac{12}{x} = \dfrac{8}{40}$

or, $x = \dfrac{12 \times 40}{8} = 60$ m

or, height of tower = 60 m. **1**

(ii) Similar triangles.

Commonly Made Error

- Most candidates are not able to prove ΔABC ~ ΔDEF.

Answering Tip

- Candidates should know about AAA criteria for similarity of triangles.

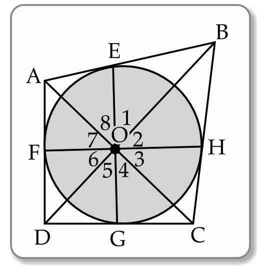
29. **Given:** A circle with centre O is inscribed in a quadrilateral ABCD.

In ΔAEO and ΔAFO,

$OE = OF$ (radii of circle)

$\angle OEA = \angle OFA = 90°$

(radius is $\perp^r$ to tangent)



The point of contact is perpendicular to the tangent.

$AE = AF$ (common side)

$\Delta AEO \cong \Delta AFO$

(SAS congruency)

$\angle 7 = \angle 8$ (i) (*CPCT*)

Similarly,

$\angle 1 = \angle 2$ (ii)

$\angle 3 = \angle 4$ (iii)

$\angle 5 = \angle 6$ (iv)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

(angle around a point is 360°)

$2\angle 1 + 2\angle 8 + 2\angle 4 + 2\angle 5 = 360°$

$\angle 1 + \angle 8 + \angle 4 + \angle 5 = 180°$

$(\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180°$ **3**

$\angle AOB + \angle COD = 180°$

Hence proved.

30. $$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta} \quad \left(\because \tan\theta = \frac{1}{\cot\theta}\right)$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} + \frac{1}{(1-\tan\theta)\tan\theta} \quad \mathbf{1}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{(\tan\theta - 1)\tan\theta}$$

$$= \frac{\tan^3\theta - 1}{(\tan\theta - 1)\tan\theta}$$

$[\because a^3 - b^3 = (a-b)(a^2 + b^2 + ab)]$ ½

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{(\tan\theta - 1)(\tan\theta)} = \frac{\tan^2\theta + \tan\theta + 1}{\tan\theta} \quad ½$$

$= \tan\theta + 1 + \cot\theta$

= RHS. **Hence proved. 1**

Commonly Made Error

- Sometimes students do not apply this formula : $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$. They directly simplify equation which leads to incorrect result.

Answering Tip

- Learn basic formulae like $(a-b)^2$, $(a+b)^2$, (a^2-b^2), (a^3-b^3) etc.

OR

$$LHS = 1+\frac{\cot^2\alpha}{1+\text{cosec}\,\alpha} = 1+\frac{\cos^2\alpha/\sin^2\alpha}{1+1/\sin\alpha}$$

$$\left[\because \cot\theta = \frac{\cos\theta}{\sin\theta} \text{ and } \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$$

$$= 1+\frac{\cos^2\alpha}{\sin\alpha\,(1+\sin\alpha)} = \frac{\sin\alpha(1+\sin\alpha)+\cos^2\alpha}{\sin\alpha\,(1+\sin\alpha)}$$

$$= \frac{\sin\alpha+(\sin^2\alpha+\cos^2\alpha)}{\sin\alpha\,(1+\sin\alpha)} \qquad [\because \sin^2\theta+\cos^2\theta = 1]$$

$$= \frac{(\sin\alpha+1)}{\sin\alpha\,(\sin\alpha+1)} = \frac{1}{\sin\alpha} \qquad \left[\because \text{cosec}\,\theta = \frac{1}{\sin\theta}\right]$$

$= \text{cosec}\,\alpha = RHS.$

31. $P(\text{red ball}) = \frac{x}{18}$

(i) $P(\text{no red ball}) = 1-\frac{x}{18} = \frac{18-x}{18}$ **1**

(ii) Total number of balls $= 18 + 2 = 20$

red balls are $= x + 2$

$P(\text{red balls}) = \frac{x+2}{20}$ ½

Now, according to the question,

$$\frac{x+2}{20} = \frac{9}{8}\times\frac{x}{18}$$

$\Rightarrow \quad 180x = 144x + 288$

$\Rightarrow \quad 36x = 288$

$\Rightarrow \quad x = \frac{288}{36} = 8$ **1½**

[CBSE Marking Scheme, 2015]

Section - D

32. Let ₹ x be the cost price of saree & ₹ y be the list price of sweater.

Case I: Sells a saree at 8% profit + Sells a sweater at 10% discount = ₹ 1,008

$\Rightarrow (100+8)\%$ of $x+(100-10)\%$ of $y = 1{,}008$

$\Rightarrow 108\%$ of $x + 90\%$ of $y = 1{,}008$

$\Rightarrow 1.08x+0.9y = 1{,}008$ **...(i)**

Case II : Sold the saree at 10% profit + Sold the sweater at 8% discount = ₹ 1,028

$\Rightarrow (100+10)\%$ of $x+(100-8)\%$ of $y = 1{,}028$

$\Rightarrow 110\%$ of $x+92\%$ of $y = 1{,}028$

$\Rightarrow 1.1x+0.92y = 1{,}028$ **...(ii)**

On putting the value of y from Eq. (i) into Eq. (ii), we get

$$1.1x+0.92\left(\frac{1{,}008-1.08x}{0.9}\right) = 1{,}028$$

$\Rightarrow 1.1\times0.9x+927.36-0.9936x = 1{,}028\times0.9$

$\Rightarrow 0.99x-0.9936x = 9{,}252-927.36$

$\Rightarrow -0.0036x = -2.16$

$\therefore \quad x = \frac{2.16}{0.0036} = 600$

On putting the value of x in Eq. (i), we get

$1.08\times600+0.9y = 1{,}008$

$\Rightarrow 108\times6+0.9y = 1{,}008$

$\Rightarrow 0.9y = 1{,}008-648$

$\Rightarrow 0.9y = 360$

$\therefore \quad y = \frac{360}{0.9} = 400$

Hence, x = ₹ 600 and y = ₹ 400 **5**

OR

Case I: The cost of one reserved first class ticket from the stations A to B = ₹ 2,530

$\Rightarrow x+y = 2{,}530$ (i)

Case II: The cost of one reserved first class ticket and one reserved first class half ticket from stations A to B = ₹ 3,810

$\Rightarrow x+y+\frac{x}{2}+y = 3{,}810$

$\Rightarrow \frac{3x}{2}+2y = 3{,}810$

$\Rightarrow 3x+4y = 7{,}620$ (ii)

Now, multiplying Eq. (i) by 4 and then subtracting from eq. (ii), we get

$$\begin{array}{l} 3x+4y = 7{,}620 \\ 4x+4y = 10{,}120 \\ -\quad -\quad - \\ \hline -x = -2{,}500 \end{array}$$

$\Rightarrow x = 2{,}500$

On putting the value of x in Eq. (i), we get

$2{,}500+y = 2{,}530$

$\Rightarrow y = 2{,}530-2{,}500$

$\therefore y = 30$ **5**

33. In ΔAPE and ΔBPF,

$\angle APE = \angle BPF$ [Vertically opposite angles]

$\angle AEP = \angle BFP$ [Alternate angles]

By AA similarity, $\Delta APE \sim \Delta BPF$ **1**

Thus, $\frac{AP}{BP} = \frac{PE}{PF} = \frac{AE}{BF}$ …(1)

In ΔCPE and ΔDPF,

$\angle CPE = \angle DPF$ [Vertically opposite angles]

$\angle CEP = \angle DFP$ [Alternate angles]

By AA similarity, $\Delta CPE \sim \Delta DPF$ **1**

Thus, $\dfrac{CP}{DP} = \dfrac{PE}{PF} = \dfrac{CE}{DF}$ ...(2)

In ΔAPC and ΔBPD,

$\angle APC = \angle BPD$ [Vertically opposite angles]

$\angle ACP = \angle BDP$ [Alternate angles]

By AA similarity, $\Delta APC \sim \Delta BPD$ **1**

Thus, $\dfrac{AP}{BP} = \dfrac{PC}{PD} = \dfrac{AC}{BD}$ ...(3)

From equations (1), (2) and (3), we get **1**

$\dfrac{AE}{BF} = \dfrac{AC}{BD} = \dfrac{CE}{FD}$ **Hence Proved. 1**

Section - D

34. Given, the side of the square = 28 cm.

Area of the square = 28 × 28 = 784 cm^2 ½

Radius of each circle $= \dfrac{28}{2} = 14$ cm **1**

$\therefore$ Area of two circles $= 2 \times \dfrac{22}{7} \times 14 \times 14$

$= 1232$ cm^2 1½

$\therefore$ Area the shaded region

$= \text{Area of square} + 2 \times \dfrac{270°}{360°} \times \text{Area of circles}$

$- \dfrac{1}{2}$ Area of circles **1**

$= 784 + \dfrac{3}{4} \times 1232 - \dfrac{1}{2} \times \pi \times 14 \times 14$

= (784 + 924) – 308

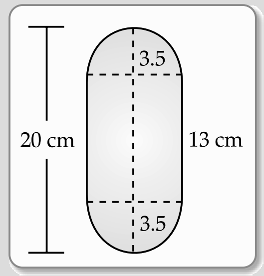
= (1708 – 308) cm^2

= 1400 cm^2 **1**

[CBSE Marking Scheme, 2017]

OR

Height of cylinder = 20 – 7 = 13 cm. **1**



Total volume $= \pi\left(\dfrac{7}{2}\right)^2 . 13 + \dfrac{4}{3}\pi\left(\dfrac{7}{2}\right)^3$ cm^3 **2**

$= \dfrac{22}{7} \times \dfrac{49}{4}\left(13 + \dfrac{4}{3} . \dfrac{7}{2}\right)\text{cm}^3$ **1**

$= \dfrac{77 \times 53}{6} = 680.17\text{cm}^3$ **1**

[CBSE Marking Scheme 2019]

Detailed Answer

Total height = 20 cm

Height of cylinder = 20 – 2 × 3.5

= 20 – 7 = 13 cm. **1**

Radius of cylinder = 3.5 cm

Radius of hemisphere = 3.5 cm

Volume of solid = volume of two hemisphere + volume of cylinder **1**

$= 2 \times \dfrac{2}{3}\pi r^3 + \pi r^2 h$ **1**

$= \dfrac{4}{3}\pi\left(\dfrac{7}{2}\right)^3 + \pi\left(\dfrac{7}{2}\right)^2 \times 13 = \left(\dfrac{7}{2}\right)^2 \pi\left[\dfrac{4}{3} \times \dfrac{7}{2} + 13\right]$

$= \dfrac{49}{4} \times \dfrac{22}{7}\left(\dfrac{14}{3} + 13\right) = \dfrac{7 \times 11}{2}\left(\dfrac{14+39}{3}\right)$ **1**

$= \dfrac{77 \times 53}{6} = 680.17\text{cm}^3$ **1**

35.

C. I.	f_i	$c.f.$	x_i	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
05 – 07	70	70	6	–3	–210
07 – 09	120	190	8	–2	–240
09 – 11	32	222	10	–1	–32
11 – 13	100	322	12 = a	0	0
13 – 15	45	367	14	1	45
15 – 17	28	395	16	2	56
17 – 19	5	400	18	3	15
	$\Sigma f = 400$				**$\Sigma f_i u_i = -366$**

Let a = assumed mean = 12 **2**

$$\text{Mean, } \bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$$

$$\text{Mean} = 12 + \frac{-366}{400} \times 2 = 12 - \frac{183}{100} = 12 - 1.83 = 10.17$$ 1

$$\text{Median class} = \frac{N}{2}$$

$$= \frac{400}{2} = 200 = 09 - 11$$

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f}\right) \times h$$

$$\therefore \quad \text{Median} = 9 + \frac{200 - 190}{32} \times 2 = 9 + \frac{10}{32} \times 2 = 9 + 0.625 = 9.625$$ 2

[CBSE Marking Scheme, 2015]

Section - E

36. **(i)** Let AD be x can, then DB = $(12 - x)$ cm

$\because AD = AF, CF = CE, DB = BE$

[tangents to a circle from an external point]

$\therefore \quad AF = x$ cm,

then $\quad CF = (10 - x)$ cm

$BE = (12 - x)$ cm,

then $\quad CE = 8 - (12 - x) = (x - 4)$cm

Now $\quad CF = CE$

$10 - x = x - 4$

$2x = 14$

$\Rightarrow \quad x = 7.$

Hence, $\quad AD = 7$cm. 1

Since,

$\because \quad BE = (12 - x)$ cm

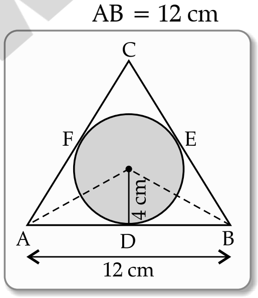
$= (12 - 7)$ cm

[$\because x = 7$ proved]

$BE = 5$ cm 1

(ii) Radius, $OD = 4$ cm

and AB = 12 cm



Then, area of ΔOAB

$$= \frac{1}{2} \times OD \times AB$$

$$= \frac{1}{2} \times 4 \times 12$$

$= 24$ cm^2. 1

(iii) Perimeter of ΔABC $= AB + BC + CA$

$= (12 + 8 + 10)$ cm

$= 30$ cm. 1

OR

Since, 100 cm cost = ₹ 1500

So, $\quad$ 30 cm cost $= \frac{1500 \times 30}{100}$

= ₹ 450

37. **(i)** Let the fixed charge for two days be ₹ x and additional charge be ₹ y per day.

As Radhika has taken book for 4 days.

It means that Radhika will pay fixed charge for first two days and pays additional charges for next two days.

$$x + 2y = 16$$

(ii) As the fixed charge for two days be ₹ x and additional charge be ₹ y per day

It means that Amruta will pay fixed charge for first two days and pays additional charges for next four days.

$$x + 4y = 22$$

(iii)

$x + 6y = 22$ ----------(i)

$x + 4y = 16$ ----------(ii)

On solving (i) and (ii)

Therefore, additional charges is y = ₹ 3.

OR

For two more days price charged will be

$$2y = 2 \times 3 = 6$$

Total money paid by Amruta and Radhika is 22 + 16 + 6 + 6 = ₹ 50

38. **(i)** Distance between two points (x_1, y_1) and (x_2, y_2).

$$d = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

Now, distance between house and bank,

$= \left|\sqrt{(5-2)^2+(8-4)^2}\right|$

$= \left|\sqrt{(3)^2+(4)^2}\right|$

$= \left|\sqrt{9+16}\right|$

$= \left|\sqrt{25}\right|$

$= 5$ km 1

(ii) Distance between bank and daughter's school,

$= \left|\sqrt{(13-5)^2+(14-8)^2}\right|$

$= \left|\sqrt{(8)^2+(6)^2}\right|$

$= \left|\sqrt{64+36}\right|$

$= \left|\sqrt{100}\right|$

$= 10$ km **1**

(iii) Distance between house to office,

$= \left|\sqrt{(13-2)^2+(26-4)^2}\right|$

$= \left|\sqrt{(11)^2+(22)^2}\right|$

$= \left|\sqrt{121+484}\right|$

$= \left|\sqrt{605}\right|$

$= 24.59$

$= 24.6$ km **1**

OR

Distance between daughter's school and office,

$= \left|\sqrt{(13-13)^2+(26-14)^2}\right|$

$= \left|\sqrt{0+(12)^2}\right|$

$= 12$ km ½

Total distance (House + Bank + School + Office) travelled = 5 + 10 + 12 = 27 km ½

■■■

SOLUTIONS
Sample Question Paper-3
Mathematics Standard (041)

Section - A

1. **Option (C) is correct.**

Explanation: According to the property, HCF of two numbers is also a factor of LCM of same two numbers.

Out of all the options, only (C) 500 is not a factor of 2400.

Therefore, 500 cannot be the HCF. **1**

[CBSE Marking Scheme, 2021]

2. **Option (C) is correct.**

***Explanation*:** Given that,

Product of HCF and LCM = 50

One of the numbers = 10

we know that,

Product of numbers = product of their HCF and LCM

so we have,

$\Rightarrow$ 10 × other number = 50

$\Rightarrow$ Other number = 5

3. **Option (B) is correct.**

***Explanation*:** Given that

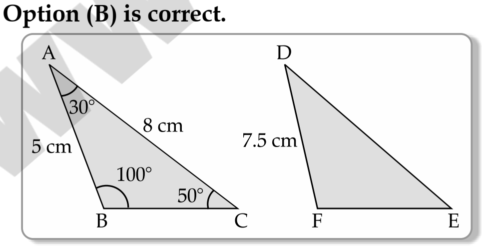
Two zeroes are 1 and 1

so the quadratic polynomial satisfying these roots is:

$\Rightarrow (x-1)(x-1)$

$\Rightarrow x^2 - 2x + 1$

4. **Option (B) is correct.**



Explanation:
$$\angle B = 180° - \angle A - \angle C$$
$$= 180° - 30° - 50°$$
$$= 100°$$

In $\Delta ABC \sim \Delta DFE$

$\therefore \quad \angle B = \angle F = 100°$

$$\frac{AB}{DF} = \frac{AC}{DE}$$

$$\frac{5}{7.5} = \frac{8}{DE}$$

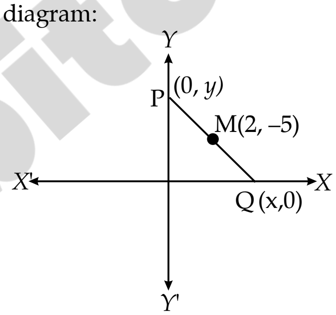
$$DE = \frac{8 \times 7.5}{5}$$

$$= 12 \text{ cm}$$ **1**

5. **Option (D) is correct.**

Explanation: Suppose the coordinates of P and Q are $(0, y)$ and $(x, 0)$ respectively.

The given situation can be represented by the following diagram:



Mid-point of a line segment having points (x_1, y_1) and (x_2, y_2)

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Now, the mid-point of P $(0, y)$ and Q $(x, 0)$ is $M\left(\frac{0+x}{2}, \frac{y+0}{2}\right)$

But it is given that, mid-point of PQ is (2, –5),

$$\therefore \ 2 = \frac{x+0}{2}$$

and

$$-5 = \frac{y+0}{2}$$

$\Rightarrow x = 4$ and $y = -10$

Hence, the coordinates of P and Q are (0, –10) and (4, 0).

6. **Option (C) is correct.**

***Explanation*:** Given polynomial is $293x^2 - 293x$

$\Rightarrow 293x(x-1)$

For the property of zeroes,

$$293x(x-1) = 0$$

Either, $293x = 0 \Rightarrow x = 0$ **1**

or, $x - 1 = 0 \Rightarrow x = 1$

Hence, it has two zeroes.

Commonly Made Error

- Students often make mistakes in analysing the zeroes as they get confused with the different terms.

Answering Tip

- Understand the different cases for zeroes.

7. Option (D) is correct.

Explanation: It is given that

$$\cot\theta = \frac{1}{\sqrt{3}} = \cot 60^\circ$$

$$\Rightarrow \quad \theta = 60^\circ$$

Substituting the value of θ

$$\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2 60^\circ + \operatorname{cosec}^2 60^\circ$$

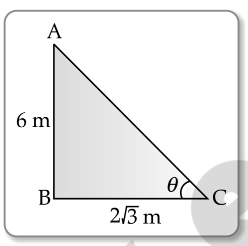
$$= (2)^2 + \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 4 + \frac{4}{3} = \frac{16}{3} = 5\frac{1}{3} \qquad 1$$

8. Option (A) is correct.

Explanation: In ΔABC, $\angle B = 90^\circ$

$$\tan\theta = \frac{6}{2\sqrt{3}} = \sqrt{3} = \tan 60^\circ \Rightarrow \theta = 60^\circ$$



1

9. Option (D) is correct.

Explanation: Since, quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle.

$\therefore \quad \angle AOB + \angle COD = 180^\circ$

$125^\circ + \angle COD = 180^\circ$

$\angle COD = 180^\circ - 125^\circ = 55^\circ \qquad 1$

10. Option (D) is correct.

Explanation: Since $\Delta ABC \sim \Delta DFE$, we have

$\angle A = \angle D = 30^\circ, \angle B = \angle F = 100^\circ, \angle C = \angle E = 50^\circ,$

And $\dfrac{AB}{DF} = \dfrac{AC}{DE} = \dfrac{BC}{FE} \Rightarrow DE = \dfrac{AC \times DF}{AB} = \dfrac{8 \times 7.5}{5} = 12.$

11. Option (B) is correct.

Explanation: Given that $AB \,||\, PQR$

$\angle B = \angle BQR = 70^\circ$ [Alternate interior angles]

$\angle OQR = \angle AMQ$ [Alternate interior angles]

As PQR and OQ are tangent and radius at contact point Q

$\therefore \quad \angle OQR = 90^\circ$

$\Rightarrow \angle 1 + \angle 70^\circ = 90^\circ$

$\Rightarrow \quad \angle 1 = 90^\circ - 70^\circ = 20^\circ$

$\therefore \quad \angle AMO = 90^\circ$

$\because$ Perpendicular from centre to chord bisect the chord

$\therefore \quad MA = MB$

$\angle QMA = \angle QMB = 90^\circ$

$MQ = MQ$ [Common]

$\therefore \Delta QMA \cong \Delta QMB$ [SAS congruence]

$\Rightarrow \quad \angle A = \angle B$

$\Rightarrow \quad \angle A = 70^\circ$ [$\because \angle B = 70^\circ$]

$\therefore \angle A + \angle AMQ + \angle 2 = 180^\circ$

[Angle sum property of a triangle]

$\Rightarrow 70^\circ + 90^\circ + \angle 2 = 180^\circ$

$\Rightarrow \quad \angle 2 = 180^\circ - 160^\circ$

$\Rightarrow \quad \angle 2 = 20^\circ$

$\therefore \quad \angle AQB = \angle 1 + \angle 2 = 20^\circ + 20^\circ = 40^\circ$

12. Option (D) is correct.

Explanation: Let $r_1 = 24$ cm and $r_2 = 7$ cm

Area of first circle $= \pi r_1^2 = \pi(24)^2 = 576\pi\ cm^2$

Area of second circle $= \pi r_2^2 = \pi(7)^2 = 49\pi$ cm^2

According to question,

Area of circle = area of first circle + area of second circle

$$\pi R^2 = 576\pi + 49\pi$$

[where, R be radius of circle]

$$R^2 = 625 \Rightarrow R = 25 \text{ cm}$$

Diameter of a circle $= 2R = 2 \times 25 = 50$ cm. 1

13. Option (A) is correct.

Explanation: When two hemispheres of equal radii are joined base to base, new solid becomes sphere and curved surface area of sphere is $4\pi r^2$. 1

14. Option (B) is correct.

Explanation: In grouping the data from ungrouped data, all the observations between lower and upper limits of class marks are taken in one group then, mid-value or class mark is taken for further calculation.

Therefore, frequencies or observations must be centred at the class marks of the classes. 1

15. Option (C) is correct.

Explanation: It is given that circumference of the circle is 176 cm^2

$\Rightarrow \quad 2\pi r = 176$

$\Rightarrow \quad 2 \times \dfrac{22}{7} \times r = 176$

$\Rightarrow \quad r = \dfrac{176 \times 7}{2 \times 22} = 28$ cm

Also, in a quadrant $\theta = 90^\circ$

$$\text{Area of quadrant} = \frac{\theta}{360^\circ} \times \pi r^2$$

$$= \frac{90^\circ}{360^\circ} \times \frac{22}{7} \times 28 \times 28$$

$$= 616 \text{ cm}^2 \qquad 1$$

16. Option (B) is correct.

Explanation: When dice is rolled twice, then out of events in which the product of the numbers that turn up 12 = {(2, 6), (6, 2), (3, 4), (4, 3)}

No. of events = 4

Total no. of events = 36

Probability (product of 12) = $\frac{4}{36} = \frac{1}{9}$. **1**

17. Option (D) is correct.

Explanation: An event that cannot occur, has 0 probability, such an event is called impossible event. **1**

18. Option (D) is correct.

Explanation: Given, $\sin\alpha = \frac{1}{2} = \sin 30°$ $\left[\because \sin 30° = \frac{1}{2}\right]$

$\Rightarrow \alpha = 30°$

And, $\cos\beta = \frac{1}{2} = \cos 60°$ $\left[\because \cos 60° = \frac{1}{2}\right]$

$\Rightarrow \quad \beta = 60°$

$\therefore \quad \alpha + \beta = 30° + 60° = 90°$

19. Option (D) is correct.

Explanation: For assertion,

Product of two numbers = 2890

and HCF × LCM = 17 × 450

= 7650

Here, product of two numbers is not equal to the product of HCF and LCM.

So, assertion is false. ½

It is clear that LCM is always greater than HCF.

So, reason is true. ½

20. Option (A) is correct.

Explanation: For assertion,

As we know that the distance between two point is $\left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$.

So, assertion is true.

For reason,

Given points are $y(4, 0)$ and $z(0, 3)$

So, distance between $yz = \sqrt{(0-4)^2 + (3-0)^2}$

$= \sqrt{16+9} = \sqrt{25}$

= 5 units.

Hence, reason is also true. ½

Section - B

21. For equation,

$(2m - 1)x + 3y - 5 = 0$ (i)

$a_1 = 2m - 1$, $b_1 = 3$ and $c_1 = -5$

and for equation

$3x + (n - 1)y - 2 = 0$ (ii)

$a_2 = 3$, $b_2 = (n - 1)$ and $c_2 = -2$

For a pair of linear equations to have infinite number of solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

or $\quad \frac{2m-1}{3} = \frac{3}{n-1} = \frac{5}{2}$ **1**

or $\quad 2(2m - 1) = 15$ and $5(n-1) = 6$

Hence, $\quad m = \frac{17}{4}$ and $n = \frac{11}{5}$ **1**

OR

Given, equation $x^2 + px + 16 = 0$ have equal roots,

if $\quad D = p^2 - 4 \times 16 = 0$ **1**

$p^2 = 64$

$\Rightarrow \quad p = \pm 8$ ½

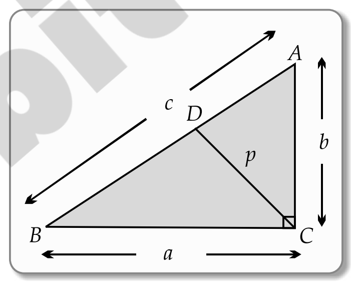
$\therefore \quad x^2 \pm 8x + 16 = 0$

$\Rightarrow \quad (x \pm 4)^2 = 0$

$x \pm 4 = 0$ **1**

$\therefore$ Roots are $x = -4$ and $x = 4$ ½

22. Let CD ⊥ AB,



then $\quad CD = p$

area of ΔABC = $\frac{1}{2}$ × base × height **1**

or, area of ΔABC with base AB

$= \frac{1}{2} \times AB \times CD = \frac{1}{2}cp$

Also, area of ΔABC with base BC

$= \frac{1}{2} \times BC \times AC = \frac{1}{2}ab$

So, $\quad \frac{1}{2}cp = \frac{1}{2}ab$

or, $\quad cp = ab.$ **Hence proved. 1**

23. Circumference of the outer circle, $2\pi r_1 = 88$ cm

$\therefore \quad r_1 = \frac{88 \times 7}{2 \times 22} = 14$ cm. ½

Circumference of the inner circle, $2\pi r_2 = 66$ cm

$\therefore \quad r_2 = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$ cm ½

= 10.5 cm

$\therefore$ Width of the ring $= r_1 - r_2$

= (14 – 10.5) cm

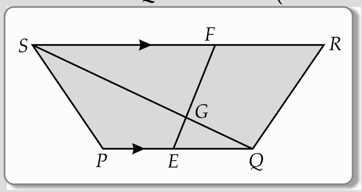
= 3.5 cm. **1**

24. In ΔGEQ and ΔGFS

$\angle EGQ = \angle FGS$ (vert. opp. angles)

$\angle EQG = \angle FSG$ (alt. angles)

$\therefore$ $\Delta GEQ \sim \Delta GFS$ (AA similarity) **1**



or, $\dfrac{EQ}{FS} = \dfrac{GQ}{GS}$

or, $EQ \times GS = GQ \times FS$. **Hence Proved 1**

[CBSE Marking Scheme, 2016]

25.
$$\text{LHS} = \frac{(\sin^4\theta + \cos^4\theta)}{1 - 2\sin^2\theta\cos^2\theta}$$
$$= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{1 - 2\sin^2\theta\cos^2\theta}$$
$$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}$$
$$= \frac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta} = 1 = \text{RHS}$$

Hence Proved

[CBSE Marking Scheme, 2015] 2

Commonly Made Error

- Students commit errors in simplification of expressions.

Answering Tip

- Follow step by step simplification to avoid errors.

OR

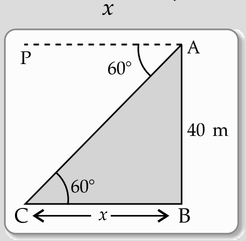
Let AB be the light house and C be the position of the boat.

Since, $\angle PAC = 60° \therefore \angle ACB = 60°$ **1**

Let BC be x m.

In ΔABC, $\dfrac{AB}{BC} = \tan 60°$

$\Rightarrow$ $\dfrac{40}{x} = \sqrt{3}$



$\Rightarrow$ $x = \dfrac{40}{\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{40\sqrt{3}}{3}$ m

Hence, the boat is $\dfrac{40\sqrt{3}}{3}$ m away from the foot of the light house.

[CBSE Marking Scheme, 2015] 1

Section - C

26. Let $\sqrt{5}$ be a rational number.

$\therefore$ $\sqrt{5} = \dfrac{p}{q}$, ½

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get

$5 = \dfrac{p^2}{q^2}$

or $p^2 = 5q^2$ ½

$\because$ p^2 is divisible by 5

$\therefore$ p is divisible by 5

Let $p = 5r$ for some positive integer r,

$p^2 = 25r^2$ ½

$\therefore$ $5q^2 = 25r^2$

or $q^2 = 5r^2$

$\because$ q^2 is divisible by 5

$\therefore$ q is divisible by 5. ½

Here p and q are divisible by 5, which contradicts the fact that p and q are co-primes.

Hence, our assumption is false

$\therefore$ $\sqrt{5}$ is an irrational number. **Hence proved. 1**

[CBSE Marking Scheme, 2020]

27. Let a and A be the first terms and d and D be the common difference of two A.P. 's

$$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a + (n-1)d]}{\frac{n}{2}[2A + (n-1)D]} \quad \mathbf{1}$$

or, $\dfrac{2a + (n-1)d}{2A + (n-1)D} = \dfrac{7n+1}{4n+27}$

or $\dfrac{a + \left(\dfrac{n-1}{2}\right)d}{A + \left(\dfrac{n-1}{2}\right)D} = \dfrac{7n+1}{4n+27}$ (i) ½

To get the ratio of 9th terms,

$\dfrac{n-1}{2} = 8$ or $n = 17$ ½

Hence, $\dfrac{t_9}{t_9'} = \dfrac{a+8d}{A+8D} = \dfrac{7\times17+1}{4\times17+27}$

$= \dfrac{120}{95} = \dfrac{24}{19}$ **1**

28. Given, $a = 1$ and $d = 4 - 1 = 3$

Let number of terms is the series be n, then

$S_n = \dfrac{n}{2}[2a + (n-1)d]$ **1**

$\Rightarrow \quad \dfrac{n}{2}[2 \times 1 + (n-1)3] = 287$

$\Rightarrow \quad \dfrac{n}{2}[2 + 3n - 3] = 287$

$\Rightarrow \quad 3n^2 - n - 574 = 0$ **1**

$\Rightarrow \quad 3n^2 - 42n + 41n - 574 = 0$

$\Rightarrow \quad 3n(n-14) + 41(n-14) = 0$

$\Rightarrow \quad (n-14)(3n+41) = 0$

Either $n = 14$ or $n = -\dfrac{41}{3}$, it is not possible.

Thus 14th term is x

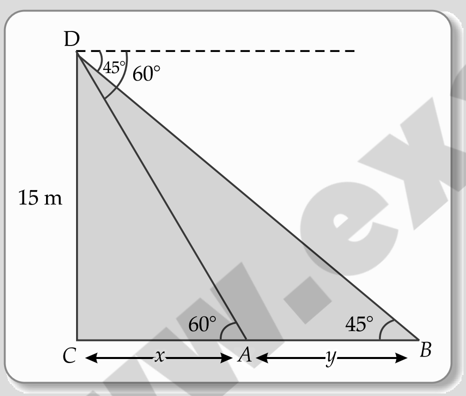
$\therefore \quad a + (n-1)d = x$

$\Rightarrow \quad x = 1 + 13 \times 3$

$= 40.$ **1**

[CBSE Marking Scheme, 2020]

29.



1

In ΔDCA, $\dfrac{DC}{CA} = \tan 60°$

$\Rightarrow \quad \dfrac{15}{x} = \sqrt{3}$

$\Rightarrow \quad x = \dfrac{15}{\sqrt{3}}$

$\Rightarrow \quad x = 5\sqrt{3}$ **1**

In ΔDCB, $\dfrac{DC}{CB} = \tan 45° = \dfrac{15}{x+y} = 1$

$\Rightarrow \quad x + y = 15$ **1**

$\Rightarrow \quad 5\sqrt{3} + y = 15$

$\Rightarrow \quad y = 15 - 5\sqrt{3}$

$= 5(3-\sqrt{3})$ m

Hence, the distance between the points

$= 5(3-\sqrt{3})$ m.

[CBSE Marking Scheme, 2017]

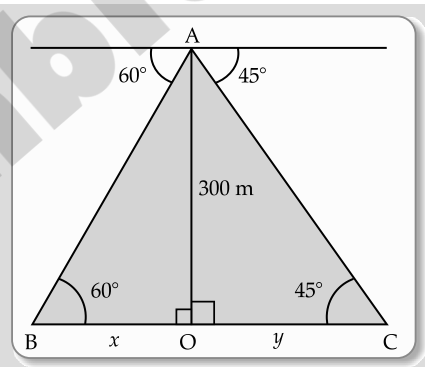
Commonly Made Error

- The concept of angle of depression and elevation is not clear to many students. That is why they are not able to draw the diagram correctly.

Answering Tip

- The concept of angle of depression and angle of elevation must be understood deeply and clearly.

OR



1

In ΔAOC, $\tan 45^\circ = \dfrac{300}{y}$

$\Rightarrow \quad 1 = \dfrac{300}{y}$ or $y = 300$ **1**

In ΔAOB, $\tan 60^\circ = \dfrac{300}{x}$

$\Rightarrow \quad \sqrt{3} = \dfrac{300}{x}$ or $x = \dfrac{300}{\sqrt{3}} = 100\sqrt{3}$

1

$\therefore \quad$ width of river $= 300 + 100\sqrt{3}$

$= 300 + 173.2$

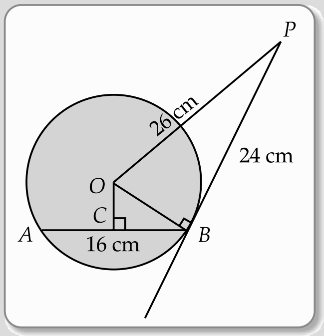
$= 473.2$ m

[CBSE Marking Scheme 2017]

30. Given, AB is a chord of circle with centre O and tangent $PB = 24$ cm, $OP = 26$ cm.

Construction: Join O to B and draw OC $\perp$ AB.

By Pythagoras theorem,



1

$$OB = \sqrt{(26)^2 - (24)^2}$$

$$= \sqrt{676 - 576} = \sqrt{100}$$

$$= 10 \text{ cm} \quad \textbf{1}$$

Now, in ΔOBC, $BC = \frac{1}{2} AB = \frac{16}{2} = 8$ cm

(perpendicular drawn from the centre to a chord bisects it.)

$$OB = 10 \text{ cm}$$
$$OC^2 = OB^2 - BC^2$$
$$= 10^2 - 8^2$$
$$OC^2 = 36$$
$$OC = 6 \text{ cm} \quad \textbf{1}$$

$\therefore$ Distance of the chord from the centre = 6 cm.

[CBSE Marking Scheme, 2014]

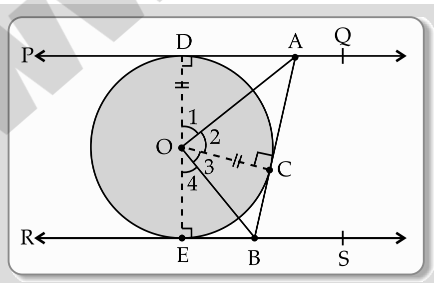
Commonly Made Error

- Some candidates are not versed with the circle properties e.g., could not well identify $\angle OBP = 90^\circ$ (angle between radius and tangent)

Answering Tip

- Candidates should be familiar with the properties of circle.

OR



½

In ΔAOD and ΔAOC

$OD = OC$ (radius of circle)

$OA = OA$ (common)

$\angle ADO = \angle OCA$

(90°, $\because$ angle between radius and tangent]

$\Delta AOD \cong \Delta AOC$ [RHS] **1**

$\Rightarrow$ $\angle 1 = \angle 2$ (by cpct) (i) ½

Similarly, $\angle 4 = \angle 3$ (ii) ½

On adding (i) and (ii), we get

$$\angle 1 + \angle 4 = \angle 2 + \angle 3 = \frac{1}{2}(180^\circ)$$

$\Rightarrow$ $\angle 2 + \angle 3 = 90^\circ$

or $\angle AOB = 90^\circ$ ½

Hence Proved

[CBSE Marking Scheme, 2019]

31. Total number of all possible outcomes = $6^2 = 36$

(i) The sum less than 7 = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)

No. of favourable outcomes = 15

P (have sum less than 7) = $\frac{15}{36} = \frac{5}{12}$ **1**

(ii) Product less than 16 = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2)

No. of favourable outcomes = 25

$\therefore$ P (have a product less than 16)

$$= \frac{25}{36} \quad \textbf{1}$$

(iii) Doublet of odd numbers = (1, 1), (3, 3), (5, 5)

No. of favourable outcomes = 3

$\therefore$ P (a doublet of odd number)

$$= \frac{3}{36} = \frac{1}{12} \quad \textbf{1}$$

[CBSE Marking Scheme, 2017]

Section - D

32. Let the smaller tap fills the tank in x hours

$\therefore$ The larger tap fills the tank in $(x - 2)$ hours. Time taken by both the taps together

$$= \frac{15}{8} \text{ h.} \quad \textbf{1}$$

Therefore, $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$ **2**

$\Rightarrow$ $4x^2 - 23x + 15 = 0$ ½

$\Rightarrow$ $(4x - 3)(x - 5) = 0$

$\Rightarrow$ $x = \frac{3}{4}$ or $x = 5$

Now, $x \neq \frac{3}{4}$ therefore $x = 5$ **1**

Hence, smaller and larger tap can fill the tank separately is 5 h and 3 h, respectively. ½

[CBSE Marking Scheme, 2017]

OR

Let speed of the boat in still water = x km/h
and speed of the current = y km/h
Downstream speed = $(x+y)$ km/h ½
Upstream speed = $(x - y)$ km/h ½

$$\frac{24}{x+y} + \frac{16}{x-y} = 6 \quad \text{(i) ½}$$

$$\frac{36}{x+y} + \frac{12}{x-y} = 6 \quad \text{(ii) ½}$$

Let $\frac{1}{x+y} = u$

and $\frac{1}{x-y} = v$ ½

Put in the above equation, we get

$24u + 16v = 6$

or, $12u + 8v = 3$ (iii) ½

$36u + 12v = 6$

or, $6u + 2v = 1$ (iv)

Multiplying (iv) by 4, we get

$24u + 8v = 4$ (v) ½

Subtracting (iii) by (v), we get

$12u = 1$

$\Rightarrow u = \frac{1}{12}$

Putting the value of u in (iv), we get, $v = \frac{1}{4}$ ½

$\Rightarrow \frac{1}{x+y} = \frac{1}{12}$

and $\frac{1}{x-y} = \frac{1}{4}$

$\Rightarrow x + y = 12$ and $x - y = 4$

Thus, speed of the boat in still water = 8 km/h ½
Speed of the current = 4 km/h ½

33. $PT = \sqrt{169 - 25} = 12$ cm

and $TE = OT - OE = 13 - 5$
$= 8$ cm ½ + ½

Let $PA = AE = x$. (tangents)

Then, $TA^2 = TE^2 + EA^2$ 1

or, $(12 - x)^2 = 8^2 + x^2$ 1

$24x = 80$

or, $x = 3.3$ cm. (approx.) 1

Thus $AB = 2 \times x = 2 \times 3.3$
$= 6.6$ cm. (approx.) 1

[CBSE Marking Scheme, 2016]v

34. Volume of water in cone

$= \frac{1}{3}\pi r^2 h$ ½

$= \frac{1}{3}\pi \times (5)^2 \times 8$ ½

$= \frac{200}{3}\pi$ cm^3 ½

Volume of water flown out

$= \frac{1}{4} \times \frac{200}{3}\pi = \frac{50}{3}\pi$ cm^3 1

Let the radius of one spherical ball be r cm 1½

$\therefore \frac{4}{3}\pi r^3 \times 100 = \frac{50}{3}\pi$

$r^3 = \frac{50}{4 \times 100} = \frac{1}{8}$

or, $r = \frac{1}{2} = 0.5$ cm 1

[CBSE Marking Scheme, 2015]

35.

Marks	Number of students (f_i)	Mid values (x_i)	$u_i = \frac{x_i - A}{h}$	$f_i u_i$
30 - 35	14	32.5	–3	–42
35 - 40	16	37.5	–2	–32
40 - 45	28	42.5	–1	–28
45 - 50	23	47.5	0	0
50 - 55	18	52.5	1	18
55 - 60	8	57.5	2	16
60 - 65	3	62.5	3	9
	N = Σf_i = 110			$\Sigma f_i u_i$ = **–59**

2

Mean $= 47.5 + \frac{(-59)}{110} \times 5$ 1

$= 47.5 - 2.682$ 1

$= 44.82$ 1

[CBSE Marking Scheme, 2019]

OR

(a) Total number of cards = 52

Number of non face cards = 52 – 12
= 40

P (non-face cards) $= \frac{40}{52} = \frac{10}{13}$ 1

(b) Number of black kings = 2

Number of red queens = 2

P (a black king or a red queen) $= \frac{4}{52} = \frac{1}{13}$ 2

(c) Number of spade cards = 13

P (Spade cards) $= \frac{13}{52} = \frac{1}{4}$ 2

[CBSE Marking Scheme, 2016]

Section - E

36. **(i)** Mid-point of J(6, 17) and I(9, 16) is

$x = \frac{6+9}{2}$ and $y = \frac{17+16}{2}$

$x = \frac{15}{2}$ and $y = \frac{33}{2}$ **1**

(ii) The distance of the point P from the Y-axis = 4. **1**

(iii) The co-ordinates of $A = (1, 8)$

The coordinates of $B = (4, 10)$

Also, $m = 1$ and $n = 3$

then, $(x, y) = \left(\frac{1\times4+3\times1}{1+3}, \frac{1\times10+3\times8}{1+3}\right)$

$= \left(\frac{7}{4}, \frac{34}{4}\right)$

$= (1.75, 8.5)$ **2**

OR

Let point be P(x, y)

$PQ^2 = PS^2$

or, $(x-9)^2 + (y-8)^2 = (x-17)^2 + (y-8)^2$

or, $x - 13 = 0$

$x = 13$

37. **(i)** For cuboid

$l = 15$ cm, $b = 10$ cm and $h = 3.5$ cm

Volume of the cuboid $= l \times b \times h$

$= 15 \times 10 \times 3.5$

$= 525 \text{ cm}^3$ **1**

(ii) For conical depression:

$r = 0.5$ cm,

$h = 1.4$ cm

Volume of conical depression

$= \frac{1}{3}\times\frac{22}{7}\times0.5\times0.5\times1.4$

$= \frac{11}{30} \text{ cm}^3$ **2**

(iii) Volume of four conical depressions

$= 4\times\frac{11}{30} = 1.47 \text{ cm}^3$

OR

Volume of the wood in the entire stand

= Volume of cuboid

– Volume of 4 conical depressions

= 525 – 1.47

= 523.53 cm^3

38. **(i)** (4, – 2) **[CBSE Marking Scheme, 2020]**

Detailed Solution:

$x^2 - 2x - 8 = 0$

or, $x^2 - 4x + 2x - 8 = 0$

or, $x(x-4) + 2(x-4) = 0$

or, $(x-4)(x+2) = 0$

or, $x = 4, x = -2$ **1**

(ii) We know that the number of zeroes of polynomial is equal to number of points where the graph of polynomial intersects X-axis. **1**

(iii) $x^2 - 36$ **[CBSE Marking Scheme, 2020]**

Detailed Solution:

Give: One root $(\alpha) = 6$

Sum of roots $(\alpha + \beta) = 0$

Hence, the required equation

$x^2 (\alpha + \beta) x + \alpha\beta = 0$

$x^2 - 0 + (6)(-6) = 0$

$x^2 - 36 = 0$

OR

We have,

$f(x) = (x-2)^2 + 4$

$= x^2 + 4 - 4x + 4$

$= x^2 - 4x + 8.$

Since, $D = b - 4ac < 0$

Hence no real value of x is possible, *i.e.*, no zeros. **1**

SOLUTIONS

Sample Question Paper-4

Mathematics Standard (041)

Section - A

1. **Option (C) is correct.**

Explanation: Any odd integer can be written as $2m + 1$.

Put $n = 2m + 1$ in $n^2 - 1$

$n^2 - 1 = (n+1)(n-1) = (2m+2)(2m) = 4m(m+1)$

The product of two consecutive numbers is divisible by 2. Thus, $m(m + 1)$ is divisible by 2.

Let $m(m + 1) = 2k$

$$n^2 - 1 = 4m(m + 1) = 4 \times 2k = 8k$$

Thus, if n is an odd integer then $n^2 - 1$ is divisible by 8. **1**

2. **Option (B) is correct.**

Explanation: Since $a = x^3y^2 = x \times x \times x \times y \times y$

$$b = xy^3 = x \times y \times y \times y$$

$\therefore$ HCF $(a, b) = x \times y \times y = xy^2$ **1**

3. **Option (C) is correct.**

Explanation: For equal roots $b^2 - 4ac = 0$ or $b^2 = 4ac$, b^2 is always positive, so, $4ac$ must be positive, i.e., product of a and c must be positive, i.e., a and c must have same sign either positive or negative. **1**

4. **Option (B) is correct.**

Explanation: $-x^2 + 3x - 3 = 0$

On comparing with $ax^2 + bx + c = 0$, we get

$a = -1, b = 3, c = -3$

$\therefore$ Sum of the roots $= \dfrac{-b}{a} = \dfrac{-3}{-1} = 3.$ **1**

5. **Option (D) is correct.**

Explanation: Let (x, y) be the required point

Then, $x = \dfrac{mx_2 + nx_1}{m+n}$ and $y = \dfrac{my_2 + ny_1}{m+n}$ ...(i)

Here, $x_1 = 7, y_1 = -6, x_2 = 3, y_2 = 4, m = 1$ and $n = 2$

$\therefore x = \dfrac{1(3)+2(7)}{1+2} \Rightarrow x = \dfrac{3+14}{3} = \dfrac{17}{3}$

and $y = \dfrac{1(4)+2(-6)}{1+2} \Rightarrow y = \dfrac{4-12}{3} = \dfrac{-8}{3}$

So, the required point $(x, y) = \left(\dfrac{17}{3}, \dfrac{-8}{3}\right)$ lies in IV quadrant. **1**

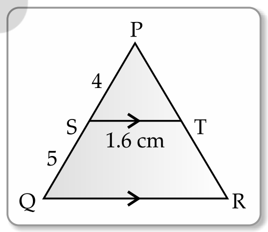
6. **Option (C) is correct.**

Explanation: $ST \parallel QR$ (given)

$$\frac{PS}{PQ} = \frac{ST}{QR} \quad \text{(corr. B.P.T.)}$$

$$\Rightarrow \frac{4}{9} = \frac{1.6}{QR}$$

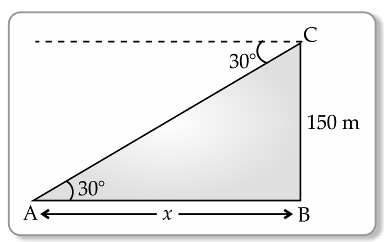
$$\therefore QR = \frac{9}{4} \times 1.6 = 3.6 \text{ cm.}$$



7. **Option (B) is correct.**

Explanation: Let the distance of the car from the tower BC be x metre.

In ΔABC, $\angle B = 90^\circ$



$$\tan\theta = \frac{CB}{AB}$$

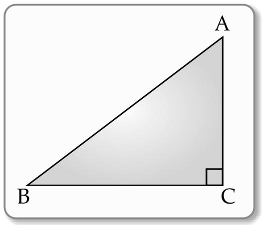
$$\tan 30^\circ = \frac{150}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{150}{x}$$

$$x = 150\sqrt{3} \text{ m}$$

8. Option (A) is correct.

Explanation: We know that, in ΔABC,



Sum of three angles = 180°

i.e., $\angle A + \angle B + \angle C = 180°$

$\angle C = 90°$ [given]

$\angle A + \angle B + 90° = 180°$

$\Rightarrow \quad A + B = 90°$

$\therefore \quad \cos(A + B) = \cos 90° = 0$

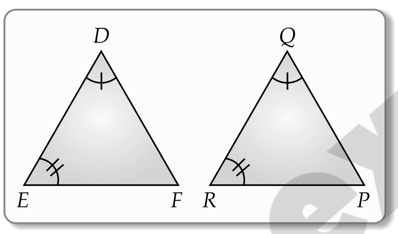
9. Option (B) is correct.

Explanation: In ΔDEF and ΔPQR, $\angle D = \angle Q$ and $\angle R = \angle E$. By AA similarity, we get $\Delta DEF \sim \Delta QRP$.

Hence, $\frac{DE}{QR} = \frac{EF}{RP} = \frac{DF}{QP}$.

From this we get,

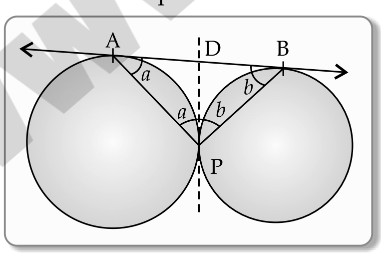
$$\frac{DE}{DF} \neq \frac{FE}{RP}$$ 1



So, it is not true.

10. Option (D) is correct.

Explanation: Let $\angle BAP$ be a and $\angle ABP$ be b and tangent at a contact point P cut AB at D.



and $\angle APB = a + b$

In ΔAPB,

$\angle BAP + \angle APB + \angle PBA = 180°$

(Angle sum property of a triangle)

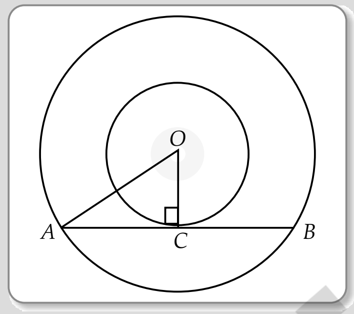
$a + (a + b) + b = 180°$

$2(a + b) = 180° \Rightarrow a + b = 90°$

So, $\angle APB = 90°$. 1

11. Option (B) is correct.

Explanation: Here, $AO = 5$ cm and $OC = 3$ cm



$$AC = \sqrt{(5)^2 - (3)^2}$$

$$= \sqrt{25 - 9}$$

$$= \sqrt{16} = 4 \text{ cm}$$

Length of chord, $AB = 8$ cm. 1

[CBSE Marking Scheme, 2012]

12. Option (D) is correct.

Explanation: Area of sector of angle θ

$$= \frac{\theta}{360°} \times \pi R^2$$

Area of sector of angle, p

$$= \frac{p}{360°} \times \pi R^2 = \frac{p}{720°} \times 2\pi R^2$$ 1

13. Option (C) is correct.

Volume = 12π

$\frac{\cancel{4}}{3}\cancel{\pi}r^3 = \cancel{12}^{3}\cancel{\pi}$

$r^3 = 9$

$r = 3^{\frac{2}{3}}$ cm.

Ans: (c) $3^{\frac{2}{3}}$ cm

[Topper Answer, 2020] 1

14. Option (C) is correct.

Explanation: In the given formula, a is assumed mean from class marks (x_i) and $d_i = x_i - a$

Therefore, d_i is the deviation of class mark (mid-value) from the assumed mean 'a'.

15. Option (D) is correct.

Explanation:

Each side of Savita's square yard = 20 m

$\therefore$ Area of square yard $= (\text{side})^2$

$= (20)^2 = 400 \text{ m}^2$

Radius of circle, where light of lamp covers

$= 10$ m

$\therefore$ Area of circle $= \pi r^2$

$= \pi (10)^2 = 100\,\pi \text{ m}^2$

Hence, the area of the yard not lit by the lamp

= area of square yard

– area of circle

$= (400 - 100\pi) \text{ m}^2$.

16. **Option (C) is correct.**

Explanation: Probability of an event + probability of its complementary event = 1

$\therefore$ p + probability of complement = 1

Probability of complement $= 1 - p$

17. **Option (B) is correct.**

***Explanation*:** Probability lies between 0 and 1 and when it is converted into percentage it will be between 0 and 100. So, cannot be negative. **1**

18. **Option (B) is correct.**

Explanation:

$$9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$$
$$= 9\,(1) \qquad [\because \sec^2 A - \tan^2 A = 1]$$
$$= 9$$

19. **Option (B) is correct.**

Explanation: For assertion,

Smallest number divisible by 306 and 657

= LCM (306, 657)

= 22338.

Hence, assertion is true.

For reason,

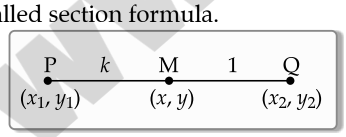
HCF (306, 657) = 9.

So, reason is also true.

20. **Option (A) is correct.**

Explanation: For assertion,

It is clear that the line, segment joining two any points is divided by the y-axis *i.e.*, $x = 0$ is the ratio k : 1 is called section formula.



$\therefore$ Assertion is true.

For reason,

Let the point on Y-axis which divides the line PQ is M(0, y) and the ratio be k : 1.

According to the section formula,

$$M(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$M(0, y) = \left(\frac{5k + (-3)}{k+1}, \frac{k(7) + 1(2)}{k+1}\right)$$

On comparing, we get

$$0 = \frac{5k - 3}{k+1}$$

or $\quad 5k - 3 = 0$

or $\quad k = \dfrac{3}{5}$

So the required ratio is 3 : 5

Hence, reason is also true.

Section - B

21. Given quadratic polynomial $= x^2 - 6x + k$

$\therefore$ Comparing it with $ax^2 + bx + c$, we get

$a = 1, b = -6$ and $c = k$ ½

The sum of zeroes $= -\dfrac{b}{a}$

i.e., $\quad \alpha + \beta = -\left(\dfrac{-6}{1}\right) = 6$ ½

and product of zeroes,

$$\alpha\beta = \frac{c}{a} = \frac{k}{1} = k$$

Now, $\quad \alpha - \beta = 2 \quad$ (given)

$\Rightarrow \quad (\alpha - \beta)^2 = 4$

$\Rightarrow \quad (\alpha + \beta)^2 - 4\alpha\beta = 4$ ½

$\Rightarrow \quad 36 - 4k = 4$

$\Rightarrow \quad 4k = 32$

$\Rightarrow \quad k = 8$ ½

OR

Let $p(y) = 7y^2 - \dfrac{11}{3}y - \dfrac{2}{3}$

$$= \frac{1}{3}\left(21y^2 - 11y - 2\right)$$
$$= \frac{1}{3}\left[21y^2 - 14y + 3y - 2\right]$$
$$= \frac{1}{3}\left[7y(3y-2) + 1(3y-2)\right]$$
$$= \frac{1}{3}\left[(7y+1)(3y-2)\right] \quad ½$$

$\therefore$ Zeroes are $\dfrac{2}{3}$ and $\dfrac{-1}{7}$ ½

Verification:

Now, $\quad$ sum of zeroes $= \dfrac{2}{3} - \dfrac{1}{7}$

$= \dfrac{11}{21} \Rightarrow \dfrac{-b}{a} = \dfrac{11}{21}$

$\therefore \quad$ sum of zeroes $= \dfrac{-b}{a} = \dfrac{11}{21}$ ½

Product of zeroes $= \left(\dfrac{2}{3}\right)\left(-\dfrac{1}{7}\right) = \dfrac{-2}{21}$

$\dfrac{c}{a} = -\dfrac{2}{3}\left(\dfrac{1}{7}\right) = \dfrac{-2}{21}$ $\therefore$ product of zeroes $= \dfrac{c}{a} = -\dfrac{2}{21}$

½

[CBSE Marking Scheme 2019]

22. $\dfrac{\sin 90^\circ}{\cos 45^\circ}+\dfrac{1}{\text{cosec } 30^\circ} = \dfrac{1}{\frac{1}{\sqrt{2}}}+\dfrac{1}{2}$ **1**

$= \sqrt{2}+\dfrac{1}{2} = \dfrac{2\sqrt{2}+1}{2}$ **1**

23. Volume of cylinder: volume of cone: volume of hemisphere

$= \pi r^2 h : \dfrac{1}{3}\pi r^2 h : \dfrac{2}{3}\pi r^3$ **1**

$= \pi r^2 h : \dfrac{1}{3}\pi r^2 h : \dfrac{2}{3}\pi r^2 \times h$ $(\because h = r)$

$= 1 : \dfrac{1}{3} : \dfrac{2}{3}$

or, 3 : 1 : 2 is the required ratio. **1**

OR

Area of square = $(\text{side})^2 = 121 \text{ cm}^2$

Side of square = $\sqrt{121}$ = 11 cm **1**

Perimeter of square = 4 × 11 = 44 cm.

Circumference of the circle

= Perimeter of the square

= 44 cm **1**

[CBSE Marking Scheme, 2012]

24. Given: AP and BP are tangents of circle having centre O. ½

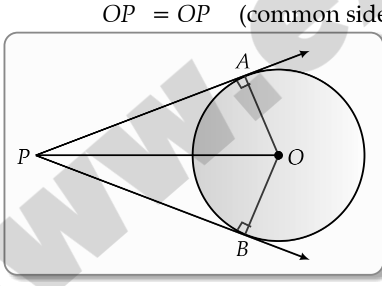
To Prove: $AP = BP$

Construction: Join OP, AO and BO.

Proof: ΔOAP and ΔOBP

$OA = OB$ (radius of circle)

$OP = OP$ (common side)



½

∠OAP = ∠OBP = 90° (tangent angle)

ΔOAP ≅ ΔOBP (RHS congruency rule)

∴ $AP = BP$ (CPCT) **1**

Hence proved.

25. In ΔPBC and ΔPDE, we have

$\dfrac{PB}{PD} = \dfrac{5}{10} = \dfrac{1}{2}$

and $\dfrac{PC}{PE} = \dfrac{6}{12} = \dfrac{1}{2}$

$\Rightarrow \dfrac{PB}{PD} = \dfrac{PC}{PE}$ **1**

And $\angle BPC = \angle EPD$ as vertically opposite angles are equal.

Thus, by using *SAS* similarity criterion, Hence, $\Delta PBC \sim \Delta PDE$. **1**

Section - C

26. Let us assume, to the contrary, that $2\sqrt{5}-3$ is a rational number

$\therefore 2\sqrt{5}-3 = \dfrac{p}{q}$, where p and q are integers and $q \neq 0$

$\Rightarrow \sqrt{5} = \dfrac{p+3q}{2q}$ **1**

Since p and q are integers

$\therefore \dfrac{p+3q}{2q}$ is a rational number **1**

$\therefore \sqrt{5}$ is a rational number which is contradiction as $\sqrt{5}$ is an irrational number

Hence, our assumption is wrong and hence $2\sqrt{5}-3$ is an irrational number.

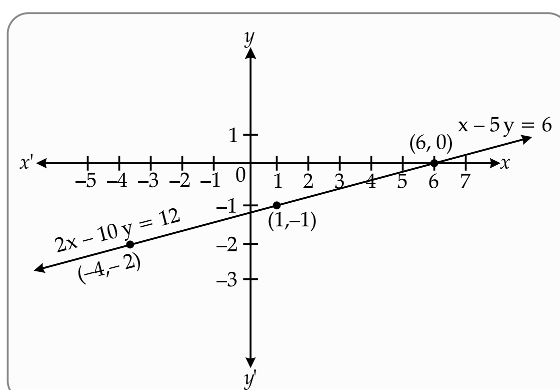
[CBSE SQP Marking Scheme, 2020] 1

27. Given, $x - 5y = 6 \Rightarrow y = \dfrac{x-6}{5}$

x	6	1	– 4
y	0	– 1	– 2

and $2x - 10y = 12 \Rightarrow y = \dfrac{x-6}{5}$ **1**

x	6	1	– 4
y	0	– 1	– 2



1

Since, the lines are coincident, so the system of linear equations is consistent with infinitely many solutions. **1**

OR

Let the fraction be $\frac{x}{y}$

$\therefore \quad \frac{x-2}{y} = \frac{1}{3}$ (i) **1**

and $\quad \frac{x}{y-1} = \frac{1}{2}$ (ii) **1**

Solving Eqs. (i) and (ii), we get

$x = 7$ and $y = 15$ **1**

$\therefore$ Required fraction is $\frac{7}{15}$

Commonly Made Error

- Candidates do error in simplifying this type of equations.

Answering Tip

- Adequate practice is necessary for simplifying these type of equations.

28. We have, $A = (a^2 + b^2)$, $B = 2(ac + bd)$ and $C = (c^2 + d^2)$

For no real roots, $D < 0$ **1**

i.e., $D \Rightarrow b^2 - 4ac < 0$

$$b^2 - 4ac = [2(ac+bd)]^2 - 4(a^2+b^2)(c^2+d^2)]$$ ½

$$= 4[a^2c^2 + 2abcd + b^2d^2] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$$

$$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$$

$$= -4[a^2d^2 + b^2c^2 - 2abcd]$$ **1**

$$= -4[ad - bc]^2$$

Since, $\quad ad \neq bc$

Therefore, $\quad D < 0$

Hence, the equation has no real roots. ½

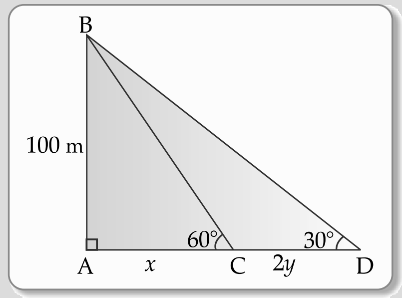
Commonly Made Error

- Students often make mistakes in analyzing the nature of roots as they get confused with the conditions.

Answering Tip

- Understand the different conditions for nature of roots.

29.



½

Let the speed of the boat by y m/min

$\therefore \quad CD = 2y$

In ΔBAC, $\tan 60° = \sqrt{3} = \frac{100}{x} \Rightarrow x = \frac{100}{\sqrt{3}}$ ½

In ΔDAB, $\tan 30° = \frac{1}{\sqrt{3}} = \frac{100}{x+2y}$

$\Rightarrow x + 2y = 100\sqrt{3}$ **1**

$\therefore \quad \Rightarrow \frac{100}{\sqrt{3}} + 2y = 100\sqrt{3}$

$$y = 50\sqrt{3} - \frac{50}{\sqrt{3}}$$

Speed of boat $= \frac{y}{2} = 57.73$

or speed of boat = 57.73 m/min **1**

[CBSE Marking Scheme 2019]

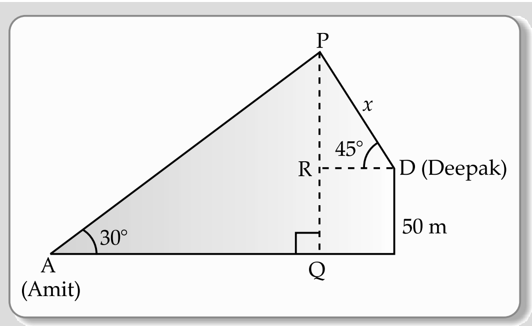
Commonly Made Error

- Most candidates are unable to draw the diagram as per the given data and lose their marks.

Answering Tip

- Do sufficient practice of drawing correct diagrams for problem based on the questions.

OR



½

In ΔAPQ,

$$\frac{PQ}{AP} = \sin 30° = \frac{1}{2} \qquad \tfrac{1}{2}$$

$$PQ = (200)\left(\frac{1}{2}\right) = 100 \text{ m} \qquad \tfrac{1}{2}$$

$$PR = 100 - 50 = 50 \text{ m} \qquad \tfrac{1}{2}$$

In ΔPRD,

$$\frac{PR}{PD} = \sin 45° = \frac{1}{\sqrt{2}}$$

$$PD = (PR)(\sqrt{2}) = 50\sqrt{2}\text{m} \qquad 1$$

The distance between Deepak and bird is $50\sqrt{2}$ m.

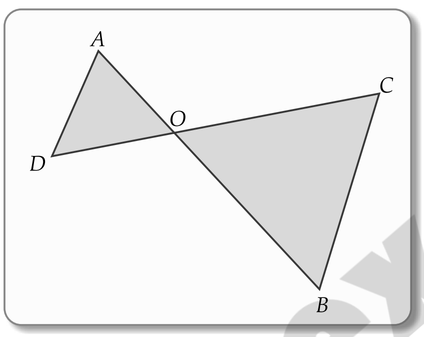
[CBSE Marking Scheme 2019]

30. Since, $OA \times OB = OC \times OD$

$$\therefore \quad \frac{OA}{OD} = \frac{OC}{OB} \qquad 1$$

$$\angle AOD = \angle COB$$

(vertically opposite angles)



$$\therefore \quad \Delta AOD \sim \Delta COB \qquad 1$$

(*SAS* similarity)

$\therefore$ So we have, $\angle A = \angle C$ and $\angle B = \angle D$. 1

(corresponding angles of similar triangles)

31. Total number of outcomes = 8 ½

Favourable number of outcomes (HHH, TTT) = 2 ½

Probability (getting success) = $\frac{2}{8}$ or $\frac{1}{4}$ 1

$\therefore$ Probability (losing the game) = $1 - \frac{1}{4} = \frac{3}{4}$ 1

[CBSE Marking Scheme, 2019]

Commonly Made Error

- Most of the students commit errors in writing correct total outcomes. Sometimes, favorable outcomes are also incorrectly written by students.

Answering Tip

- Remember to list the favorable outcomes of tossing a coin 3 times.

Section - D

32. **(i)** Since, distance between the first potato and the bucket = 5 m

and also there are 10 potatoes which are 3 m apart.

Distance covered by the competitor in first pick

$= 2 \times 5 = 10$ m

Distance covered by the competitor in second pick

$= 2 \times (5 + 3 \times 1) = 2 \times 8$

$= 16$ m

Distance covered by the competitor in third pick

$= 2 \times (5 + 3 \times 2)$

$= 2 \times (5 + 6) = 22$ m 1

Similarly, distance covered by the competitor in 10th pick

$= 2 \times (5 + 3 \times 9)$

$= 2 \times (5 + 27) = 64$ m

Therefore, the sequence becomes,

10, 16, 22, ..., 64 1

Let S be the total distance covered by the competitor. i.e.,

$$S = 10 + 16 + 22 + \ldots.. + 64$$

Here,

$$a = 10, d = 16 - 10 = 6, n = 10, l = 64$$

Now, $$S_n = \frac{n}{2}[a + l]$$

$$\therefore \quad S_{10} = \frac{10}{2}[10 + 64] = 5(74)$$

$$= 370 \text{ m} \qquad 2$$

Hence, the total distance covered by the competitor = 370 m.

(ii) Arithmetic progression. 1

OR

Let Arun's marks in Hindi be x

Then, marks in English $= 30 - x$ 1

$\therefore \quad (x + 2)(30 - x - 3) = 210$ 1

$\Rightarrow \quad x^2 - 25x + 156 = 0$

$\Rightarrow \quad (x - 13)(x - 12) = 0$

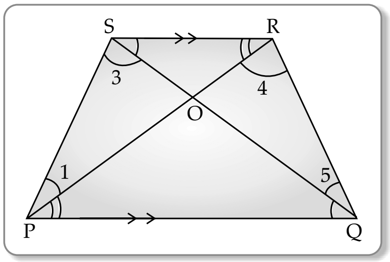
$\Rightarrow \quad x = 13$ or $x = 12$ 1

$\Rightarrow$ Marks in English : $30 - 13 = 17$

or $\quad 30 - 12 = 18$ 1

$\therefore$ Marks in Hindi and English are (13, 17) or (12, 18) 1

33.



1

In ΔPOQ and ΔROS

$\left.\begin{aligned}\angle P &= \angle R\\ \angle Q &= \angle S\end{aligned}\right\}$ Alternate angles **1**

$\therefore$ $\Delta POQ \sim \Delta ROS$ (AA similarity) **1**

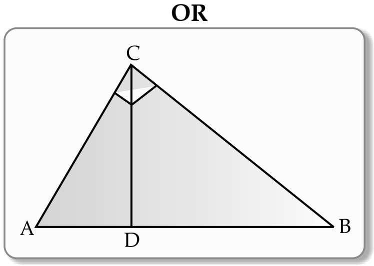
Now, in ΔPOS and ΔQOR

(i) $\angle POS = \angle QOR$ (vertically opposite angle)

(ii) $\angle 3 = \angle 4$

(iii) $\therefore$ $\Delta POS \sim \Delta QOR$ (By AA similarity)

OR



In ΔACB and ΔADC

$\angle C = \angle D$ (both 90°)

$\angle A = \angle A$ (common) **1**

$\therefore \Delta ACB \sim \Delta ADC$ (AA similarity)

$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{AD}{CD}$ **...(i) 1**

Also, $\Delta ACB \sim \Delta CDB$ (AA similarity)

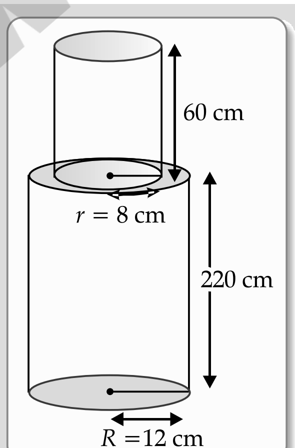
$\Rightarrow \quad \dfrac{AC}{BC} = \dfrac{CD}{BD}$ **...(ii) 1**

using Eqs. (i) and (ii), we get

$\dfrac{AD}{CD} = \dfrac{CD}{BD}$ **1**

$\Rightarrow \quad CD^2 = AD \times BD$ **Hence proved 1**

34.



1

Radius of lower cylinder $= R = 12$ cm

Radius of upper cylinder $= r = 8$ cm

Height of upper cylinder $= h = 60$ cm

Height of lower cylinder $= H = 220$ cm **1**

Volume of solid iron pole $= \pi R^2 H + \pi r^2 h$ **1**

$= 3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$

$= 111{,}532.8 \text{ cm}^3$ **1**

Mass of the pole $= 111{,}532.8 \times 8$ g

$= 892.2624$ kg. **1**

[CBSE Marking Scheme, 2019]

Commonly Made Error

- Some students do not know how to find the mass of the pole from the volume.

Answering Tip

- Students must practice more questions to learn more concepts.

35. Here, Modal class = 45 – 55 **1**

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h \quad \textbf{1}$$

and $l = 45$, $f_0 = 31$, $f_1 = 33$

$f_2 = 17$ and $h = 10$ **1**

$$\text{Mode} = 45 + \frac{33-31}{66-31-17} \times 10 \quad \textbf{1}$$

$$= 45 + \frac{2}{18} \times 10$$

$$\text{Mode} = 46.1 \quad \textbf{1}$$

[CBSE Marking Scheme, 2015]

Section - E

36. **(i)** First term $(a) = 51$

common difference $(d) = -2$

So, A.P. $= 51, 49, 47 \ldots\ldots$. **1**

(ii) Given: Goal = 31 s

n = number of days

$\therefore \quad a_n = 31$

$a + (n-1)d = 31$

$51 + (n-1)(-2) = 31$

$51 - 2n + 2 = 31$

$-2n = 31 - 53$

$-2n = -22$

$n = 11$ days **1**

(iiii) $a_n = 2n + 3$ (given)

$\therefore \quad a_1 = 2 \times 1 + 3 = 5$

$a_2 = 2 \times 2 + 3 = 7$

$a_3 = 2 \times 3 + 3 = 9$ **1**

So, common difference $= a_2 - a_1 = a_3 - a_2$

$= 7 - 5 = 9 - 7$

$= 2.$

So, $d = 2$ **1**

OR

Since, $2x$, $x + 10$, $3x + 2$ are in A.P., this common difference will remain same. **1**

$$x + 10 - 2x = (3x + 2) - (x + 10)$$
$$10 - x = 2x - 8$$
$$3x = 18$$
$$x = 6$$ **1**

37. **(i)** Point A lies in $x = 3$ and $y = 4$.

$\therefore$ A (3, 4) is the correct position. ½

Point D lies at $x = 6$ and $y = 1$.

So, the correct position of D is (6, 1). ½

(ii) Position of A = (3, 4)

Position of B = (6, 7)

$\therefore$ Distance of AB $= \left|\sqrt{(3-6)^2 + (4-7)^2}\right|$

$= \left|\sqrt{(-3)^2 + (-3)^2}\right|$

$= \sqrt{18} = 3\sqrt{2}$ unit. **1**

(iii) $\therefore$ Equation of line CD = equation of line through C (9, 4) and D(6, 1)

i.e., $y - 4 = \dfrac{1-4}{6-9}(x-9)$ **1**

$\Rightarrow$ $y - 4 = x - 9$

$\Rightarrow$ $x - y - 5 = 0.$ **1**

OR

Position of B = (6, 7)

and position of C = (9, 4)

$\therefore$ Mid-point of B and C $= \left(\dfrac{6+9}{2}, \dfrac{7+4}{2}\right)$

$= \left(\dfrac{15}{2}, \dfrac{11}{2}\right)$.

38. **(i)** From figure, the electrician is required to reach at the point B on the pole AD.

So, $BD = AD - AB$

$= (5 - 1.3)$ m $= 3.7$ m **1**

(ii) In ΔBDC,

$\sin 60° = \dfrac{BD}{BC}$

$\Rightarrow$ $\dfrac{\sqrt{3}}{2} = \dfrac{3.7}{BC}$

$BC = \dfrac{3.7 \times 2}{\sqrt{3}} = \dfrac{3.7 \times 2 \times \sqrt{3}}{3}$

$\Rightarrow$ $BC = 4.28$ m (approx.) **1**

(iii) In ΔBDC,

$\therefore$ $\cot 60° = \dfrac{DC}{BD}$ **1**

$\Rightarrow$ $\dfrac{1}{\sqrt{3}} = \dfrac{DC}{3.7}$

$\Rightarrow$ $DC = \dfrac{3.7}{\sqrt{3}} = \dfrac{3.7\sqrt{3}}{3}$

$\Rightarrow$ $DC = 2.14$ m (approx.) **1**

OR

In ΔBDC,

$\therefore$ $\sin 30° = \dfrac{BD}{BC}$ **1**

$\Rightarrow$ $\dfrac{1}{2} = \dfrac{3.7}{BC}$

$\Rightarrow$ $BC = 3.7 \times 2 = 7.4$ m. **1**

SOLUTIONS
Sample Question Paper-5
Mathematics Standard (041)

Section - A

1. Option (C) is correct.

Explanation: a and b are two positive integers such that the least prime factor of a is 3 and the least prime factor of b is 5. Then, least prime factor of $(a + b)$ is 2. **1**

2. Option (A) is correct.

Explanation: Since, $1200 = 4 \times 3 \times (2 \times 5)^2$
$= 2^4 \times 3 \times 5^2$

Hence, the required smallest natural number is 3. **1**

3. Option (B) is correct.

Explanation: Let given quadratic polynomial be

$p(x) = x^2 + 99x + 127$

On comparing $p(x)$ with $ax^2 + bx + c$. we get

$a = 1, b = 99$ and $c = 127$

Since,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-99 \pm \sqrt{(99)^2 - 4\times1\times127}}{2\times1}$$
$$= \frac{-99 \pm \sqrt{9801 - 508}}{2}$$
$$= \frac{-99 \pm \sqrt{9293}}{2} = \frac{-99 \pm 96.4}{2}$$
$$= \frac{-99 + 96.4}{2}, \frac{-99 - 96.4}{2}$$
$$= \frac{-2.6}{2}, \frac{-195.4}{2}$$
$$= -1.3, \ -97.7$$

Hence, both zeroes of the given quadratic polynomial $p(x)$ are negative. **1**

4. Option (B) is correct.

Explanation:

Given, $a = -1$ and $d = 4 - (-1) = 5$

$$a_n = -1 + (n-1) \times 5 = 129$$

or $(n - 1) \times 5 = 130$

$$n - 1 = 26$$
$$n = 27$$

Hence, 27th term = 129 **1**

[CBSE Marking Scheme 2017]

5. Option (B) is correct.

Explanation: Distance between two points (x_1, y_1) and (x_2, y_2) is given as,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where, $(x_1, y_1) = (4, p)$, $(x_2, y_2) = (1, 0)$ and, $d = 5$

Put the values, we have

$$5^2 = (1 - 4)^2 + (0 - p)^2$$
$$25 = (-3)^2 + (-p)^2$$
$$25 - 9 = p^2$$
$$16 = p^2$$

or, $p = \pm 4$ **1**

6. Option (C) is correct.

Sol. $A(m, -n)$
$B(-m, n)$

$AB = \sqrt{(m+m)^2 + (-n-n)^2}$

$AB = \sqrt{4m^2 + 4n^2}$

$AB = 2\sqrt{m^2 + n^2}$

Ans- (c) $2\sqrt{m^2+n^2}$

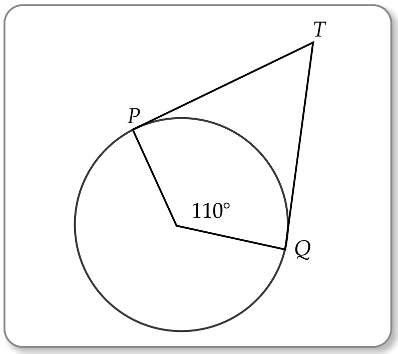
[Topper Answer, 2020]

7. Option (B) is correct.

Explanation: Given that, TP and TQ are tangents,

Since, the radius drawn to the tangents will be perpendicular on it.

$\therefore$ $OP \perp TP$ and $OQ \perp TQ$



$\angle OPT = 90^\circ$ and $\angle OQT = 90^\circ$

In quadrilateral $POQT$,

Sum of all interior angles = 360°

$$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$$

$$90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$$

$$\angle PTQ = 70^\circ$$

1

8. **Option (C) is correct.**

Explanation: $A(4, -2)$, $B(7, -2)$ and $C(7, 9)$ are the vertices of a triangle.

Using distance formula,

$$d = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

$$AB = \left|\sqrt{[7-4]^2 + [-2-(-2)]^2}\right|$$

$$= |\sqrt{3^2 + 0}| = 3 \text{ units}$$

$$BC = \left|\sqrt{[7-7]^2 + [9-(-2)]^2}\right|$$

$$= |\sqrt{0 + 11^2}| = 11 \text{ units}$$

$$AC = \left|\sqrt{[7-4]^2 + [9-(-2)]^2}\right|$$

$$= |\sqrt{3^2 + 11^2}|$$

$$= |\sqrt{9 + 121}| = \sqrt{130} \text{ units}$$

Clearly, they are neither equilateral nor isosceles.

Also, $AC^2 = AB^2 + BC^2$

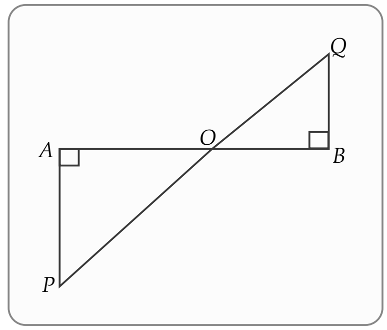
which mean it is following Pythagoras theorem.

$\therefore$ ΔABC is a right angled triangle.

9. **Option (B) is correct.**

Explanation: In ΔPAO and ΔQBO,

$\angle A = \angle B = 90^\circ$ (given)



$\angle POA = \angle QOB$

(vertically opposite angle)

Since, $\Delta PAO \sim \Delta QBO$, (by AA criterion)

Then, $\dfrac{OA}{OB} = \dfrac{PA}{QB}$

or, $\dfrac{6}{4.5} = \dfrac{4}{QB}$

or, $QB = \dfrac{4 \times 4.5}{6}$

$\therefore$ $QB = 3$ cm **1**

10. **Option (A) is correct.**

Explanation: Here, $r = 7$ cm, $h = 10$ cm,

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 7^2 \times 10$$

$= 1540 \text{ cm}^3$ **1**

11. **Option (A) is correct.**

Explanation: According to question,

Circumference of circle = circumference of first circle + circumference of second circle

$$2\pi R = 2\pi R_1 + 2\pi R_2$$

$$R = R_1 + R_2$$

1

12. **Option (B) is correct.**

Explanation: Diameter of wheel = 42 cm

Radius of the wheel $= \dfrac{42}{2} = 21$ cm

Distance in 1 revolution = circumference of the wheel

$= 2\pi r$

$= 2 \times \dfrac{22}{7} \times 3$

$= 132$ cm

Total distance covered by the wheel = 132 km = 132 × 100,000 cm = 13,200,000 cm

(number of revolutions)

$$= \frac{\text{total distance covered by wheel}}{\text{distance covered in 1 revolution}}$$

$$= \frac{13,200,000}{132} = 100,000 = 10^5$$

13. Option (B) is correct.

Explanation:

Class	Frequency	Cumulative frequency
0.5 – 5.5	13	13
5.5 – 11.5	10	23
11.5 – 17.5	15	38
17.5 – 23.5	8	46
23.5 – 29.5	11	57

The median of 57 (odd) observations $= \frac{(57+1)}{2}$

$$= \frac{58}{2} = 29^{th} \text{ term}$$

29th term lies in class 11.5 – 17.5.
So, upper limit is 17.5.

14. Option (A) is correct.

Explanation: Give $\tan\theta = \frac{15}{8}$

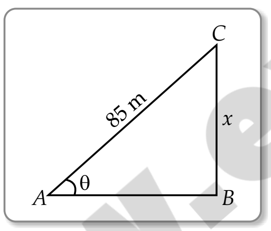
So, $\sin\theta = \frac{15}{17}$ (i)

Now, $\sin\theta = \frac{x}{85}$ (ii)

From, Eqs. (i) and (ii),

$$\frac{15}{17} = \frac{x}{85}$$

$$x = 75 \text{ m}$$



15. Option (B) is correct.
Explanation: Total number of marbles in a bag = 12 + 8 = 20
Number of blue marbles = 8
i.e., favourable outcomes = 8
and total possible outcomes = 20

$\therefore P(\text{drawing a blue marble}) = \frac{\text{favourable outcomes}}{\text{possible outcomes}}$

$$= \frac{8}{20} = \frac{2}{5}.$$

16. Option (D) is correct.

Explanation:

Here, $\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}$, $\frac{b_1}{b_2} = \frac{-1}{3}$, $\frac{c_1}{c_2} = \frac{5}{1}$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

So, the system of linear equations has no solution. **1**

17. Option (B) is correct.

Explanation: $a\cot\theta + b\,\text{cosec}\,\theta = p$ and $b\cot\theta + a\,\text{cosec}\,\theta = q$ are the given equations.

Taking, $p^2 - q^2$

$= (a\cot\theta + b\,\text{cosec}\,\theta)^2 - (b\cot\theta + a\,\text{cosec}\,\theta)^2$

$= a^2\cot^2\theta + b^2\text{cosec}^2\theta + 2ab.\cot\theta.\text{cosec}\,\theta$
$\qquad - b^2\cot^2\theta - a^2\text{cosec}^2\theta - 2ab.\cot\theta.\text{cosec}\,\theta$

$= a^2(\cot^2\theta - \text{cosec}^2\theta) + b^2(\text{cosec}^2\theta - \cot^2\theta)$

[using, $\text{cosec}^2\theta - \cot^2\theta = 1$]

$= a^2(-1) + b^2(-1)$

$= b^2 - a^2$

18. Option (A) is correct.

Explanation: Number of days in non-leap year

= 365

Number of weeks $= \frac{365}{7} = 52\frac{1}{7} = 52$ weeks

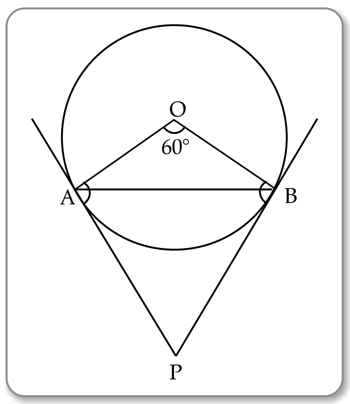
Number of days left = 1 For example, it may be any of 7 days which from Sunday, Monday, Tuesday, Wednesday, Thursday, Friday and Saturday; so, $T(E) = 7$

$F(E) = 1$ (Sunday)

$$P(F) = \frac{F(E)}{T(E)} = \frac{1}{7}$$

19. Option (D) is correct.

Explanation: In case of assertion:
Chord AB subtends ∠60° at O.



$\therefore \quad \angle OAP = 90°$

Similarly, $\angle OBP = 90°$

In quadrilateral $OAPB$,

$\angle O + \angle P + \angle OAP + \angle OBP = 360°$

$\Rightarrow \quad 60° + \angle P + 90° + 90° = 360°$

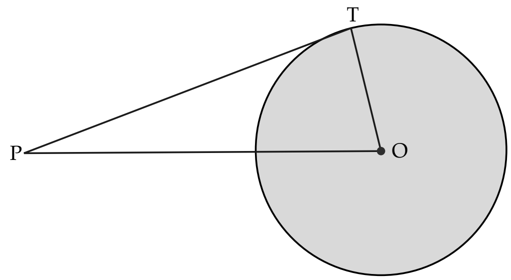
$\Rightarrow \quad \angle P = 360° - 240°$

$\Rightarrow \quad \angle P = 120°$

$\therefore$ Assertion is incorrect.

In case of reason:

PT and OT are the tangent and radius, respectively at contact point T.



So, $\angle OTP = 90°$

$\Rightarrow \Delta OPT$ is right angled triangle.

Again, in ΔOPT

$\because \quad \angle T > \angle O$

$\therefore \quad OP > PT$

[Side opposite to greater angle is larger]

$\therefore$ Reason is correct.

Hence, assertion is incorrect but reason is correct.

20. Option (B) is correct.

Explanation: In case of assertion:

$$\sqrt{3x^2+6} = 9$$
$$3x^2 + 6 = 81$$
$$3x^2 = 81 - 6 = 75$$
$$x^2 = \frac{75}{3} = 25$$

$\therefore \quad x = \pm 5$

Hence, positive root $= 5$.

$\therefore$ Assertion is correct.

In case of reason:

Substituting $x = \frac{2}{3}$ in $ax^2 + 7x + b = 0$

$$\frac{4}{9}a + \frac{14}{3} + b = 0$$

$\Rightarrow \quad 4a + 42 + 9b = 0$

$\Rightarrow \quad 4a + 9b = -42$ ...(i)

again, substituting $x = -3$ in $ax^2 + 7x + b = 0$

$$9a - 21 + b = 0$$

$\Rightarrow \quad 9a + b = 21$ ...(ii)

Solving (i) and (ii), we get

$$a = 3 \text{ and } b = -6$$

$\therefore$ Reason is correct

Hence, both assertion and reason are correct but reason is not the correct explanation for assertion.

Section - B

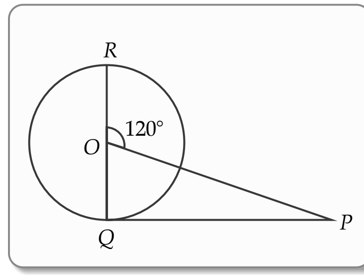
21. α and β are the roots of $2x^2 + 2x + 1$

So, Sum of roots $(\alpha+\beta) = -\frac{2}{2} = -1$ **1**

Product of roots $(\alpha\beta) = \frac{1}{2}$ **1**

22. In ΔOQP, $\angle POR = \angle OQP + \angle OPQ$

(exterior angle) **1**



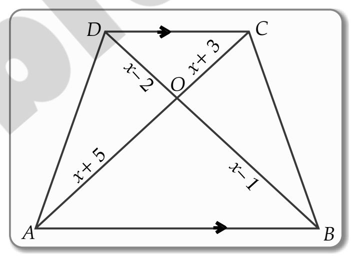
$\therefore \quad \angle OPQ = \angle POR - \angle OQP$
$= 120° - 90°$
$= 30°$ **1**

OR

Since, the diagonals of a trapezium divide each other proportionally, so we have

$$\frac{OA}{OC} = \frac{BO}{OD}$$

or, $\frac{x+5}{x+3} = \frac{x-1}{x-2}$ **1**



or, $(x + 5)(x - 2) = (x - 1)(x + 3)$

or, $x^2 - 2x + 5x - 10 = x^2 + 3x - x - 3$

or, $3x - 2x = 10 - 3$

$\therefore \quad x = 7.$ **1**

23. Area of sector $OAPB = \frac{5}{36}$ times the area of circle

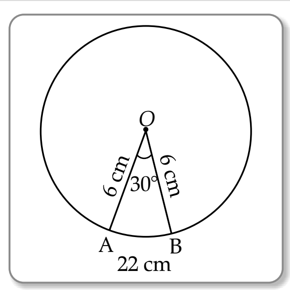
$\therefore \quad \pi r^2 \times \frac{x}{360°} = \frac{5}{36}\pi r^2$

or, $\frac{x}{360°} = \frac{5}{36}$

or, $x = 50°$ **2**

[CBSE Marking Scheme, 2012]

24.



Given, radius of a circle,

$OA = OB = 6$ cm

(assuming in figure) **1**

and central angle $\theta = \angle AOB = 30°$

By using formula,

area of the sector of a circle

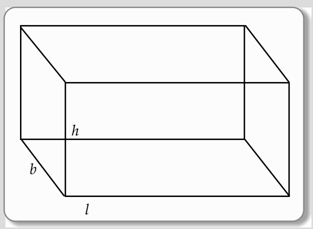
$$= \frac{\theta}{360°} \times \pi r^2$$

$$= \frac{30°}{360°} \times 3.14 \times 6 \times 6$$

$$= 9.42 \text{ cm}^2 \qquad \mathbf{1}$$

OR

Let the length, breadth and height of the cuboid be l, b and h respectively.



$$X = l \times b$$

$$Y = b \times h$$

$$Z = l \times h$$

$$XYZ = l^2 b^2 h^2$$

Volume of cuboid $= l \times b \times h$

$\therefore \quad l^2 b^2 h^2 = XYZ$

or, $\quad lbh = \sqrt{XYZ} \qquad \mathbf{1}$

[CBSE Marking Scheme, 2012]

25. $$\text{LHS} = \frac{\tan\theta}{1-\frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta} \qquad \mathbf{1}$$

$$= \frac{\tan^2\theta}{\tan\theta - 1} - \frac{1}{\tan\theta(\tan\theta - 1)}$$

$$= \frac{\tan^3\theta - 1}{\tan\theta(\tan\theta - 1)} \qquad \mathbf{1}$$

$$= \frac{(\tan\theta - 1)(\tan^2\theta + \tan\theta + 1)}{\tan\theta(\tan\theta - 1)} \qquad \mathbf{1}$$

$$= \tan\theta + 1 + \cot\theta$$

$$= 1 + \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \qquad \tfrac{1}{2}$$

$$= 1 + \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$

$$= 1 + \frac{1}{\sin\theta\cos\theta}$$

$$= 1 + \operatorname{cosec}\theta \sec\theta = \text{RHS} \qquad \tfrac{1}{2}$$

[CBSE Marking Scheme 2019]

OR

$$\text{L.H.S.} = \frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} - 1}{\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta} + 1} \qquad \tfrac{1}{2}$$

$$= \frac{\sin\theta(\cos\theta - \sin\theta + 1)}{\sin\theta(\cos\theta + \sin\theta - 1)} \qquad \tfrac{1}{2}$$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta + \sin\theta}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{\sin\theta\cos\theta + \sin\theta - (1 - \cos^2\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)} \qquad \mathbf{1}$$

$$= \frac{\sin\theta(\cos\theta + 1) - [(1 - \cos\theta)(1 + \cos\theta)]}{\sin\theta(\cos\theta + \sin\theta - 1)} \qquad \tfrac{1}{2}$$

$$= \frac{(1 + \cos\theta)(\sin\theta - 1 + \cos\theta)}{\sin\theta(\cos\theta + \sin\theta - 1)}$$

$$= \frac{1 + \cos\theta}{\sin\theta}$$

$= \text{RHS}$ **Hence proved.** ½

Section - C

26. **(i)** In order to arrange the books as required, we have to find the largest number that divides 96, 240 and 336 exactly, clearly, such a number is their *HCF*.

We have,

$$96 = 2^5 \times 3$$

$$240 = 2^4 \times 3 \times 5$$

and $$336 = 2^4 \times 3 \times 7$$

$\therefore$ HCF of 96, 240, and 336 is $2^4 \times 3 = 48$

So, there must be 48 books in each stack.

Number of stacks of English books

$$= \frac{96}{48} = 2$$

Number of stacks of Hindi books

$$= \frac{240}{48} = 5$$

Number of stacks of Sociology books

$$= \frac{336}{48} = 7 \qquad \mathbf{1}$$

(ii) HCF of numbers. **1**

27. For system of equation has infinitely many solutions.

$$\therefore \quad \frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c} \qquad \mathbf{1}$$

$\Rightarrow \quad c^2 = 36 \Rightarrow c = 6$ or $c = -6$ (i) ½

Also, $-3c = 3c - c^2 \Rightarrow c = 6$ or $c = 0$ (ii) **1**

From Eqs. (i) and (ii), we get

$c = 6$

[CBSE Marking Scheme 2019] ½

Commonly Made Error

- Many candidates are not able to solve this problem because of not having the basic idea of unique solution, infinitely many solutions and no solution.

Answering Tip

- Candidates must remember the conditions for the no solution, unique solution and infinitely many solutions.

28. Let the original speed of train be x km/h

Therefore, $\frac{300}{x} - \frac{300}{x+5} = 2$ **1½**

$\Rightarrow \quad x^2 + 5x - 750 = 0$

$\Rightarrow \quad (x + 30)(x - 25) = 0$

$\Rightarrow \quad x = 25$ or $x = -30$

Hence, speed of train is 25 km/h **1½**

[CBSE Marking Scheme 2017]

Commonly Made Error

- Candidates do error in framing and simplifying these type of equations.

Answering Tip

- Adequate practice is necessary for simplifying these type of quadratic equations.

OR

Given, $S_4 = 40$

$\Rightarrow \quad 2(2a + 3d) = 40$

$\Rightarrow \quad 2a + 3d = 20$ (i) ½

$S_{14} = 280$

$\Rightarrow \quad 7(2a + 13d) = 280$

$\Rightarrow \quad 2a + 13d = 40$ (ii) ½

On solving (i) and (ii), we get

$d = 2$ and $a = 7$ **1**

$\therefore \quad S_n = \frac{n}{2}[14 + (n-1) \times 2]$

$= n(n + 6)$ or $n^2 + 6n$ **1**

29. LHS $= (\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta.\,\text{cosec}\,\theta)$

$+ \cos^2\theta + \sec^2\theta + 2\cos\theta.\sec\theta$ ½

$= (\sin^2\theta + \cos^2\theta) + (\sec^2\theta + \text{cosec}^2\theta)$

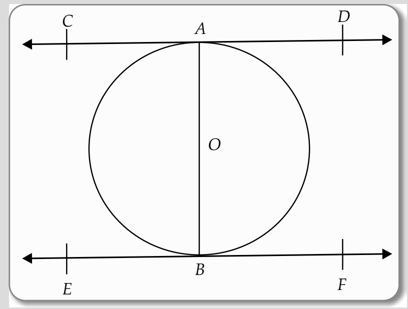
$+ \frac{2\sin\theta}{\sin\theta} + \frac{2\cos\theta}{\cos\theta}$ ½

$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 2 + 2$

$= 7 + \cot^2\theta + \tan^2\theta =$ RHS **Hence proved**

[CBSE Marking Scheme 2019] 1

30.



Let AB be the diameter of a given circle and let CD and EF be the tangents drawn to the circle at A and B, respectively.

$AB \perp CD$ and $AB \perp EF$ **1**

$\therefore \angle CAB = 90°$ and $\angle ABF = 90°$ ½

$\angle CAB = \angle ABF$

and $\angle ABE = \angle BAD$ **1**

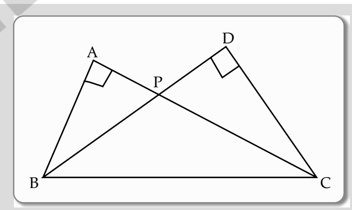
$\angle CAB$ and $\angle ABF$ also $\angle ABE$ and $\angle BAD$ are alternate interior angles. **1**

$\therefore \quad CD \parallel EF$

Hence proved

[CBSE Marking Scheme, 2012]

OR



1

In ΔAPB and ΔDPC

$\angle BAP = \angle CDP$ (90º each)

$\angle APB = \angle CPD$

(vertically opposite angle)

$\therefore \quad \Delta APB \sim \Delta DPC$ (AA similarity) **1**

Therefore, $\frac{AP}{DP} = \frac{BP}{PC}$ **1**

$\Rightarrow \quad AP \times PC = BP \times DP$ **Hence Proved**

[CBSE Marking Scheme 2019]

31. Peter throws two dice together

$\therefore$ Total number of possible outcomes

$= 6^2 = 36$ ½

He get 25 only when he gets (5, 5)

$\therefore$ No. of favourable outcomes = 1 ½

P(getting the numbers of product 25) $= \frac{1}{36}$ **1**

Rina throws one dice

$\therefore$ Total number of all possible outcomes = 6

The number where square is 25 is 5

$\therefore$ No. of favourable outcomes = 1 ½

P(getting a number whose square is 25) $= \frac{1}{6}$

$\because \qquad \frac{1}{6} > \frac{1}{36}$ ½

Hence, Rina has better chances to get the number square 25.

[CBSE Marking Scheme 2017]

Section - D

32. Let the speed of the boat in still water be x km/h and speed of the stream be y km/h.

Given, $$\frac{30}{x-y}+\frac{44}{x+y} = 10 \qquad \text{(i)} \; \mathbf{1}$$

and $$\frac{40}{x-y}+\frac{55}{x+y} = 13 \qquad \text{(ii)} \; \mathbf{1}$$

Solving Eqs. (i) and (ii), we get

$$x + y = 11 \qquad \text{(iii)}$$

and $$x - y = 5 \qquad \text{(iv)}$$

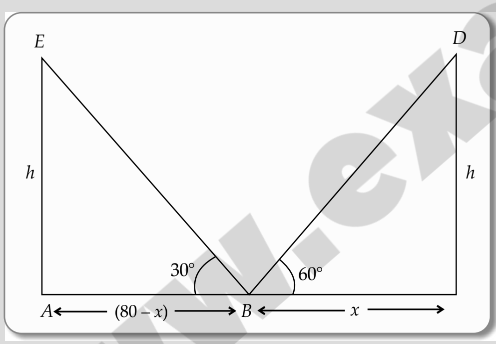
Solving Eqs. (iii) and (iv), we get

$x = 8$ and $y = 3$ **1 + 1**

Hence, speed of boat = 8 km/h and speed of stream = 3 km/h.

[CBSE Marking Scheme 2019]

OR



1

Let $BC = x$, $AB = 80 - x$, where AC is the road.

In ΔACB, $\tan 60° = \sqrt{3} = \frac{h}{x}$

$\Rightarrow \qquad h = \sqrt{3}\,x$ **1**

In ΔBAE, $\tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{80-x}$

$\Rightarrow \qquad h\sqrt{3} = 80 - x$ **1**

Solving equation to get **1**

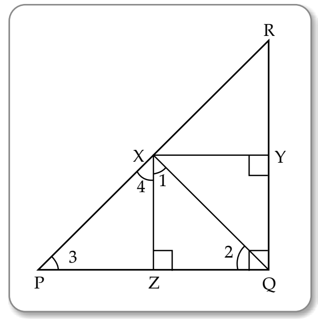
$$x = 20, h = 20\sqrt{3}$$

$\therefore$ AB = 60 m, BC = 20 m and

$$h = 20\sqrt{3} \text{ m} \qquad \mathbf{2}$$

[CBSE Marking Scheme 2019]

33.



1

Here, $RQ \perp PQ$ and $XZ \perp PQ$

or, $\qquad XZ \parallel YQ$

$\therefore$ Similarly, $\qquad XY \parallel ZQ$

XYQZ is a rectangle. ($\because \angle PQR = 90°$) **1**

In ΔXZQ, $\quad \angle 1 + \angle 2 = 90°$ (i)

and in ΔPZX, $\quad \angle 3 + \angle 4 = 90°$ (ii)

$XQ \perp PR$ or, $\quad \angle 2 + \angle 3 = 90°$ (iii)

By Eqs. (i) and (iii), we get ½

$$\angle 1 = \angle 3$$

By Eqs. (ii) and (iii), we get ½

$$\angle 2 = \angle 4$$

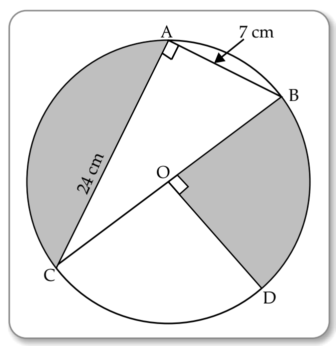
$\therefore \qquad \Delta PZX \sim \Delta XZQ$ (AA similarity) **1**

$\therefore \qquad \frac{PZ}{XZ} = \frac{XZ}{ZQ}$

Thus, $\qquad XZ^2 = PZ \times ZQ$ **1**

Hence proved.

34.



Diameter $BC = \sqrt{24^2 + 7^2} = 25$ cm **1**

Area $\Delta CAB = \frac{1}{2} \times 24 \times 7 = 84$ cm² **1**

Area of shaded region

$$= \frac{\pi}{2}\left(\frac{25}{2}\right)^2 - 84 + \frac{\pi}{4}\left(\frac{25}{2}\right)^2 \qquad \mathbf{1}$$

$$= \left(\frac{1875\pi}{16} - 84\right)\text{cm}^2$$

$$= (117.18\pi - 84)\text{ cm}^2 \qquad \mathbf{1}$$

$$= 283.94 \text{ cm}^2 \qquad \mathbf{1}$$

OR

Volume of ice-cream in the cylinder

$= \pi (6)^2 \times 15 \text{ cm}^3$ **1**

Volume of ice-cream in one cone

$= \frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3$

(Given, $h = 4r$) 1

$= 2\pi r^3 \text{ cm}^3$

$\Rightarrow \quad 10(2\pi r^3) = \pi(6)^2 \times 15$ **1**

$\Rightarrow \quad r^3 = 3^3 \Rightarrow r = 3 \text{ cm}$ **2**

So the radius of ice-cream cone is 3cm.

[CBSE Marking Scheme 2019]

Detailed Answer:

Given, radius of right cylinder (r) = 6 cm

height of right cylinder (h) = 15 cm

$\therefore$ Volume of ice-cream in the cylinder (V_1)

$= \pi r^2 h_1$

$= \pi(6)^2 \times 15$

$= 540\pi \text{ cm}^3$ **1**

Also given, height of the conical portion is four times its base radius i.e.,

height of cone, $h = 4r$

$\therefore$ Volume of ice-cream in one cone

= Volume of cone + volume of hemispherical shape on the top

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3}\pi r^2 (4r) + \frac{2}{3}\pi r^3$

$= 2\pi r^3 \text{ cm}^3$ **1**

$[\because h = 4r]$

Now, volume of ice-cream in the cylinder = 10 × (volume of ice-cream in one cone) **1**

i.e., $540\pi = 10 \times 2\pi r^3$

$\Rightarrow \quad r^3 = \frac{540}{10 \times 2}$

$\Rightarrow \quad r^3 = 27$

$\Rightarrow \quad r^3 = 27 \text{ cm}$

$\Rightarrow \quad r = 3 \text{ cm}$

Therefore, radius of ice cream cone is 3 cm. **2**

35. Maximum frequency = 50,

class (model) = 35 – 40 1

$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ **1**

$= 35 + \frac{50 - 34}{100 - 34 - 42} \times 5$

$\text{Mode} = 35 + \frac{16}{24} \times 5$ **1**

$= 38.33$ **2**

[CBSE Marking Scheme 2019]

Section - E

36. **(i)** No. of rose plants = 135

No. of marigold plants = 225

The maximum number of columns in which they can be planted

= HCF of 135 and 225

$\therefore$ Prime factors of 135 = 3 × 3 × 3 × 5

and 225 = 3 × 3 × 5 × 5

$\therefore$ Prime factors of 135 = 3 × 3 × 5 = 45.

OR

Total number of plants 135 + 225 = 360 plants

(ii) We have proved that the maximum number of columns = 45

So, prime factors of 45 = 3 × 3 × 5

$= 3^2 \times 5^1$

$\therefore$ Sum of exponents = 2 + 1 = 3.

(iii) Number of rows of Rose plants

$= \frac{135}{45} = 3$

Number of rows of marigold plants = $\frac{225}{45} = 5$

Total number of rows = 3 + 5 = 8

37. **(i)** Relative speed of both cars while they are travelling in same direction = $(u - v)$ km/h.

OR

Relative speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = $(u + v)$ km/h.

(ii) Let the speeds of first car and second car be u km/h and v km/h, respectively.

According to the given information.

$5(u - v) = 100$

i.e,. $\quad u - v = 20$ (i)

and $\quad u + v = 100$ (ii)

Solving Eqs. (i) and (ii), we get $u = 60$ km/h.

(iii) From above part ii, referring to the solution of both equations

$v = 40$ km/h.

38. **(i)** Area of two bedrooms = $10x$ sq. m

Area of kitchen = $5y$ sq. m

$10x + 5y = 95$

$2x + y = 19$

Also, $\quad x + 2 + y = 15$

$x + y = 13$

OR

Length of outer boundary= 12 + 15 + 12 + 15

= 54 m

(ii) On solving two equation part (i)

$x = 6$ m and $y = 7$ m

area of bedroom = 5 × 6 = 30 m

area of kitchen = 5 × 7 = 35 m

(iii) Area of living room = (15 × 7) – 30

= 105 – 30

= 75 sq. m

HINTS

Mathematics Standard (041)

Self Assessment Paper–1

Section - A

1. Factorize 525 and 3000 to get the HCF.

OR

Use formula: product of two numbers = HCF × LCM

2. LCM of 40, 42 and 45 in the required result.

3. Since roots are reciprocal to each other, therefore product of roots = 1.

4. The two digit number divisible by 3 are 12, 15, 18, ..., 99.

6. Use mid-point formula.

8. Apply property AA of similar triangle to get the required result.

9. Angle between radius and tangent is 90º. Also the sum of 3 angles of a triangle is 180º.

10. Write $\sec A = \dfrac{1}{\cos A}$ and $\tan A = \dfrac{\sin A}{\cos A}$ and simplify it.

11. Area of triangle $= \dfrac{1}{2} \times$ base × height

Here, base = diameter of semi – circle.
and height = radius of semi circle.

14. Using: perimeter of circle

$$= \frac{1}{2} \times \text{perimeter of square}$$

$$\Rightarrow \quad 2\pi r = \frac{1}{2}(4a)$$

16. Use Formula:
Quadratic polynomial = x^2 – (sum of zeroes)x + Product of Zeroes.

17. Write prime factorisation of 484.

18. ∵ Rate of smaller and larger opening be u and v m^3/h

∴ $u + v = 30$ and $3u + 2v = 70$ and solve it.

Section - B

21. Let son's present age be x years then Sumit's present age is $3x$ year.

According to questions;

$3x + 5 = \dfrac{5}{2}(x+5)$ and solve it.

22. A divides the line segment XY in the ratio 2 : 3. Use section formula to get coordinates of A.

24. Find side of the cube using formula:
Volume of cube = (side)^3

OR

Apply formula, volume of cone, $V = \dfrac{1}{3}\pi r^2 h$

Then apply formula: Surface area of cuboid = $2(lb + bh + lh)$

25. Using identities $1 + \tan^2\theta = \sec^2\theta$ and $(a + b)(a - b) = a^2 - b^2$.

Section - C

27. Using $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$ for unique solution.

28. Here, $a_m = \dfrac{1}{n} \Rightarrow a + (m-1) = \dfrac{1}{n}$ and $a_n = \dfrac{1}{m}$

$\Rightarrow a + (n-1)d = \dfrac{1}{m}$, find a and d, then apply

$$S_{mn} = \frac{mn}{2}[2a + (mn-1)d]$$

OR

Here, $S_n = 636$, first term, $(a) = 9$, common difference, $(d) = 8$, apply formula,

$S_n = \dfrac{n}{2}[2a + (n-1)d]$ to get value of n.

29. Get simultaneous equation: A + 2B = 60º and A + 4B = 90º. Solve both the equation to find value of A and B.

30. Using property: the tangents drawn to a circle from and exterior point are equal in length.

Section - D

32. Form the quadratic equation using given condition,

i.e., $\frac{192}{x} - \frac{192}{x+16} = 2$

Solve it to get the value of x i.e., speed of the passenger train.

OR

Make quadratic equation,

$$(l+5)\left(\frac{200}{l} - 2\right) = \frac{200}{l}$$

simplify it, to get the value of l i.e., length of the cloth.

33. Using Basic proportionality Theorem in ΔABX and in ΔAXC.

34. Volume of material used = Volume of cuboid.

OR

Area of shaded region = Area of sector ODACB – Area of ΔOBD

Self Assessment Paper-2

Section - A

1. The product of the greatest power of each prime factor, involved in the numbers in LCM.

2. Here, $\beta = -\alpha$ where α and β are the roots of quadratic polynomial.

Also, Sum of roots = 0 $\Rightarrow a = 0$ i.e., polynomial has no linear term.

3. Putting $n = 2$ in $2(7^n + 8^n)$, we get 226.

4. Put $x = \frac{1}{2}$ to get the value of k.

For real roots, D ≥ 0.

5. Here, points are (– 4, – 7) and (0, – 7)

6. ∠PQO = 90° and area of QORP = 2 × ar (ΔOPR)

7. Diameter of circle = diagonal of square.

8. Volume of sphere, V = $\frac{4}{3}\pi r^3$

9. No. of favourable event, F(E) = 1.

10. Using distance formula of AB and BC and putting $AB = BC$.

11. Here, ∠OPR = 90°

and ∠OPQ = ∠OPR – ∠QPR = 90° – 50° = 40°

To get the required angle use sum of all angles of a triangle is 180°.

13. Use $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\cot\theta = \frac{\cos\theta}{\sin\theta}$

16. Common difference $d = t_2 - t_1$, where t_1 and t_2 are the given terms.

17. Use trigonometric ratio, $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

Section - B

21. Here, $\frac{1}{x-6} + \frac{1}{x} = \frac{1}{4}$ and solve it.

22. Use Basic Proportionality theorem i.e., $\frac{AD}{DB} = \frac{AE}{EC}$.

23. Distance covered in 1 revolution = circumference of wheel.

24. Use section formula, for internal division.

25. $\frac{4}{11} = \frac{\sin\theta}{\cos\theta}$

i.e., $\tan\theta = \frac{4}{11}$

Section - C

26. Use method of contradiction to prove the required result.

27. Put $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$ and simplify.

28. Formed equations: $x + 20y = 3000$

$x + 25y = 3500$

where, x be fixed charge and y be food cost per day.

OR

$a_p = q \Rightarrow a + (p-1)d = q$

and $a_q = p \Rightarrow a + (q-1)d = p$

Find a and d by solving these equations and further find the required result.

29. Squaring both sides and simplify.

30. **(OR Part)**

Since, ΔABC ~ ΔPQR $\Rightarrow \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

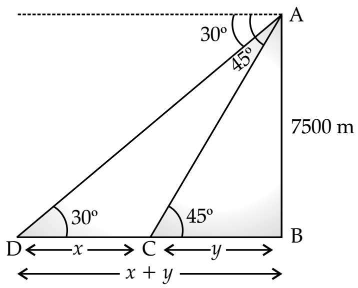
31. P (drawing black ball) = 3 × P(drawing white ball)

i.e., $$\frac{x}{15+x} = 3\times\left(\frac{15}{15+x}\right)$$

simplify to get value of x.

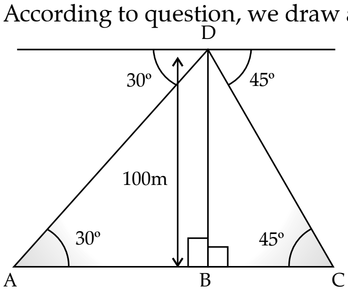
Section - D

32. According to question, we draw a figure:



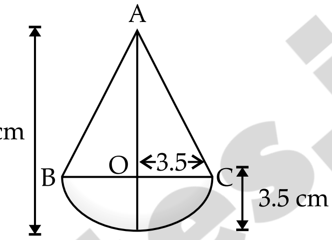
OR

According to question, we draw a figure:



33. Use Pythagoras theorem in ΔOAM and in ΔOAP.

34.



TSA of toy = CSA of cone + CSA of hemisphere.

Self Assessment Paper–3

Section - A

1. Use prime factorisation method to get HCF.

5. Use section formula to get the value of k.

6. $\angle$OAP = 90° and use sin 30° = $\frac{a}{OP}$.

9. Put $\sec^2 A - \tan^2 A = 1$.

10. Diameter of circle = side of square.

12. Use distance formula.

14. Use surface area formula of a cube = 6 (side)^2.

16. Put $x = 2$ does not satisfy the given equation.

17. Use formula: $a_n = a + (n-1)d$.

18. Putting sin 30° = $\frac{1}{2}$ and tan 45° = 1.

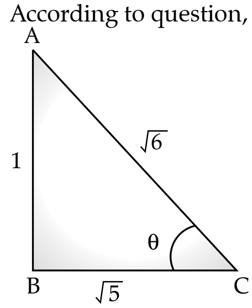
Section - B

21. Equations are: $7(10y + x) = 4(10x + y)$ and $x - y = 3$ and solve it.

22. Use section formula to find co-ordinates of point of intersection.

25. Volume of n cubes = Volume of cuboid

32. According to question, we draw a figure:



Section - C

26. Use section formula.

27. Sum of roots $(\alpha + \beta) = \frac{-2}{3}$

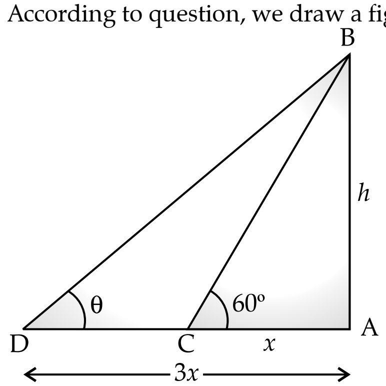
product of roots $(\alpha\beta) = \frac{1}{3}$

Required polynomial

$= x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$

$$= x^2 - \left[\left(\frac{1-\alpha}{1+\alpha}\right)+\left(\frac{1-\beta}{1+\beta}\right)\right]x + \left[\left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{1-\beta}{1+\beta}\right)\right]$$

28. According to question, we draw a figure:



OR

Using $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$

29. Change $\tan A = \frac{\sin A}{\cos A}$, $\operatorname{cosec} A = \frac{1}{\sin A}$ and $\sec A = \frac{1}{\cos A}$. Further simplify it.

30. Use pythagoras theorem in ΔABC.

31. **(i)** Prime numbers are 2, 3, 5.

(ii) Numbers between 2 and 6 are 3, 4, 5.

Section - D

32. $\dfrac{30}{15-x}+\dfrac{30}{15+x} = \dfrac{9}{2}$

OR

$\dfrac{1}{x}+\dfrac{1}{x+3} = \dfrac{3}{10}$

33. Use basic proportionality theorem.

34. **(OR)** Length of the longest rod is equal to HCF of 850, 625 and 475.

Self Assessment Paper–4

Section - A

2. Irrational number

3. Putting product of zeroes = k to get the value of k.

4. $y = 0$ and $y = -7$ represent parallel lines, hence no solution.

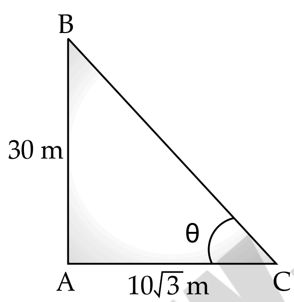
5. Use mid-point formula i.e., $x = \dfrac{x_1+x_2}{2}$ and $y = \dfrac{y_1+y_2}{2}$ where (x, y) are the coordinates of mid-point.

6. Use distance formula to get the value of k.

7. All tangents of a circle are equal in measure.

8. AC = BC = 4 cm and $AB = \sqrt{AC^2+BC^2}$

9.



11. Area of shaded region

= Area of outer ring – Area of inner ring.

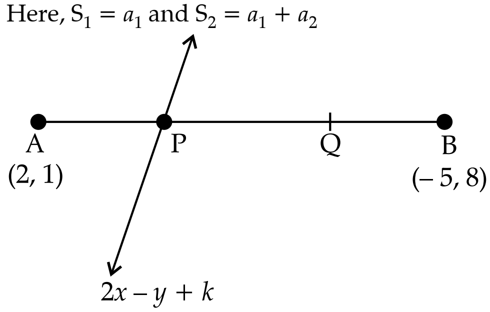
13. Use, area of circle = $\pi(\text{radius})^2$.

15. Volume of capsule = Volume of two hemisphere + volume of cylinder.

Section - B

21. Here, $S_1 = a_1$ and $S_2 = a_1 + a_2$

22.



AP : PB = 1 : 2

Use section formula for internal division to find value of k.

$$x=\frac{mx_2+nx_1}{m+n},\ y=\frac{my_2+ny_1}{m+n}$$

24. Volume of sphere = Volume of cone

OR

Volume of cylinder = $\pi r^2 h$

25. Substitute values of standard angles and simplify to get the answer.

Section - C

26. Use method of contradiction to prove required.

27. For equal roots $b^2 - 4ac = 0$

28. $S_n = 180 = \dfrac{n}{2}[90+(n-1)(-6)]$

OR

$a_m = a + (m-1)d = \dfrac{1}{n}$

and $a_n = a + (n-1)d = \dfrac{1}{m}$

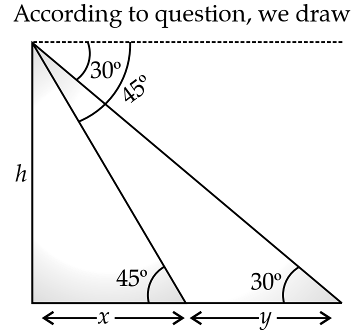
31. P(5 will not come either) = 1 – P(5 will come at least once)

30. **(OR)** $\angle OAD = \angle OCD = 90^\circ$

and sum of all angles of quadrilateral AOCD = 360°.

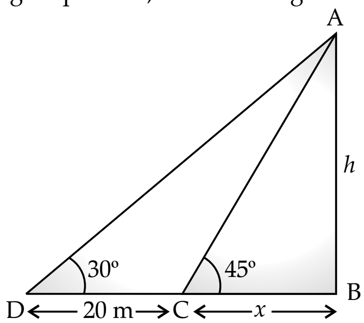
Section - D

32. According to question, we draw a figure:



OR

According to question, we draw a figure:



34. Volume of water in cylindrical tank = Volume of water in the park.

OR

Area of minor segment = $\frac{\theta}{360°}\pi r^2 - \frac{1}{2}r^2 \sin\theta$

and Area of major segment = area of circle – area of minor segment

Self Assessment Paper–5

Section - A

1. Use prime factorisation method to get HCF.

6. Use section formula.

7. Use distance formula.

8. Use basic proportionality theorem to get AD.

10. Use $\sin 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$ and $\cot 45° = 1$.

12. Use formula of volume of cylinder = $\pi r^2 h$ to get height.

15. Use formula total surface area of a cone = $\pi r(r + l)$ to get radius.

16. Putting $x = 3$ in $x^2 + 5x + k = 0$, we get $k = -24$.

Section - B

21. Use formula for required polynomial

$$= x^2 - \text{(sum of zeroes) } x + \text{product of zeroes.}$$

22. Use mid point theorem to get the point D and apply distance formula to get AD.

23. Sum of angles of a quadrilateral is 360°.

24. Use formula surface area of a cube = 6 $(\text{side})^2$.

Section - C

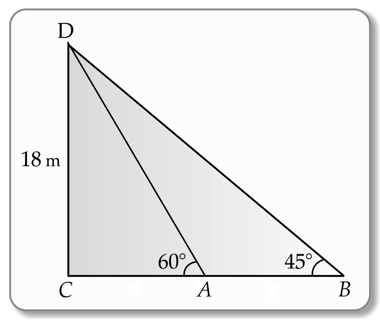
28. Use discriminant formula = $b^2 - 4ac$.

29. Use $\cos^2\beta = (1 - \sin^2\beta)$ and $\tan^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha}$

30. Use formula $S_n = \frac{n}{2}(a + a_n)$ to get $n = 20$.

Section - D

32.



33. Use basic proportionality theorem, to get

$$\frac{AE}{AD} = \frac{PE}{BD} \text{ and } \frac{AE}{AD} = \frac{EQ}{DC}.$$

34. Use volume of water flowing in 1 h = $\pi r^2 h$.

EXCLUSIVE SCHOOL BOOKS SUPPLIERS

	ANDHRA PRADESH
VIJAYAWADA	Sri Vikas Book Centre, 9848571114, 9440715700, Sri Kanka Durga Book Stall, 9603491009
	ASSAM
WEST KAMENG	Dutta Book Stall, 8729948473
	JHARKHAND
DEOGHAR	Bharti International, 06432359136, 9431132051
	KARNATAKA
TUMKUR	Palasandra Promoters, 9448214377, (0816) 2273220
BANGLORE	Sree byraveshawara book Depot, 9880561081,
	MADHYA PRADESH
INDORE	Adithi Distributor, 9229171880
	MAHARASHTRA
KOLHAPUR	Ashish Book Depot, 7798420420
PUNE	Pravin Sales, 9890683475, Sai Shubham, 9975687687
DHULE	Navjeevan Book Stall, 9422296434
JALNA	Anil Paper Mart, 9422722522, (02482) 230733
	UTTAR PRADESH
AGRA	Maheshwari Pustak Bhandar, 9760602503
	TAMIL NADU
CHENNAI	Bookmark-IT, 7305151653
	TELANGANA
HYDERABAD	Sri Balaji Book Depot , 9676996199, (040) 27613300, Shri Sharada Store & Stationers, 9000400069
	WEST BENGAL
KOLKATA	United Book House, 9831344622
WEST KEVVERY	Schoolwale & Company, 9731715655

OUR DISTRIBUTORS

	ANDHRA PRADESH
VISAKHAPATHAM	JBD Educational, 9246632691, 9246633693
	ANDAMAN & NICOBAR ISLAND
PORTBLAIR	Krishna Book Centre, 9474205570
	ASSAM
GUWAHATI	Book Emporium, 9675972993, 6000763186, Ashok Publication, 7896141127
	Kayaan Enterprises, (0361) 2630443, Central Book Agency, 99547107165, Orchid Book house, 9864624209
	BIHAR
PATNA	Bokaro Student Friends, (0612) 2300600, 2618400, Gyan Ganga Ltd., 9304826651
	Nova Publisher & Distributors, (0612) 2666404,
	Shri Durga Pustak Mandir, 9334477386, Vikas Book Depot, 9504780402
	Sharda Pustak Bhandar, 9334259293
MADHUBANI	Krishna Book Agency, 9801019292
PURNEA	Chaurasia book centre, 7004456102
	CHATTISGARH
RAIPUR	Shri Ramdev Traders, 9981761797
RAIGARH	Sindhu Book Depot, 9425275763, 9981935763
DELHI	**DELHI**
	Mittal Books, (011) 23288887, 9899037390, Shyam Brother, 9313619935
	Bokaro,Student Friends Pvt Ltd., 7004074900, R. D. Chawla & Sons, 9899445522
	GOA
GOA	Golden Heart Emporium, (0832) 2725208, 9370273479
	GUJARAT
AHMEDABAD	Patel Book, 9898184248, 9824386112, 9825900335
	Shalibhadra Stationers, 9904477855, Zaveri Agency, 9979897312, 9979890330
VAPI	Goutam Book Sellers, 9081790813
VALSAD	Mahavir Stationers, 9429474177
NAVSARI	College Store, 8141913750, 9825099121
SURAT	Kazi Brothers, 7984880995, Shopping Point, 0261-2230097
	HARYANA
ROHTAK	Swami Kitab Ghar, 9255121948
BALLABGARH	Kashi Ram Kishan lal, 9289504004, 8920567245
FARIDABAD	Agarwal Book House, 9810657456
	JHARKHAND
BOKARO	Bokaro Student Friends, (0654) 2233094, 7360021503
DHANBAD	Bokaro Student Friends, (0326) 3590527
RANCHI	Bokaro Student Friends, 9234628152, Gyan Ganga Ltd., 9117889900, Crown Book Distributor & Publishers, (0651) 2213735, 9431173904 Pustak Mandir, 9431115138
DUMKA	Vidyanthi Pustak Bhandar, 9431310228
DEOGHAR	Navyug Sahitya Mandir, 9334375509, 9431367981
	KARNATAKA
HUBLI	Renuka Book Distributor, (0836) 2244124
BANGLORE	Hema Book world, 8040105110, Krishna book house, 9739847334
BELLARY	Chaitanya Exhibition, 9886393971
KARWAR	Akshile Marketing, 9945891555
	KERALA
ERNAKULAM	Asad Book Centre, (0484) 2370431, 9447314548, Academic Book House, (0484) 2376613
	Surya Book Centre, 9847238314, H & C Store, 9864196344, Surya Book House, 9847124217, 9847238314
KOTTAYAM	Book Centre, (0481) 2566992
TRIVANDRUM	Academic Book House, (0471) 2333349, 9447063349, T.B.S. Publisher Distributors, (0471) 2570504
KOZHIKODE	T.B.S. Publisher Distributors, 755982223
CALICUT	Aman Book Stall, (0495) 2721282,
KOLLAM	Vasantha Book Stall, 9895072023
PERUMBAVOOR	Adithya book house, 8606005600
	MADHYA PRADESH
GWALIOR	Agarwal Book Depot, 9425116210
INDORE	Sushil Prakashan,(0731) 2503333, 2535892, 9425322330, Bhaiya Store, 9425318103
JABALPUR	Vinay Pustak Sadan, 8962362667, New Radhika Book Palace, 9425411533-66
	MAHARASHTRA
PUNE	Natraj Book Depot, (020) 24485054, 9890054092, Vikas Book Depot, 9921331187, Sai Shubham, 9975687687, 9890043496, Pravin Sales, 9890683475, New Saraswati Granth Bhandar, 9422323859, Akshar Books & Stationary, 7385089789, Vandhaman Educational, 9860574354 The Wisdom Store, 9822588775, 02025433444
AHMED NAGAR	Hira Stationers, 9423755472
MUMBAI	Vidyarthi Sales Agencies, 9819776110, New Student Agencies, 7045065799, Shivam Books & Stationery, (022) 28230897, 9892935799
LATUR	Yash Book House, 9637936999
KOLHAPUR	Granth the Book World, 9922295522
NASHIK	New India book House, 9623123458, 7448123456
NAGPUR	Laxmi Pustakalay and Stationers, (0712) 2727354, Vijay Book Depot, 9860122094, (0712) 2534217, Renuka Book distributor, 9765406133, Novelty Book Depot, 9657690220
DHULE	Navjeevan Book Stall, 7020525561
YAVATMAL	Shri Ganesh Pustkalaya, 9423131275
	ODISHA
CUTTACK	A. K. Mishra Agencies, 9437025991, 9437081319
BHUBANESHWAR	M/s Pragnya, 8847888616, 9437943777, Padmalaya, 9437026922 Bidyashree, 9937017070
BARIPADA	Trimurti Book World, 9437034735
KEONJHAR	Students corner, 7008435418
RAIGARH	Sindhu book Depot, 9425275763, 9981935763
BRAHMAPUR	Sibani Book Store, 8763340822
	PUNJAB
JALANDHAR	Cheap Book Store, 9872223458, 9878258592, City Book Shop, 9417440753 Gaurav Book World, 9478821183, Subhash Book Depot, 9876453625
LUDHIANA	Gupta Brother, 988200206
CHANDIGARH	Mahindra Book Depot, 9814920226
	RAJASTHAN
KOTA	Perfect Stationers & Gen. Shoppe, 9829863904
	Vardhman Book Depot, 9571365020, Bhandari Stationers, 9001094271
BHILWARA	Nakoda Book Depot, (01482) 243653, 9214983594
JAIPUR	J K Enterprises, 9460983939, 9414782130, Ravi Enterprises, 9829060694
	Saraswati Book House, (0141) 2610823, 9829811155
	TRIPURA
AGARTALA	Book Corner, 8794894165, 8984657146
	TAMIL NADU
COIMBATORE	Majestic Book House, (0422) 2384333
CHENNAI	Arraba Book Traders, (044) 25387868, 9841459105, M.R. Book Store (044) 25364596, Indian Book House, (044) 24327784, 9094129595, Kalaimagal Store, (044) 5544072, 9940619404, Vijaya Stores, 9381037417
	Bookmark It-Books & Stat. Store, 7305151653, M.K. Store, 9840030099
	Tiger Books Pvt. Ltd., 9710447000
PUDUCHERRY	Sri Saraswathi Book Stall, (04132) 222283, 9092626287
SALEM	Pattu book centre, 9894816280
TRICHY	Rasi Publication, 9894816280
THENI	Maya Book Centre, 9443929274
	TELANGANA
HYDERABAD	Sri Balaji Book Depot, (040) 27613300, 9866355473, Central Book Shop, 9121152053, Shah Book House, 9849564564
SECUNDERABAD	Shraddha Books & Stationery, 9849451558
	UTTAR PRADESH
AGRA	Sparsh Book Agency, 9412257817, Manav Book world, 9358429663
	Om Pustak Mandir, (0562) 2464014, 9319117771, Panchsheel Books, 9412257961, 9412257962,
ALLAHABAD	Mehrotra Book Agency, (0532) 2266865, 9415636890
AZAMGARH	Sasta Sahitya Sadan, 9450029674
LUCKNOW	Vyapar Sadan, 7607102462, Om Book Depot, 7705871398 Vidyarthi kendra Publisher & Distributor, 9554967415 Azad Book Depot Pvt. Ltd., 7317000250, Book Sadan, 9839487327, Universal Books, 8881093333, 7652043110
JHANSI	Bhanu Book Depot, 9415031340
KANPUR	Raj Book Dist., 9235616506
MEERUT	Ideal Book Depot, (0121) 4059252, 9837066307
NOIDA	Prozo (Global Edu4 Share Pvt. Ltd), 9318395520 Goyal Books Overseas, (0120) 4655555, 9873387003
VARANASI	Bokaro Student Friends, (0542) 2401250, 8299344607
	Bookman & Company, 9935194495, Gupta Books, 9918155500
	UTTARAKHAND
DEHRADUN	Brother Pustak Bhandar, 9897833882
	WEST BENGAL
KOLKATA	Oriental Publishers & Distributor (033) 40628367, Katha 'O' Kahini, (033) 22196313, 22419071, Saha Book House, (033), 22193671, 9333416484, United Book House, 9831344622, Bijay Pustak Bhandar, 8961260603, Krishna Book House, 8420431083
COOCH BEHAR	S.B. Book Distributor, Cooch behar, 9002670771

0709

SOLUTIONS

Self Assessment Paper-1

Mathematics Standard (041)

Section - A

1. **Option (A) is correct.**

Explanation: $525 = 5 \times 5 \times 3 \times 7 = 3 \times 5^2 \times 7$

and $3000 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 5$

$= 2^3 \times 3^1 \times 5^3$

$\therefore$ HCF $= 3^1 \times 5^2 = 75$. **1**

2. **Option (B) is correct.**

Explanation: We have to find minimum distance (*i.e.*, LCM) covered by steps.

$$40 = 2^3 \times 5$$
$$42 = 2 \times 3 \times 7$$
$$45 = 3^2 \times 5$$
$$\text{LCM }(40, 42, 45) = 2^3 \times 3^2 \times 5 \times 7$$
$$= 2520 \text{ cm}$$ **1**

3. **Option (A) is correct.**

Explanation: Roots of the equation $3x^2 - 10x + k = 0$ are reciprocal to each other.

$\Rightarrow$ Product of roots = 1

$\Rightarrow$ $\dfrac{k}{3} = 1 \Rightarrow k = 3$ **1**

[CBSE Marking Scheme 2019]

4. **Option (C) is correct.**

Explanation: Numbers are 12, 15, 18, ..., 99 ½

$\therefore$ $99 = 12 + (n - 1) \times 3$

$\Rightarrow$ $99 - 12 = 3n - 3$

$\Rightarrow$ $3n = 87 + 3$

$\Rightarrow$ $3n = 90$

$\Rightarrow$ $n = 30$ ½

[CBSE Marking Scheme 2019]

5. **Option (B) is correct.**

Explanation: Let the point A be (x, y)

$\therefore$ $\dfrac{x+3}{2} = -2$ and $\dfrac{y+4}{2} = 2$

$\Rightarrow$ $x = -7$ and $y = 0$

$\therefore$ Point is $(-7, 0)$ **1**

6. **Option (C) is correct.**

A (−6, 3)

B (6, 4)

$O = \left(\frac{-6+6}{2}, \frac{3+4}{2}\right)$

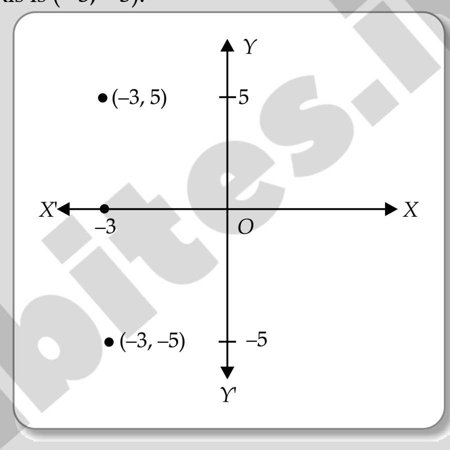
$O = \left(0, \frac{7}{2}\right)$

Ans − (c) $\left[0, \frac{7}{2}\right]$

[Topper Answer, 2020] 3

7. **Option (C) is correct.**

Explanation: By using the graph of coordinate plane, we have the reflection of point (– 3, 5) is x-axis is (– 3, – 5).



[CBSE Marking Scheme, 2020]

8. **Option (B) is correct.**

Explanation: In ΔABC and ΔDEF, $\angle B = \angle E$ and $\angle F = \angle C$. *By AA* similarity, we get $\Delta ABC \sim \Delta DEF$. Thus, the triangles are similar but not congruent. **1**

9. **Option (A) is correct.**

Explanation: OP is radius and PR is tangent at P.

So, $\angle OPR = 90°$

$\Rightarrow \angle OPQ + 50° = 90°$

$\Rightarrow$ $\angle OPQ = 90° - 50°$

$\Rightarrow$ $\angle OPQ = 40°$

In ΔOPQ,

$OP = OQ$ [Radii of same circle]

$\therefore \angle Q = \angle OPQ = 40°$

[Angle opposite to equal sides are equal]

But, $\angle POQ = 180° - \angle P - \angle Q$

$= 180° - 40° - 40° = 180° - 80°$

$= 100°$

$\Rightarrow$ $\angle POQ = 100°$. **1**

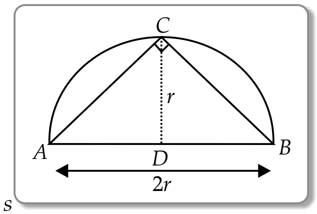
10. **Option (D) is correct.**

Explanation:

$$(\sec A + \tan A)(1 - \sin A) = \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$
$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$
$$= \left(\frac{1 - \sin^2 A}{\cos A}\right) = \frac{\cos^2 A}{\cos A}$$
$$= \cos A$$ **1**

11. Option (A) is correct.

Explanation: Take a point C on the circumference of the semi-circle and join it by the end points of diameter AB.



$\angle C = 90^\circ$ [Angle in a semi-circle is right angle]

So $\Delta ABC = \frac{1}{2} \times AB \times CD$

$= \frac{1}{2} \times 2r \times r = r^2$ sq. units **1**

12. Option (A) is correct.

Explanation: In a right circular cone, if any cut is made parallel to its base, we get a circle. **1**

13. Option (C) is correct.

Explanation: As the probability of an event lies between 0 and 1.

14. Option (D) is correct.

Explanation: Let radius of the circle be r cm and side of the square is a cm.

According to the question, perimeter of the circle is half of perimeter of the square.

$\Rightarrow \quad 2\pi r = \frac{1}{2}(4a)$

$\Rightarrow \quad r = \frac{2a}{2\pi}$

or $\quad \frac{r}{a} = \frac{1}{\pi}$

$$\frac{\text{Area of the circle}}{\text{Area of the square}} = \frac{\pi r^2}{a^2}$$

$$= \pi \times \frac{1}{\pi^2} = \frac{1}{\pi} \text{ or } \frac{7}{22}$$

15. Option (B) is correct.

Explanation:

Class	Frequency	Cumulative frequency
0 – 5	10	10
5 – 10	15	25
10 – 15	12	37
15 – 20	20	57
20 – 25	9	66

The modal class is the class having the maximum frequency.

The maximum frequency 20 belongs to class (15–20).

Here, $\quad \Sigma f_i = n = 66$

So, $\quad \frac{n}{2} = \frac{66}{2} = 33$

33 lies in the class 10–15.

Therefore, 10–15 is the median class.

So, sum of lower limits of (15–20) and (10–15) is $(15 + 10) = 25$

16. Option (B) is correct.

Explanation:

Given,

Sum of zeroes $= \frac{21}{8}$

and Product of zeroes $= \frac{5}{16}$ **½**

So, quadratic polynomial

$= x^2 - (\text{Sum of zeroes})\, x + \text{Product of zeroes}$

$= x^2 - \left(\frac{21}{8}\right) x + \frac{5}{16}$

$= \frac{1}{16}(16x^2 - 42x + 5)$ **½**

17. Option (B) is correct.

Explanation: Prime factors of $484 = 2^2 \times 11^2$

Hence, the exponent of 2 in $2^2 \times 11^2 = 2$.

18. Option (B) is correct.

Explanation: Total volume of water in a vessel $= 30$ m^3

Rate of smaller opening $= U$ m^3/h

Rate of larger opening $= V$ m^3/h

In 1 hour water displaced by smaller opening $= U$ m^3

Similarly in 1 hour, water displaced by larger opening $= V$ m^3

There fore, $\quad U + V = 30$ (i)

and $\quad 3U + 2V = 70$ (given) (ii)

Multiplying by 3 in eq. (i) and subtracting with eq. (ii),

$$\begin{array}{r} 3U + 3V = 90 \\ 3U + 2V = 70 \\ (-) \quad (-) \quad (-) \\ \hline v = 20 \end{array}$$

Hence U is 20 m^3/h.

19. Option (D) is correct.

Explanation: In case of assertion:

Let α and β be the roots of the quadratic polynomial. If α and β are positive then

$\alpha + \beta = \frac{-b}{a}$ it shows that $\frac{-b}{a}$ is negative but sum of two positive numbers (α, β) must be +ve *i.e.*, either b or a must be negative. So, a, b and c will have different signs.

$\therefore$ Given statement is incorrect.

In case of reason:

Let $\beta = 0, \gamma = 0$

$$f(x) = (x - \alpha)(x - \beta)(x - \gamma)$$
$$= (x - \alpha)\, x \cdot x$$

$\Rightarrow \quad f(x) = x^3 - \alpha x^2$

which has no linear (coefficient of x) and constant terms.

$\therefore$ Given statement is correct.

Thus, assertion is incorrect but reason is correct.

20. **Option (B) is correct.**

Explanation: In case of assertion:

$\because \quad a_n = (2n + 1)$

$\therefore \quad a_1 = 2 \times 1 + 1 = 3$

$l = a_3 = 2 \times 3 + 1 = 7$

Since, $\quad S_n = \frac{n}{2}[a + l]$

Hence, $\quad S_3 = \frac{3}{2}[3 + 7]$

$S_3 = 15$

$\therefore$ Assertion is correct.

In case of reason:

Here, $a = 10$, $d = 6 - 10 = -4$ and $n = 16$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{16} = \frac{16}{2}[2 \times 10 + (16-1)(-4)]$

$= 8[20 + 15 \times (-4)]$

$= 8[20 - 60]$

$= 8 \times (-40)$

$= -320$

$\therefore$ Reason is correct.

Hence, both assertion and reason are correct but reason is not the correct explanation for assertion.

Section - B

21. Let Son's present age be x years

Then, Sumit's present age $= 3x$ years

$\therefore$ 5 year later, we have,

$$3x + 5 = \frac{5}{2}(x + 5)$$

$\Rightarrow \quad 6x + 10 = 5x + 25$

$\Rightarrow \quad x = 15$ **1**

Hence, Sumit's present age $= 3x = 3 \times 15$

$= 45$ years

and Son's present age $= 15$ years **1**

[CBSE Marking Scheme 2019]

22. Given, $\frac{XA}{XY} = \frac{2}{5} \Rightarrow \frac{XA}{AY} = \frac{2}{3}$

$\therefore$Coordinates of A are $\left(\frac{-8+18}{5}, \frac{-2-18}{5}\right)$

i.e., $(2, -4)$ **1**

A lies on $3x + k(y + 1) = 0$

$\Rightarrow \quad 6 + k(-3) = 0$

$\Rightarrow \quad k = 2$ **1**

[CBSE Marking Scheme 2019]

Detailed Answer

Given, A lies on the line segment XY joining X(6, – 6) and Y(– 4, – 1) in such a way that

$$\frac{XA}{XY} = \frac{2}{5}$$

or $\quad \frac{XA}{(XA + AY)} = \frac{2}{5}$

or $\quad 5\,XA = 2\,(XA + AY)$

or $\quad 3\,XA = 2\,AY$

or $\quad \frac{XA}{AY} = \frac{2}{3}$

It means, A divides the line XY in the ratio of 2 : 3

Using section formula,

Co-ordinates of A

$= \left[\frac{2(-4)+3(6)}{2+3}, \frac{2(-1)+3(-6)}{2+3}\right]$

$= \left(\frac{-8+18}{5}, \frac{-2-18}{5}\right)$

$= (2, -4)$ **1½**

Now, A lies on the line $3x + k(y + 1) = 0$

Put $(2, -4)$ in $3x + k(y + 1) = 0$

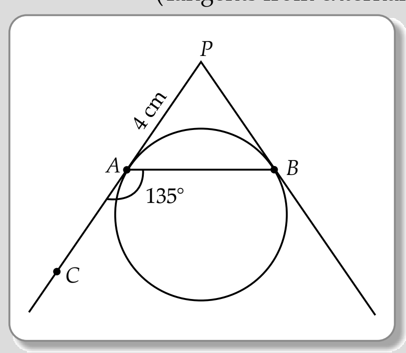
or $\quad 3 \times 2 + k(-4 + 1) = 0$

or $\quad 6 - 3k = 0$

or $\quad k = 2$ ½

23. $\quad PA = PB = 4$ cm

(Tangents from external point) ½



$\angle PAB = 180° - 135° = 45°$

(Supplementary angles)

$\angle ABP = \angle PAB = 45°$

(Opposite angles of equal sides) ½

$\therefore \quad \angle APB = 180° - 45° - 45°$

$= 90°$ ½

So, ΔABP is an isosceles right angled triangle.

$\Rightarrow \quad AB^2 = 2AP^2$

$\Rightarrow \quad AB^2 = 32$ ½

Hence, $\quad AB = \sqrt{32} = 4\sqrt{2}$ cm

[CBSE Marking Scheme 2019]

24. Side of the cube, $a = \sqrt[3]{8} = 2$ cm

Now the length of cuboid

$$l = 4 \text{ cm}$$
$$\text{breadth, } b = 2 \text{ cm}$$
$$\text{height, } h = 2 \text{ cm}$$

Surface area of cuboid

$= 2(l \times b + b \times h + h \times l)$ **1**

$= 2(4 \times 2 + 2 \times 2 + 2 \times 4)$

$= 2 \times 20 = 40 \text{ cm}^2$ **1**

[CBSE Marking Scheme 2012]

OR

Volume of the upper cone $= \frac{1}{3}\pi r^2 h$

Volume of the lower cone $= \frac{1}{3}\pi r^2 H$

Total volume of both the cones $= \frac{1}{3}\pi r^2 h + \frac{1}{3}\pi r^2 H$

$= \frac{1}{3}\pi r^2 (h + H)$ **1**

Thus, the quantity of water displaced will be $\frac{1}{3}\pi r^2(h+H)$ units^3. **1**

[CBSE Marking Scheme, 2012]

25. $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$[\because 1 + \tan^2\theta = \sec^2\theta]$

$= \sec^2\theta(1 - \sin\theta)(1 + \sin\theta)$

$= \sec^2\theta(1 - \sin^2\theta)$

$[\because (a - b)(a + b) = a^2 - b^2]$ **1**

$= \sec^2\theta \times \cos^2\theta$ $[\because 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta$ $[\because \sec\theta = \frac{1}{\cos\theta}]$

$= 1.$ **1**

[CBSE Marking Scheme, 2020]

OR

$\sin 30° + \cos B = 1$

$\frac{1}{2} + \cos B = 1$ **1**

$\therefore$ $\cos B = 1 - \frac{1}{2} = \frac{1}{2}$

i.e., $\cos B = \cos 60°$ $\left[\because \cos 60° = \frac{1}{2}\right]$

Hence, $\angle B = 60°.$ **1**

[CBSE Marking Scheme, 2020-21]

Section - C

26. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and $b \neq 0$.

So, $\sqrt{2} = \frac{a}{b}$

$\Rightarrow$ $a^2 = 2b^2$

Thus, a^2 is a multiple of 2. **1**

Let $a = 2m$, for some integer m

$\therefore$ $b^2 = 2m^2$

Thus, b^2 is multiple of 2

$\Rightarrow$ b is multiple of 2

Hence, 2 is common factor of a and b **1**

This contradicts the fact that a and b are co primes.

Hence, $\sqrt{2}$ is an irrational number. **1**

[CBSE Marking Scheme 2019]

27. For unique solution $\frac{1}{3} \uparrow \frac{2}{k}$ **1**

$\Rightarrow$ $k \neq 6$ **1**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given equations are

$x + 2y - 5 = 0$ (i)

and $3x + ky + 15 = 0$ (ii)

Comparing eq (i) with $a_1x + b_1y + c_1 = 0$ and eq (ii) with $a_2x + b_2y + c_2 = 0$, we get

$a_1 = 1, a_2 = 3, b_1 = 2, b_2 = k, c_1 = -5$ and $c_2 = 15$

Since, given equations have unique solution, So,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

i.e. $\frac{1}{3} \neq \frac{2}{k}$

$\Rightarrow$ $k \neq 6$

Hence, for all values of k except 6, the given pair of equations have unique solution.

Commonly Made Error

- Many students make mistakes while applying condition of unique solution. They forget to insert the 'not equals to' sign (≠).

Answering Tip

- It is necessary to practice more such questions for better understanding of the condition of unique solution.

28. $a_m = \frac{1}{n} \Rightarrow 2a + (m-1)d = \frac{1}{n}$ (i) ½

$a_n = \frac{1}{m} \Rightarrow 2a + (n-1)d = \frac{1}{m}$ (ii) ½

Solving (i) and (ii), we get

$a = \frac{1}{mn}$ and $d = \frac{1}{mn}$ **1**

$$S_{mn} = \frac{1}{mn}$$

$$= \frac{mn}{2}\left[2\times\frac{1}{mn}+(mn-1)\times\frac{1}{mn}\right]$$

$$= \frac{1}{2}(mn+1)$$ **1**

[CBSE Marking Scheme 2017]

OR

Here, $a = 9$, $d = 8$, $S_n = 636$ **1**

Therefore, $636 = \frac{n}{2}[18+(n-1)8]$ **1**

$\Rightarrow \quad 4n^2 + 5n - 636 = 0$

$\Rightarrow \quad (4n + 53)(n - 12) = 0$

$\Rightarrow \quad n = 12$ **1**

[CBSE Marking Scheme 2017]

29. $A + 2B = 60°$ and $A + 4B = 90°$ **1+1**

Solving to get $B = 15°$ and $A = 30°$ **1**

[CBSE Marking Scheme 2019]

Detailed Answer

Given, $\sin(A + 2B) = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin(A + 2B) = \sin 60°$

$\Rightarrow \quad A + B = 60°$ (i) **1**

and $\quad \cos(A + 4B) = 0$

$\Rightarrow \quad \cos(A + 4B) = \cos 90°$

$\Rightarrow \quad A + 4B = 90°$ (ii) **1**

Solving eqs. (i) and (ii), we get

A = 30° and B = 15° **1**

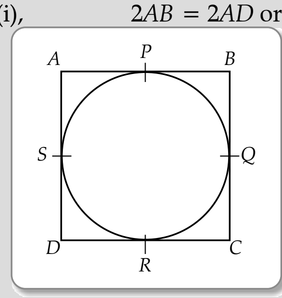
30. Let ABCD be the $||^{gm}$.

$\therefore \quad AB = CD$ and $AD = BC$ (i) ½

$AP + PB + DR + CR = AS + BQ + DS + CQ$ **1**

or, $\quad AB + CD = AD + BC$ ½

From (i), $\quad 2AB = 2AD$ or $AB = AD$



or, ABCD is a rhombus. **1**

[CBSE Marking Scheme, 2020]

Detailed Solution:

Let ABCD be the parallelogram.

$\therefore AB = CD$ and $AD = BC$ ...(i)

We know that the tangents drawn to a circle from an exterior point are equal in length.

Therefore, $AP = AS$, $BP = BQ$, $CR = CQ$ and $DR = DS$.

Adding the above equations.

$AP + BP + CR + DR = AS + BQ + CQ + DS$

$\Rightarrow (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)$

$\Rightarrow \quad AB + CD = AD + BC$

From eq. (i),

$2AB = 2AD$

or, $\quad AB = AD$

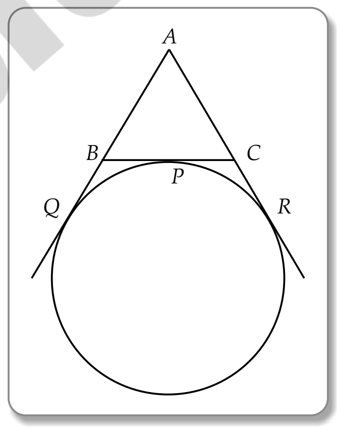
Hence, $ABCD$ is a rhombus. **Hence Proved.**

OR

$BC + CA + AB$

$= (BP + PC) + (AR - CR) + (AQ - BQ)$

$= AQ + AR - BQ + BP + PC - CR$



$\because$ From the same external point, the tangent segments drawn to a circle are equal.

From the point B, $\quad BQ = BP$

From the point A, $\quad AQ = AR$

From the point C, $\quad CP = CR$

$\therefore$ Perimeter of ΔABC,

$AB + BC + CA = 2AQ - BQ + BQ + CR - CR]$

$\Rightarrow \quad = 2AQ$

$\Rightarrow \quad AQ = \frac{1}{2}(BC + CA + AB)$

Hence proved.

31. Total possible outcomes $n(S) = 6$

(i) Let E_1 = getting event letter A, then $n(E_1) = 2$ **1**

$\therefore \quad$ Probability $= \frac{n(E_1)}{n(S)} = \frac{2}{6} = \frac{1}{3}$ **1**

(ii) Let E_2 = Getting event letter C, then $n(E_2) = 3$

$\therefore \quad$ Probability $= \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$. **1**

Section - D

32.

A 192 km B

Let speed of passenger train be x km/h. ½

$\therefore$ speed of superfast train = $(x + 16)$ km/h ½

By question, $T_{passenger} = \frac{192}{x}$

and $T_{superfast} = \frac{192}{(x+16)}$ ½

According to the question,

$$\frac{192}{x} - \frac{192}{x+16} = 2 \quad ½$$

$\Rightarrow \quad 192(x + 16) - 192x = 2(x^2 + 16x)$ ½

$\Rightarrow \quad 192x + 192 \times 16 - 192x = 2(x^2 + 16x)$ ½

$\Rightarrow \quad x^2 + 16x - 1536 = 0$

$\Rightarrow \quad x^2 + 48x - 32x - 1536 = 0$

$\Rightarrow \quad x(x + 48) - 32(x + 48) = 0$ 1

$\Rightarrow \quad (x - 32)(x + 48) = 0$

$\Rightarrow \quad x = 32$ or -48

Since, speed cannot be negative, therefore -48 is not possible.

$\therefore$ Speed of passenger train = 32 km/h 1

and speed of superfast train = 48 km/h.

[CBSE Marking Scheme 2012]

OR

Let total length of cloth = l m

$\therefore$ Rate per meter = ₹ $\frac{200}{l}$ 1

$\Rightarrow \quad (l+5)\left(\frac{200}{l} - 2\right) = 200$ 1

$\Rightarrow \quad (l + 5)(200 - 2l) = 200l$ 1

$\Rightarrow \quad l^2 + 5l - 500 = 0$

$\Rightarrow \quad (l + 25)(l - 20) = 0$ 1

$\Rightarrow l = 20$ m (neglecting $l = -25$)

$\therefore$ Rate per meter = ₹ $\frac{200}{20}$ = ₹ 10 per meter 1

[CBSE Marking Scheme 2019]

33. **(i)** Since, BC and OX bisect each other.

So, $BXCO$ is a parallelogram then $BE \parallel XC$ and $BX \parallel CF$.

In ΔABX, by B.P.T.,

$$\frac{AF}{FB} = \frac{AO}{OX} \quad \text{(i) } \mathbf{1}$$

In ΔAXC,

$$\frac{AE}{EC} = \frac{AO}{OX} \quad \text{(ii) } \mathbf{1}$$

Eqn. (i) and (ii) gives,

$$\frac{AF}{FB} = \frac{AE}{EC} \quad \mathbf{1}$$

So by converse of B.P.T.,

$$EF \parallel BC$$

(ii) Given $\frac{OX}{OA} = \frac{FB}{AF}$ 1

Adding 1 on both sides

$$\frac{OX}{OA} + 1 = \frac{FB}{AF} + 1$$

$$\frac{OX + OA}{OA} = \frac{FB + AE}{AF}$$

$$\frac{AX}{OA} = \frac{AB}{AF}$$

(from fig. given in question)

or $OA : AX = AF : AB$ **Hence Proved. 1**

34. Volume of cuboid = $4.4 \times 2.6 \times 1$ m^3 1

Inner and outer radii of cylindrical pipe

= 30 cm and 35 cm ½

$\therefore$ Volume of material used

$$= \frac{\pi}{100^2}(35^2 - 30^2) \times h \text{ m}^3 \quad 1$$

$$= \frac{\pi}{100^2} \times 65 \times 5 \times h \quad ½$$

Now, $\frac{\pi}{100^2} \times 65 \times 5h = 4.4 \times 2.6$ 1

$\Rightarrow \quad h = \frac{7 \times 4.4 \times 2.6 \times 100 \times 100}{22 \times 65 \times 5}$

$\Rightarrow \quad h = 112$ m 1

OR

Area of shaded region = Area of sector OACB – Area of triangle OBD 1

$$= \frac{90°}{360°} \times \pi \times (3.5)^2 - \frac{1}{2} \times 3.5 \times 2 \quad 1$$

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5 \times 3.5 - 3.5$$

$$= \frac{11}{2} \times 0.5 \times 3.5 - 3.5$$

$= 6.125$ cm^2 3

35.

$x_1 = 5 + 7 = 12$

$x_2 = 18 - x_1 = 18 - 12 = 6$ 2

$x_3 = 18 + 5 = 23$

and $x_4 = 30 - x_3 = 30 - 23 = 7$ 3

[CBSE Marking Scheme, 2016]

Section - E

36. **(i)** Speed of Raj's car $= x$ km/h

Speed of Ajay's car $= (x + 5)$ km/h

Distance covered by Ajay in 2 hours

$= [(x + 5) \times 2]$ km

$= 2(x + 5)$ km.

(ii) Given,

Speed of Raj's car $= x$ km/h

Speed of Ajay's car $= x + 5$ km/h

Time taken by Raj $= \dfrac{400}{x}$ h

Time taken by Ajay $= \dfrac{400}{(x+5)}$ h

Raj took 4 hours more than Ajay i.e.,

$$\frac{400}{x} - \frac{400}{x+5} = 4$$

$$\Rightarrow \quad \frac{100}{x} - \frac{100}{x+5} = 1$$

$$\Rightarrow \quad 100(x + 5) - 100x = x(x + 5)$$

$$\Rightarrow \quad x^2 + 5x - 500 = 0$$

(iii) Time taken by Ajay $= \dfrac{400}{x+5}$ h

$= \dfrac{400}{20+5}$ h

$= 16$ h

OR

Speed of Ajay's car

$= x + 5$ km/h

$= 20 + 5$

$= 25$ km/h

37. **(i)** Modal Class $= 120 – 130$

Upper limit $= 130$

(ii) Mode class frequency of the given data is 21.

(iii) No. of wrestlers with more than 120 kg weight$= 21 + 8 + 3 = 32$

OR

For class mark of 130 – 140,

$$= \frac{130+140}{2}$$

$$= \frac{270}{2} = 135$$

38. **(i)** $DR = 5$ m (given)

$\therefore$ $DR = DS$

(Length of tangents are equal)

i.e., $DS = 5$ m.

(ii) We have $AD = 23$ m.

and $DS = 5$ m [Proved in (i)]

$\therefore$ $AS = AD – DS$

$= (23 – 5)$ m $= 18$ m.

(iii) We have,

$AB = 29$ m

But $AS =$ AP (lengths of tangents are equal)

and $AS = 18$ m [Proved in (ii)]

$\therefore$ $AP = 18$ m

Now, $PB = AB – AP$

$= (29 – 18)$ m

$= 11$ m.

OR

$\because$ $PB = 11$ m [proved in (iii)]

But $PB = BQ$ (lengths of tangents are equal)

$\therefore$ $BQ = 11$ m

or $r = OQ = QB = 11$ m

Hence, diameter $= 2r = 2 \times 11 = 22$ m.

■■■

SOLUTIONS

Self Assessment Paper-2

Mathematics Standard (041)

Section - A

1. **Option (A) is correct.**

Explanation: Prime factorisation

$6 = 3 \times 2$, $72 = 2^3 \times 3^2$ and $120 = 2^3 \times 3 \times 5$

$\therefore$ LCM $(6, 72, 120) = 2^3 \times 3^2 \times 5^1 = 360$ **1**

2. **Option (A) is correct.**

Explanation: Let $f(x) = x^2 + ax + b$ and α, β are the roots of it.

Then, $\beta = -\alpha$ (Given)

$$\alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\alpha - \alpha = -\frac{a}{1} \text{ and } \alpha(-\alpha) = \frac{b}{1}$$

$$-a = 0 \text{ and } -\alpha^2 = b$$

$$a = 0 \Rightarrow b < 0 \text{ or } b \text{ is negative.}$$

So, $f(x) = x^2 + b$ shows that it has no linear term. **1**

3. **Option (D) is correct.**

Explanation: Since 2 is an even prime number, then $n = 2$

Putting the value of n is $2\,(7^2 + 8^2)$, we get

$$\begin{aligned} 2(7^2 + 8^2) &= 2(49 + 64) \\ &= 2 \times 113 \\ &= 226 \end{aligned}$$

Hence, $2(7^n + 8^n)$ ends with 6, where n is an even prime number.

4. **Option (A) is correct.**

Explanation: Since, $\frac{1}{2}$ is a root of the equation

$$x^2 + kx - \frac{5}{4} = 0$$

Then, $\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$

$$\frac{1}{4} + \frac{k}{2} - \frac{5}{4} = 0$$

$$\frac{k}{2} = \frac{5}{4} - \frac{1}{4}$$

$$\frac{k}{2} = 1$$

$$k = 2$$

1

5. **Option (C) is correct.**

Explanation: Points are $(-4, -7)$ and $(0, -7)$

$$\therefore \quad \text{Distance} = \sqrt{(0+4)^2 + (-7+7)^2}$$

$$= \sqrt{4^2 + 0} = \sqrt{16} = 4 \text{ units}$$ **1**

6. **Option (A) is correct.**

Explanation: PQ is tangent and QO is radius at contact point Q.

$\therefore$ $\angle PQO = 90°$

$\therefore$ By Pythagoras theorem,

$$\begin{aligned} PQ^2 &= OP^2 - OQ^2 \\ &= 13^2 - 5^2 = 169 - 25 = 144 \end{aligned}$$

$\Rightarrow$ $PQ = 12$ cm

$\therefore$ $\Delta OPQ \;\; \Delta OPR$ [SSS congruence]

$\therefore$ Area of ΔOPQ = area of ΔOPR

[Since, congruent figures are equal in areas]

Area of quadrilateral $QORP$ = 2 area of ΔOPR

$$\begin{aligned} &= 2 \times \frac{1}{2} \text{ base} \times \text{height} \\ &= RP \times OR \\ &= 12 \times 5 \\ &= 60 \text{ cm}^2 \end{aligned}$$ **1**

Answering Tip

- Applying the pythagoras theorem after that applying Area of quadrilateral.

Commonly Made Error

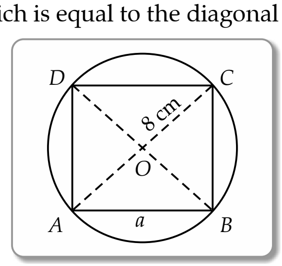
- Sometimes students unable to see and justify the right triangles and hence do not apply the pythagoras theorem.

7. **Option (B) is correct.**

Explanation: Given, radius of circle, $r = OC = 8$ cm

Diameter of the circle $= AC = 2 \times OC = 2 \times 8$

$= 16$ cm which is equal to the diagonal of a square.



Let side of square be 'a'.

Using Pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$
$$a^2 + a^2 = 16^2$$
$$2a^2 = 256$$
$$a^2 = 128 \text{ cm}^2 \quad \mathbf{1}$$

8. Option (D) is correct.

Explanation: $\frac{V_1}{V_2} = \frac{64}{27}$

$$\Rightarrow \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

[r_1 and r_2 are the radii of two spheres.]

$$\Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{64}{27}$$

$$\Rightarrow \frac{r_1}{r_2} = \frac{4}{3}$$

Now, the ratio of their surface areas,

$$\frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{16}{9} \quad \mathbf{1}$$

9. Option (A) is correct.

Explanation: Total number of outcomes favourable for event E are (1, 2, 3, 4, 5, 6), i.e., $T(E) = 6$

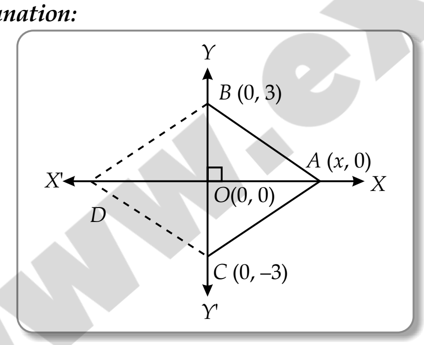
A number which is odd and less than 3 is 1 so,

$F(E) = 1$

So, probability $P(E) = \frac{F(E)}{T(E)} = \frac{1}{6}$ **1**

10. Option (C) is correct.

Explanation:



O is the midpoint of the base BC

i.e., O is the midpoint of B and $C(0, –3)$

Therefore, coordinates of point B is (0, 3)

So, $BC = 6$ units.

Let the coordinates of point A be $(x, 0)$.

Using distance formula,

$$AB = \left|\sqrt{(0-x)^2 + (3-0)^2}\right|$$
$$= |\sqrt{x^2+9}|$$
$$BC = \left|\sqrt{(0-0)^2 + (-3-3)^2}\right|$$
$$= |\sqrt{36}|$$

Also, $BC = AB$

$$|\sqrt{x^2+9}| = |\sqrt{36}|$$
$$x^2 = 27$$

or $x = \pm 3\sqrt{3}$

Coordinates of A and B are $(\pm 3\sqrt{3}, 0)$ and (0, 3) respectively.

11. Option (C) is correct.

Explanation: $\angle RPQ = 50°$

$\therefore$ $\angle OPQ = 90° - 50° = 40°$

Since, $OP = OQ$ (radii of circle)

$\because$ $\angle OPQ = \angle OQP = 40°$

$\angle POQ = 180° - (40° + 40°)$

$= 100°$. **1**

[CBSE Marking Scheme 2012]

12. Option (C) is correct.

Explanation: Here, $x_1 = a \cos \theta + b \sin \theta, y_1 = 0$

and $x_2 = 0, y_2 = a \sin \theta - b \cos \theta$

$\therefore$ Distance $= \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$

$$= \left|\sqrt{(0 - a\cos\theta - b\sin\theta)^2 + (a\sin\theta - b\cos\theta - 0)^2}\right|$$

$$= \left|\sqrt{(-1)^2(a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2}\right|$$

$$= \left|\sqrt{\begin{array}{l} a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta \\ + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta \end{array}}\right|$$

$$= \left|\sqrt{a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}\right|$$

$$= |\sqrt{a^2 \times 1 + b^2 \times 1}| = \sqrt{a^2 + b^2} \text{ units.}$$

[CBSE Marking Scheme, 2020]

13. Option (A) is correct.

Explanation:

$$\frac{1}{\tan\theta + \cot\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}}$$

$$= \frac{1}{\frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}}$$

$$= \frac{\cos\theta\sin\theta}{\sin^2\theta + \cos^2\theta}$$

$$= \cos\theta\sin\theta.$$

[$\sin^2\theta + \cos^2\theta = 1$]

14. Option (B) is correct.

Explanation: Plumbline is an instrument used to check the verticality of an object. It is a combination of a hemisphere and a cone.

15. Option (C) is correct.

Explanation:

Marks	Number of students	f_i
0 – 10	3 – 0=3	3
10 – 20	12 – 3=9	9
20 – 30	27 – 12=15	15
30 – 40	57 – 27=30	30
40 – 50	75 – 57=18	18
50 – 60	80 – 75=5	5

Modal class has maximum frequency (30) in class 30 – 40.

16. Option (B) is correct.

Explanation:

Here, $a = \frac{1}{3q}$ and $a + d = \frac{1-6q}{3q}$

$\therefore$ $d = \frac{1-6q}{3q} - \frac{1}{3q}$

$= \frac{1-6q-1}{3q} = \frac{-6q}{3q} = -2$

[CBSE Marking Scheme 2013]

17. Option (B) is correct.

Explanation: Let the length of shadow is x,

Then height of pole = $\sqrt{3}\,x$

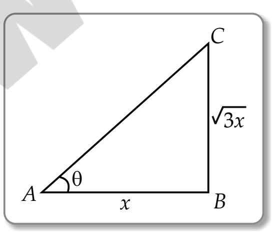
Now, $\tan\theta = \frac{CB}{AB}$

$\tan\theta = \frac{\sqrt{3}x}{x}$

$\tan\theta = \sqrt{3}$

$\tan\theta = \tan 60°$

$\theta = 60°$



18. Option (B) is correct.

Explanation: As the cylinder just encloses the sphere so the cylinder and diameter of sphere are equal, i.e., $2r$ and height $h = 2r$.

19. Option (C) is correct.

Explanation: In case of assertion:

Here, A → (0 , 5) and B → (– 3 , 1)

$$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$
$$= \sqrt{(-3-0)^2+(1-5)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5 \text{ units}$$

$\therefore$ Assertion is correct.

In case of reason:

If A (2, 7) lies on perpendicular bisector of P (6, 5) and Q (0, –4), then

$AP = AQ$

$\therefore$ By using Distance Formula,

$$AP = \sqrt{(6-2)^2+(5-7)^2}$$
$$= \sqrt{(4)^2+(-2)^2}$$
$$= \sqrt{20}$$

And $AQ = \sqrt{(0-2)^2+(-4-7)^2}$
$$= \sqrt{(-2)^2+(-11)^2}$$
$$= \sqrt{125}$$

As, $AP \neq AQ$

Therefore, A does not lie on the perpendicular bisector of PQ.

$\therefore$ Reason is incorrect.

Hence, assertion is correct but reason is incorrect.

20. Option (C) is correct.

Explanation: In case of assertion:

In the given two right triangles, both have equal right angles and one of the acute angles of one triangle is equal to an acute angle of the other triangle.

Thus, by AA similarity, the given two triangles are similar.

$\therefore$ Assertion is correct.

In case of reason:

We know that the ratio of the areas of two similar triangles is the square of the ratio of the corresponding altitudes of two similar triangles.

Thus, the ratio of the areas of two similar triangles is $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$.

$\therefore$ Reason is incorrect.

Hence, assertion is correct but reason is incorrect.

Section - B

21. Let B complete a work in x days.
Then A takes $x-6$ days to complete it.
Together they complete it in 4 days.
According to work done per day,

$$\frac{1}{x-6}+\frac{1}{x}=\frac{1}{4}$$

$$\frac{x+x-6}{x(x-6)}=\frac{1}{4}$$

$$4(2x-6)=x(x-6)$$

$$8x-24=x^2-6x$$

$$\therefore\ x^2-14x+24=0$$

$$x^2-12x-2x+24=0$$

$$x(x-12)-2(x-12)=0$$

$$(x-2)(x-12)=0$$

$$\therefore\ x=2 \text{ or } 12.$$

$x=2$ is not possible because then $x-6$ is (-4)

$\therefore\ x=12.$

So, B takes 12 days to finish the work.

[Topper' Answer, 2017] 2

22. As $DE \parallel BC$

$\therefore$ $$\frac{AD}{DB}=\frac{AE}{EC}$$

or, $$\frac{x}{x+1}=\frac{x+3}{x+5}$$

or, $x^2+5x=x^2+4x+3$

or, $x=3$ **1**

[CBSE Marking Scheme, 2016]

23. Distance covered in 1 revolution

= circumference of wheel

$= \pi d$

$= \pi \times 1.26$ m.

Distance covered in 500 revolutions **1**

$= 500 \times \pi \times 1.26$

$= 500 \times \frac{22}{7} \times 1.26$

$= 1980$ m. $= 1.98$ km **1**

[CBSE Marking Scheme 2012]

24.

m R n

P(2, −2) A$\left(\frac{24}{11}, y\right)$ Q(3, 7)

Here $x_1=2$, $y_1=-2$

$x_2=3$, $y_2=7$

Using section formula,

$$\left(\frac{24}{11}, y\right)=\left(\frac{3m+2n}{m+n}, \frac{7m-2n}{m+n}\right) \quad \text{—(1)}$$

$$\Rightarrow \frac{24}{11}=\frac{3m+2n}{m+n}$$

$$24m+24n=33m+22n$$

$$2n=9m$$

$$\frac{2}{9}=\frac{m}{n}$$

$\therefore$ The given point divides the line segment

in ratio 2 : 9.
Taking $m = 2$ and $n = 9$,

$y = \frac{7m - 2n}{m+n}$ (from ①)

$y = \frac{7(2) - 2(9)}{2+9}$

$y = \frac{14 - 18}{11}$

$\boxed{y = \frac{-4}{11}}$

[Topper's Answer, 2017] 2

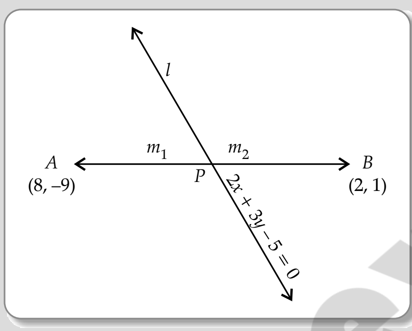
OR

P(x, y) divides AB in the ratio $m_1 : m_2$.

Let A(8, – 9) and B(2, 1)

$$x = \frac{2m_1 + 8m_2}{m_1 + m_2}$$

$$y = \frac{m_1 - 9m_2}{m_1 + m_2} \quad ½$$



½

$$2\left(\frac{2m_1 + 8m_2}{m_1 + m_2}\right) + 3\left(\frac{m_1 - 9m_2}{m_1 + m_2}\right) - 5 = 0$$

$\therefore \quad 2m_1 - 16m_2 = 0$

i.e., $\quad m_1 : m_2 = 8 : 1$ ½

$\therefore \quad x = \left(\frac{2\times 8 + 8\times 1}{8+1}\right) = \frac{8}{3}$

and $\quad y = \left(\frac{8\times 1 - 9\times 1}{8+1}\right) = \frac{-1}{9}$ ½

[CBSE Marking Scheme, 2012]

25. Given: $\quad 4\cos\theta = 11\sin\theta$

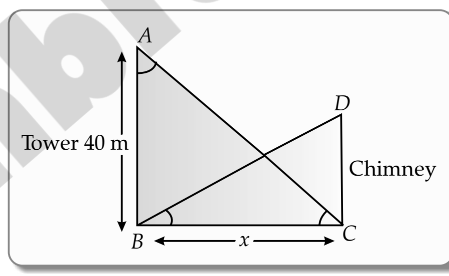
or, $\quad \cos\theta = \frac{11}{4}\sin\theta$

Now, $\frac{11\cos\theta - 7\sin\theta}{11\cos\theta + 7\sin\theta} = \frac{11\times\frac{11}{4}\sin\theta - 7\sin\theta}{11\times\frac{11}{4}\sin\theta + 7\sin\theta}$ **1**

$$= \frac{\sin\theta\left(\frac{121}{4} - 7\right)}{\sin\theta\left(\frac{121}{4} + 7\right)}$$

$= \frac{121 - 28}{121 + 28} = \frac{93}{149}$. **1**

OR



Let AB = 40 m be the height of the tower and CD be the height of smoking chimney.

Considering right angled ΔABC, we have

$$\tan 30° = \frac{BC}{AB}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{40}$

$\Rightarrow \quad BC = \frac{40}{\sqrt{3}}$ **1**

Again, considering right triangle DBC, we have

$$\tan 60° = \frac{DC}{BC}$$

$\Rightarrow \quad \sqrt{3} = \frac{DC}{BC}$ **1**

$DC = \sqrt{3} \times \frac{40}{\sqrt{3}}$

$\Rightarrow \quad DC = 40$ m

$\therefore$ The height of chimney is 40 m. **1**

Section - C

26. Let $2+5\sqrt{3}=a$, where a is a rational number

Then, $\sqrt{3}=\frac{a-2}{5}$ ½

which is a contradiction as LHS is irrational and RHS is rational. ½

$\therefore 2+5\sqrt{3}$ cannot be rational

Hence, $2+5\sqrt{3}$ is irrational. ½

Alternative method:

Let $2+5\sqrt{3}$ be rational number.

$\therefore 2+5\sqrt{3}=\frac{p}{q}$, p, q are integers, $q \neq 0$. ½

$\Rightarrow \quad \sqrt{3} = \left(\frac{p}{q}-2\right)\div 5$

$\Rightarrow \quad \sqrt{3} = \frac{p-2q}{5q}$ ½

LHS is irrational and RHS is rational

which is contradiction

$\therefore 2+5\sqrt{3}$ is rational. ½

Note: Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

You can prove it in similar way.

[CBSE Marking Scheme 2019]

27. Substituting $\frac{1}{\sqrt{x}} = X$ and $\frac{1}{\sqrt{y}} = Y$

$2X + 3Y = 2$ (i)

and $4X - 9Y = -1$ (ii) ½

Multiplying eqn. (i) by 3, and add in (ii), we get

$$\begin{aligned} 4X - 9Y &= -1 \\ \underline{6X + 9Y} &\underline{= 6} \\ 10X &= 5 \end{aligned}$$

$$10X = 5 \text{ or } X = \frac{5}{10} \Rightarrow \frac{1}{2}$$

$$\frac{1}{\sqrt{x}} = \frac{1}{2} \quad \therefore \quad x = 4$$ ½

Putting the value of X in eqn. (i), we get

$$2\cdot\frac{1}{2}+3Y = 2$$

$$3Y = 2 - 1$$

$$Y = \frac{1}{3}$$ 1

$\Rightarrow \quad Y = \frac{1}{3}$ or $\frac{1}{\sqrt{y}} = \frac{1}{3} \Rightarrow y = 9$

Hence, $x = 4$ and $y = 9$. 1

28. Let fixed charge be ₹ x and per day food cost be y

Then, $x + 20y = 3000$ (i)

and $x + 25y = 3500$ (ii) **1**

Subtracting (i) from (ii), we get

$$\begin{aligned} x + 25y &= 3500 \\ x + 20y &= 3000 \\ - \quad - \quad & \quad - \\ \hline 5y &= 500 \\ \hline y &= 100 \end{aligned}$$ 1

Substituting this value of y in (i), we get

$$\begin{aligned} x + 20(100) &= 3000 \\ x &= 1000 \end{aligned}$$

$\therefore \quad x = 1000$ and $y = 100$ 1

Hence, fixed charge and cost of food per day are ₹ 1,000 and ₹ 100.

Commonly Made Error

- Equations are formed incorrectly, leading to incorrect answer.

Answering Tip

- Verify the answer by substituting the obtained values of x and y in both the equations.

OR

$a_p = q \Rightarrow a + (p-1)d = q$ (i) ½

$a_q = p \Rightarrow a + (q-1)d = p$ (ii) ½

Solving eqs (i) & (ii), we get

$a = p + q - 1$ and $d = -1$ 1

$$\begin{aligned} \therefore \quad a_n &= a + (n-1)d \\ &= (p + q - 1) + (n-1)(-1) \\ &= p + q - n \end{aligned}$$ 1

[CBSE Marking Scheme 2017]

29. Given, $\sec\theta = x + \frac{1}{4x}$

$$\sec^2\theta = \left(x+\frac{1}{4x}\right)^2$$ 1

$$\Rightarrow \quad \sec^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \quad \sec^2\theta - 1 = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$ 1

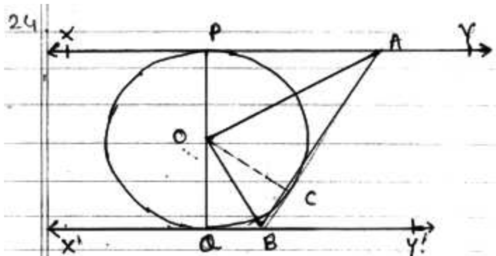
$$\Rightarrow \quad \tan^2\theta = \left(x-\frac{1}{4x}\right)^2$$

$$\Rightarrow \quad \tan\theta = \left(x-\frac{1}{4x}\right) \text{ or } \left(\frac{1}{4x}-x\right)$$

Hence, $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$ 1

[CBSE Marking Scheme 2019]

30.



Given: $XY \parallel X'Y'$ — tangents.
POQ is diameter, OC is radius.
Tangent ACB touches XY at A and $X'Y'$ at B.
To prove: $\angle AOB = 90°$.

Proof: $XY \parallel X'Y'$ and AB is transversal

$\therefore \angle XAB + \angle ABX' = 180°$ — cointerior angles

or $\angle PAB + \angle ABQ = 180°$ — (1)

It is known that tangents from a same point are equally inclined to the line joining centre to that point.

$\Rightarrow \angle PAO = \angle CAO$ and $\angle QBO = \angle CBO$

In (1),

$2\angle CAO + 2\angle CBO = 180°$

or $2\angle BAO + 2\angle ABO = 180°$

$\angle BAO + \angle ABO = 90°$ — (2)

In ΔAOB,

$\angle BAO + \angle ABO + \angle AOB = 180°$ — angle sum.

from (2), $90° + \angle AOB = 180°$

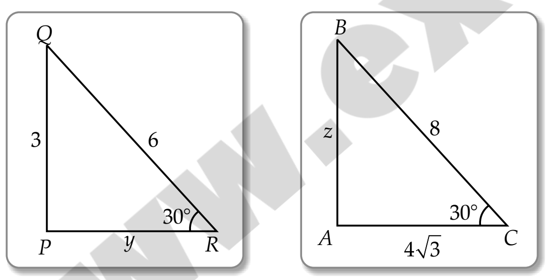
$\therefore \angle AOB = 90°$

Hence, proved.

[Topper's Answer, 2017] 3

OR

Given, $\Delta ABC \sim \Delta PQR$ (Given)



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

or, $$\frac{z}{3} = \frac{8}{6} = \frac{4\sqrt{3}}{y}$$

or, $$\frac{z}{3} = \frac{8}{6} \text{ and } \frac{8}{6} = \frac{4\sqrt{3}}{y}$$ **1**

or, $$z = \frac{8\times 3}{6} \text{ and } y = \frac{4\sqrt{3}\times 6}{8}$$

$\therefore$ $$z = 4 \text{ and } y = 3\sqrt{3}$$

$\therefore$ $$y + z = 3\sqrt{3} + 4$$ **2**

31. Let there be x black balls and 15 white balls.

Total balls $= n(S) = 15 + x$

P(drawing black ball) = $3 \times$ P(drawing white ball).

$\Rightarrow \dfrac{x}{(15+x)} = 3 \times \dfrac{15}{(15+x)}$

$x = \dfrac{3\times 15}{\cancel{(15+x)}} \times \cancel{(15+x)}$

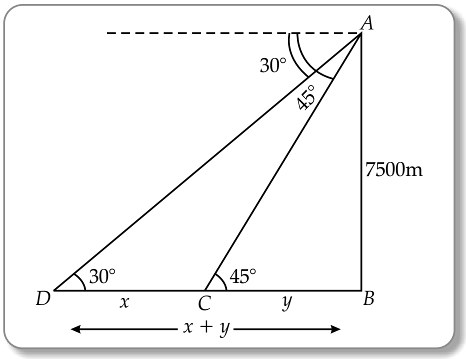
$x = 45$

$\therefore$ There are 45 black balls in the bag.

Topper's Answer, 2017] 3

Section - D

32.



In ΔABC, $\frac{AB}{BC} = \tan 45°$ **1**

$\Rightarrow \frac{7500}{y} = 1$

$\Rightarrow y = 7500$ **1**

In ΔABD, $\frac{AB}{BD} = \tan 30°$

$\frac{7500}{x+y} = \frac{1}{\sqrt{3}}$

$\Rightarrow x + y = 7500\sqrt{3}$ **1**

$x + 7500 = 7500\sqrt{3}$

$x = 7500\sqrt{3} - 7500$

$= 7500(\sqrt{3} - 1)$ **1**

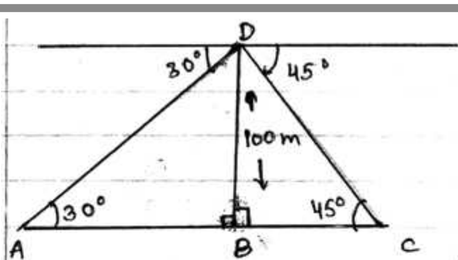
$= 7500(1.73 - 1)$

$= 7500 \times 0.73$

$= 5475$ m

Hence, the distance between two ships = 5475 m **1**

OR



To find : AC

Solution:

~~In ΔABD,~~ $\angle DAB = 30°$

~~In ΔBDC,~~ $\angle BCD = 45°$.

also, BD = 100m.

In right ΔABD,

$\tan 30° = \frac{DB}{AB}$

$\frac{1}{\sqrt{3}} = \frac{100}{AB}$

$AB = 100\sqrt{3}$ 100×1.732

173.2 m

In right ΔDBC,

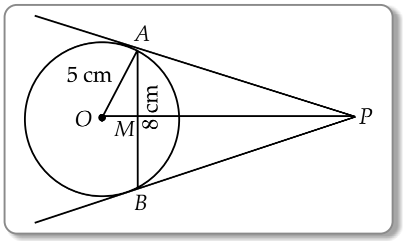
$\tan 45° = \frac{DB}{BC}$

$1 = \frac{100}{BC} \Rightarrow BC = 100m.$

Now, AC = AB + BC = 100 + 173.2 m = $\underline{273.2m}$

[Topper's Answer 2017] 3

33.



Given, $AB = 8$ cm

$\Rightarrow$ $AM = 4$ cm.

$\therefore$ $OM = \sqrt{OA^2 - AM^2}$ [By Pythagoras theorem]

$OM = \sqrt{5^2 - 4^2} = 3$ cm.

Let $AP = y$ cm, $PM = x$ cm.

$\because$ ΔOAP is a right angle triangle.

$\therefore$ $OP^2 = OA^2 + AP^2$ [By Pythagoras theorem]

$(x+3)^2 = y^2 + 25$

$\Rightarrow$ $x^2 + 9 + 6x = y^2 + 25$ (i)

Also, $x^2 + 4^2 = y^2$ (ii)

$x^2 + 6x + 9 = x^2 + 16 + 25$

$6x = 32$

$\Rightarrow$ $x = \frac{32}{6}$ or $\frac{16}{3}$ cm

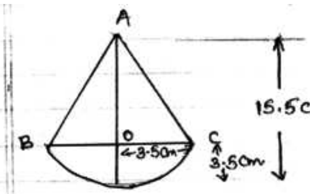
$y^2 = x^2 + 16 = \frac{256}{9} + 16$

$= \frac{400}{9}$

$y = \frac{20}{3}$ cm or $6\frac{2}{3}$ cm.

34.

18.



Height of hemisphere = r = 3.5 cm

height of cone = 15.5 cm − 3.5 cm = 12 cm = h.

Slant height of cone = $\sqrt{r^2 + h^2}$

$= \sqrt{12.25 + 144}$

$= \sqrt{156.25}$

$= 12.5$ cm

TSA of toy = CSA of cone + CSA of hemi-sphere.

$= \pi r l + 2\pi r^2$

$= \frac{22}{7} \times 12.5 \times 3.5 + 2 \times \frac{22}{\cancel{7}} \times \overset{0.5}{\cancel{3.5}} \times 3.5$

$= 22 \times 12.5 \times 0.5 + 22 \times 3.5$

$= 22\left(12.5 \times \frac{5}{10} + 3.5\right)$

$= 22\left(12.5 \times \frac{1}{2} + 3.5\right)$

$= 22(6.25 + 3.5)$

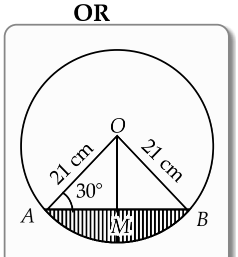
$= 22(9.75)$

$= 214.5$ cm^2

$\therefore$ Total surface area of toy is 214.5 cm^2

[Topper's Answer, 2017] 5

OR



$\angle OAB = \angle OBA = 30°$

$\sin 30° = \frac{1}{2} = \frac{OM}{21} \Rightarrow OM = \frac{21}{2}$

$\cos 30° = \frac{\sqrt{3}}{2} = \frac{AM}{21} \Rightarrow AM = \frac{21}{2}\sqrt{3}$

Area of $\Delta AOB = \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times 21\sqrt{3} \times \frac{21}{2}$ $\quad (\because AB = 2AM)$

$= \frac{441}{4}\sqrt{3}\text{ cm}^2$ **2**

$\therefore$ Area of shaded region

= Area (sector OACB) – Area (ΔOAB) **1**

$= \left(\frac{22}{7} \times 21 \times 21 \times \frac{120°}{360°}\right) - \frac{441}{4}\sqrt{3}$

$= \left(462 - 441\frac{\sqrt{3}}{4}\right)\text{cm}^2$

$= 271.3\text{ cm}^2$ (approx) **2**

35.

Class Interval	Frequency	Cumulative frequency
0 – 100	2	2
100 – 200	5	7
200 – 300	x	$7 + x$
300 – 400	12	$19 + x$
400 – 500	17	$36 + x$
500 – 600	20	$56 + x$
600 – 700	y	$56 + x + y$
700 – 800	9	$65 + x + y$
800 – 900	7	$72 + x + y$
900 – 1000	4	$76 + x + y$
	$N = 100$	

1

Hence, $76 + x + y = 100$

$\Rightarrow$ $x + y = 100 - 76 = 24$ ...(i)

Given, Median = 525, which lies between class 500 – 600.

$\Rightarrow$ Median class = 500 – 600

Now, Median $= l + \frac{\frac{n}{2} - c.f.}{f} \times h$ **1**

$\Rightarrow$ $525 = 500 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 100$

$\Rightarrow$ $25 = (50 - 36 - x)\,5$

$\Rightarrow$ $(14 - x) = \frac{25}{5} = 5$ **1**

$\Rightarrow$ $x = 14 - 5 = 9$ **1**

Substituting the value of x in equation (i),

$y = 24 - 9 = 15$ **1**

Hence, $x = 9$ and $y = 15$.

Commonly Made Error

- Mostly candidates do error in calculation, when data is bigger in digits.

Answering Tip

- When data is big they should do calculations carefully and accurately.

Section - E

36. **(i)**

$a = 1000$

$d = 100$

$a_{80} = a + (n - 1)d$

$= 1000 + (30 - 1)100$

$= 1000 + 2900 = 3900$

(ii) Sum of 30 instalments

$= \frac{n}{2}[2a + (n - 1)d]$

$= \frac{30}{2}[2 \times 1000 + (30 - 1)100]$

$= 15[2000 + 2900]$

$= 15 \times 4900$

$= 73500$

Total amount paid in 30 instalments = ₹ 73500

(iii) Amount paid in 40th instalment, a_{40}

$= a + (n - 1)d$

$= 1000 + (40 - 1)100$

$= 1000 + 3900$

$= 4900$

OR

According to question 4,

Amount paid in last instalment

$= ₹\ 4900$

Principal for Ist instalment = ₹ 1000

Hence, Ratio of Ist instalment to the last instalment

$$= \frac{1000}{4900}$$

$$= 10 : 49.$$

37. **(i)** We know that the lengths of tangents drawn from an external point to a circle are equal. So SK and SC are tangents to a circle with centre O.

$\therefore \quad SK = SC$

(ii) In question 1, we have proved

$$SK = SC$$

Then ΔSKC is an isosceles triangle and SO is the angle bisector of $\angle$KSC.

So, $\quad$ OS $\perp$ KC.

$\therefore$ OS bisects KC, gives $KR = RC = 4$ cm.

Now, $\quad OR = \sqrt{OK^2 - KR^2}$

(By using Pythagoras theorem)

$$= \sqrt{5^2 - 4^2} = \sqrt{25 - 16}$$

$$= \sqrt{25 - 16}$$

$$= \sqrt{9}$$

$$= 3 \text{ m.}$$

OR

$$\angle SKR + \angle OKR = \angle OKR = 90°$$

(Radius is $\perp^r$ to tangent)

(iii) ΔSKR and ΔRKO,

$$\angle RKO = \angle KSR$$

and $\quad \angle SRK = \angle ORK$

$\therefore \quad \Delta KSR \sim \Delta OKR$ (By AA Similarity)

Then $\quad \dfrac{SK}{KO} = \dfrac{RK}{RO}$

$\Rightarrow \quad \dfrac{SK}{5} = \dfrac{4}{3}$

(RO = 3 m, proved in Q.2.)

$\Rightarrow \quad 3SK = 20$

$\Rightarrow \quad SK = \dfrac{20}{3}$

Hence, the distance between Kanabh and Shubhi is $\dfrac{20}{3}$ m. **1**

38. **(i)** We have to find the LCM of 32 and 36.

$$\text{LCM}(32, 36) = 2^5 \times 9 = 288$$

Hence, the minimum number of books required to distribute equally among students of section A and section B are 288.

(ii) Given two number is 32, 36

HCF $\times$ LCM = Product of the number

HCF $\times$ LCM = 32 $\times$ 36

$$\text{HCF} = \frac{1152}{\text{LCM}} \quad (1)$$

LCM of (32, 36) is 288

LCM put in eq (1)

$$\text{HCF} = \frac{1152}{288}$$

$$\text{HCF} = 4$$

OR

$2^2 \times 3^2 = 36$

(iii) Given, $\quad p = ab^2 = a \times b \times b$

$q = a^2b = a \times a \times b$

LCM of $(p, q) = a^2b^2$

■■■

SOLUTIONS
Self Assessment Paper-3
Mathematics Standard (041)

Section - A

1. **Option (A) is correct.**

Explanation: Prime factors of 96 = $2^5 \times 3$

Prime factors of 404 = $2^2 \times 101$

$\therefore$ HCF = $2^2 = 4$.

2. **Option (C) is correct.**

Explanation: Product of two numbers = LCM × HCF.

3. **Option (C) is correct.**

Explanation: Condition for consistency:

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ has unique solution (consistent), i.e., intersecting at one point

or $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (many solutions)

(Consistent lines, coincident or dependent)

4. **Option (B) is correct.**

Explanation:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

$$S_8 = 3+6+9+12+ .... +24$$

$$= 3(1+2+3+4+ ... +8)$$

$$= \frac{3\times 8\times 9}{2}$$

$$= 108$$

$$\therefore \left\{\text{Sum of } n \text{ natural number} = \frac{n(n+1)}{2}\right\}$$

5. **Option (C) is correct.**

Explanation: By section formula, $m : n = 2 : 3$

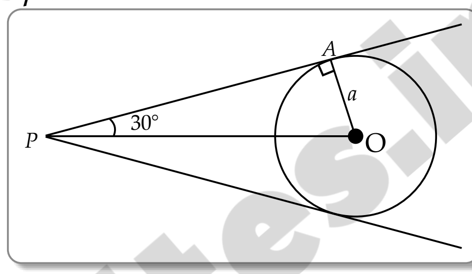
$$\frac{mx_2+nx_1}{m+n} = k, \quad \frac{my_2+ny_1}{m+n} = 4$$

Now, $\frac{2\times 5+3\times 2}{2+3} = k$

$$k = \frac{16}{5}.$$

6. **Option (B) is correct.**

Explanation:



$\angle OPA = 30°$

$\sin 30° = \frac{a}{OP}$

$\Rightarrow$ OP = $2a$

7. **Option (B) is correct.**

Explanation:

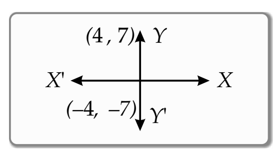
$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$$

$$\therefore AD = \frac{7.2\times 1.8}{5.4} = 2.4 \text{ cm}$$

[CBSE Marking Scheme 2019]

8. **Option (C) is correct.**

Explanation: By using the graph of coordinate plane, we have the reflection of point (– 4, 7) in x-axis is (– 4, – 7).



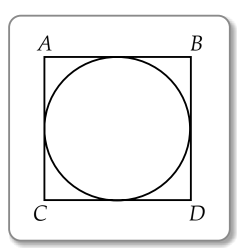
9. **Option (B) is correct.**

Explanation:

$$5\sec^2 A - 5\tan^2 A = 5(\sec^2 A - \tan^2 A)$$
$$= 5\,(1) \qquad [\because \sec^2 A - \tan^2 A = 1]$$
$$= 5$$

10. **Option (D) is correct.**

Explanation:



Given, side of square = 6 cm

Diameter of a circle, (d) = Side of square

$= 6$ cm

Radius of a circle (r) = $\frac{d}{2} = 6 = 3$ cm.

Area of circle $= \pi r^2$

$\pi(3)^2 = 9\pi \text{ cm}^2$

11. Option (C) is correct.

Explanation: Whole surface of each part

$= 2\pi r^2 + \pi r^2 = 3\pi r^2$

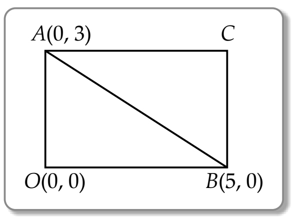
$\therefore$ Total surface of two parts

$= 3\pi r^2 + 3\pi r^2 = 6\pi r^2$

[CBSE Marking Scheme 2012]

12. Option (C) is correct.

Explanation: According to the question, a triangle can be represented as:



$\therefore$ Distance between the points A(0, 3) and B (5, 0) is

$$AB = \left|\sqrt{(5-0)^2 + (0-3)^2}\right|$$

$$= |\sqrt{25+9}|$$

$$= \sqrt{34} \text{ units}$$

Hence, the required length of diagonal is $\sqrt{34}$ units.

13. Option (C) is correct.

Explanation: In the given formula, a is assumed mean from class marks (x_i) and $d_i = x_i - a$

Therefore, d_i is the deviation of class mark (mid-value) from the assumed mean 'a'.

14. Option (D) is correct.

Explanation: Let the sides of two cubes be a and A, then

$$\frac{a^3}{A^3} = \frac{1}{64}$$

$$\Rightarrow \quad \frac{a}{A} = \frac{1}{4}$$

Since surface area of a cube is 6 (side)^2.

$$\therefore \quad \frac{6a^2}{6A^2} = \left(\frac{a}{A}\right)^2$$

$$= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

15. Option (C) is correct.

Explanation:

For jar 1, probability picking a red ball = $\frac{4}{8} = \frac{1}{2}$

For jar 2, probability picking a red ball = $\frac{3}{6} = \frac{1}{2}$

For jar 3, probability picking a red ball = $\frac{4}{6} = \frac{2}{3}$

$\therefore$ Probability picking a red ball is same in jar 1 and 2.

16. Option (D) is correct.

Explanation: Put the value of $x = 2$ in $3x^2 - 6x - 2 = 0$

$3(2)^2 - 6(2) - 2 = 0$

$12 - 12 - 2 = 0$

$12 - 14 = 0$

$-2 \neq 0$

So, $x = 2$ is not a root of $3x^2 - 6x - 2 = 0$

17. Option (A) is correct.

Explanation:

Here, $a = -4\frac{1}{2}$, $d = 1\frac{1}{2}$

$$\therefore \quad a_{21} = \frac{-9}{2} + 20\left(\frac{3}{2}\right) = \frac{51}{2}$$

[CBSE Marking Scheme 2019]

18. Option (B) is correct.

Explanation: $\sin^2 30° + \tan^2 45°$

$$= \left(\frac{1}{2}\right)^2 + (1)^2$$

$$= \frac{1}{4} + 1 = \frac{5}{4}$$

19. Option (B) is correct.

Explanation: In case of assertion:

Length of the tangent = $\sqrt{d^2 - r^2}$

$$= \sqrt{(8)^2 - (6)^2}$$

$$= \sqrt{64 - 36}$$

$$= \sqrt{28} = 2\sqrt{7} \text{ cm.}$$

$\therefore$ Assertion is correct.

In case of reason:

Since, sum of the angles between radii and between intersection point of tangent is 180°.

Angle at the point of intersection of tangents

= 180° – 130° = 50°.

$\therefore$ Reason is correct.

Hence, both assertion and reason are correct but reason is not the correct explanation for assertion.

20. Option (D) is correct.

Explanation: In case of assertion:

cot A is not the product of cot and A. It is the cotangent of $\angle$A.

$\therefore$ Assertion is incorrect.

In case of reason:

The value of sin θ increases as θ increases in interval of $0° < \theta < 90°$

$\therefore$ Reason is correct:

Hence, assertion is incorrect but reason is correct.

Section - B

21. Let the ten's and unit digit be y and x respectively.

So, the number is $10y + x$. ½

The number, when its digits are reversed, becomes $10x + y$.

So, $7(10y + x) = 4(10x + y)$ ½

$\Rightarrow 70y + 7x = 40x + 4y$

$\Rightarrow 70y - 4y = 40x - 7x$

$\Rightarrow 2y = x$ ...(i)

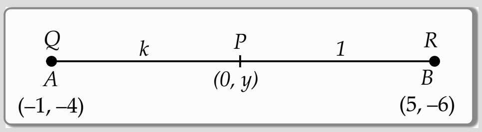
and $x - y = 3$ ...(ii)

From (i) and (ii), we get

$y = 3$ and $x = 6$

Hence, the number is 36. 1

22.



Any point on y-axis is P(0, y)

$\therefore \quad 0 = \dfrac{5k-1}{k+1}$

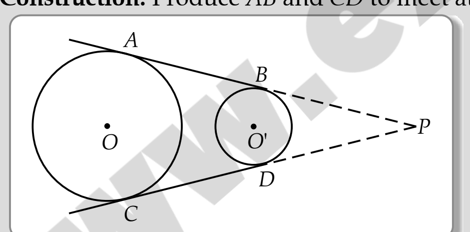
$\Rightarrow \quad k = \dfrac{1}{5}$ *i.e.*, 1 : 5 1

$\Rightarrow \quad y = \dfrac{-6k-4}{k+1} = \dfrac{-\frac{6}{5}-4}{\frac{1}{5}+1} = \dfrac{-26}{6} = \dfrac{-13}{3}$

$\therefore$ Coordinates of p is $\left(0, \dfrac{-13}{3}\right)$ 1

[CBSE Marking Scheme 2019]

23. Construction: Produce AB and CD to meet at P.



Now, $PA = PC$

and $PB = PD$ 1

Tangents to a circle from external point

Now, $PA - PB = PC - PD$

$\Rightarrow AB = CD$ 1

[CBSE Marking Scheme 2017]

24. Volume of cuboid $= 9 \times 8 \times 2\ \text{m}^3$

Volume of cube $= 2 \times 2 \times 2\ \text{m}^3$

Let number of recast cubes be n.

$\therefore$ Volume of n cubes = Volume of cuboid 1

$n \times 2 \times 2 \times 2 = 9 \times 8 \times 2$

$n = \dfrac{9 \times 8 \times 2}{2 \times 2 \times 2} = 18$

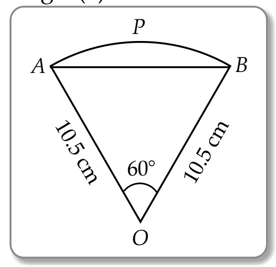
Hence, number of cubes recast = 18. 1

[CBSE Marking Scheme 2017]

OR

We have, radius (r) = 10.5 cm

and angle (θ) = 60°



Then, the length of arc APB

$= \dfrac{\theta}{360°} \times 2\pi r$

$= \dfrac{60°}{360°} \times 2 \times \dfrac{22}{7} \times 10.5$

$= 11$ cm 1

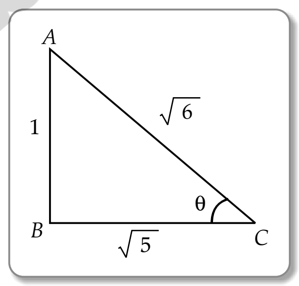
Now, the perimeter of the sector $OAPBO$

$= OA$ + length of an arc $APB + BO$

= (10.5 + 11 + 10.5) cm

= 32 cm. 1

25. Given $\tan\theta = \dfrac{AB}{BC} = \dfrac{1}{\sqrt{5}}$



In ΔABC, $AC^2 = AB^2 + BC^2 = 1 + 5 = 6$ 1

or, $AC = \sqrt{6}$

(i) $\dfrac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$,

From figure $= \dfrac{\left(\frac{\sqrt{6}}{1}\right)^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{6}}{1}\right)^2 + \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}$

$= \dfrac{6 - \frac{6}{5}}{6 + \frac{6}{5}}$

$= \dfrac{24}{36}$

$= \dfrac{2}{3}$ 1

(ii) L.H.S $= \sin^2\theta + \cos^2\theta$

$$= \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{\sqrt{5}}{\sqrt{6}}\right)^2$$

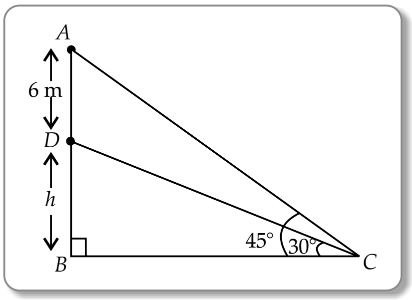
$$= \frac{1}{6} + \frac{5}{6} = \frac{6}{6} = 1 = \text{R.H.S} \qquad 1$$

Hence proved.

OR

According to question,

AD is a flagstaff and BD is a tower.



In ΔABC,

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{h+6}{BC}$$

$$\Rightarrow \quad BC = h + 6 \qquad \text{(i)}$$

In ΔDBC, $\quad \tan 30° = \frac{DB}{BC}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{h+6} \qquad \text{[from (i)]}$$

$$\Rightarrow \quad h\sqrt{3} = h + 6$$

$$\Rightarrow \quad h\sqrt{3} - h = 6$$

$$\Rightarrow \quad h(\sqrt{3} - 1) = 6$$

$$\Rightarrow \quad h = \frac{6}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$\Rightarrow \quad h = \frac{6(\sqrt{3}+1)}{2}$$

$$\Rightarrow \quad h = 3(\sqrt{3}+1)$$

$$= 3(1.73 + 1)$$

$$\Rightarrow \quad h = 3 \times 2.73$$

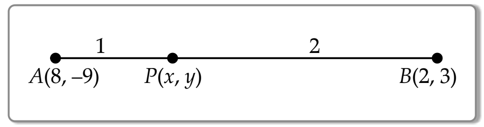
$$\Rightarrow \quad h = 8.19 \text{ m}$$

$\therefore$ The height of the tower is 8.19 m

Section - C

26. IV quadrant. **[CBSE SQP Marking Scheme, 2020]**

Detailed Solution:



½

$m = 1, n = 2$

Given, $\quad (x_1, y_1) = (8, -9)$

$\quad (x_2, y_2) = (2, 3)$

$$(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right] \qquad 1$$

$$(x, y) = \left[\frac{1\times 2 + 2\times 8}{1+2}, \frac{1\times 3 + 2\times(-9)}{1+2}\right]$$

$$(x, y) = \left[\frac{2+16}{3}, \frac{3-18}{3}\right]$$

$$(x, y) = \left[\frac{18}{3}, \frac{-15}{3}\right]$$

$$(x, y) = (6, -5)$$

Hence, the point $(6, -5)$ lies in IV quadrant. 1½

27. Since, α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\quad \alpha + \beta = -\frac{2}{3}$

and $\quad \alpha\beta = \frac{1}{3}$ ½

Now for the new polynomial,

$$\text{Sum of the zeroes} = \frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$$

$$= \frac{(1-\alpha+\beta-\alpha\beta)+(1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$$

$$\therefore \quad \text{Sum of zeroes} = \frac{4/3}{2/3} = 2 \qquad 1$$

$$\text{and product of zeroes} = \left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$$

$$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$$

$$= \frac{1-\alpha-\beta+\alpha\beta}{1+\alpha+\beta+\alpha\beta} \Rightarrow \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$$

$$\therefore \text{Product of zeroes} = \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} \Rightarrow \frac{\frac{6}{3}}{\frac{2}{3}} = 3 \qquad 1$$

Hence, required polynomial

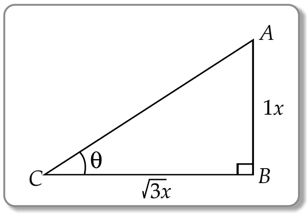
$= x^2 -$ (Sum of zeroes)x + Product of zeroes

$= x^2 - 2x + 3$ ½

28. Let AB be a vertical rod and BC be its shadow.

From the figure, $\angle ACB = \theta$.

In ΔABC,



$$\tan\theta = \frac{AB}{BC} \qquad \textbf{1}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}} \quad \left[\because \frac{AB}{BC} = \frac{1}{\sqrt{3}}\text{(given)}\right]$$

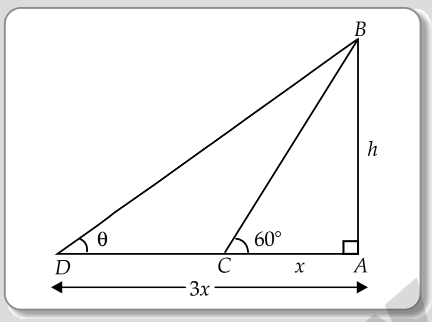
$$\Rightarrow \quad \tan\theta = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ \qquad \textbf{2}$$

Hence, the angle of elevation of the sun is 30°.

OR

28.



½

In ΔABC, $\quad \dfrac{AB}{AC} = \tan 60^\circ$

$$\frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad h = x\sqrt{3} \qquad ½$$

In ΔABD, $\quad \dfrac{AB}{AD} = \tan\theta$

$$\frac{h}{3x} = \tan\theta$$

$$\Rightarrow \quad \frac{x\sqrt{3}}{3x} = \frac{1}{\sqrt{3}} = \tan 30^\circ \qquad ½$$

$$\therefore \quad \theta = 30^\circ \qquad ½$$

[CBSE Marking Scheme, 2017]

29. $$\frac{\dfrac{\sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A}{\cos^2 A} - 1} + \frac{\dfrac{1}{\sin^2 A}}{\dfrac{1}{\cos^2 A} - \dfrac{1}{\sin^2 A}} \qquad ½$$

$$= \frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A} \qquad ½$$

$$= \frac{1}{\sin^2 A - \cos^2 A} \qquad [\because \sin^2\theta + \cos^2\theta = 1] \ \textbf{1}$$

$$= \frac{1}{1 - 2\cos^2 A} \qquad [\because \sin^2\theta = 1 - \cos^2\theta] \ \textbf{1}$$

[CBSE Marking Scheme 2019]

30. Given,

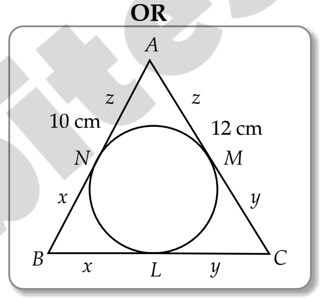
$$BD = CD = \frac{BC}{2}$$

or, $\quad BC = 2BD \qquad$ **1**

Using Pythagoras theorem in the right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= AB^2 + 4BD^2 \qquad \textbf{1}$$
$$= (AB^2 + BD^2) + 3BD^2$$
$$AC^2 = AD^2 + 3CD^2.$$
$$[\therefore BD=CD] \ \textbf{1}$$

OR



Let BL = BN = x (tangents from external points are equal) ½

$$CL = CM = y$$
$$AN = AM = z$$

$\therefore$ AB + BC + AC = $2x + 2y + 2z = 30$

$$x + y + z = 15 \qquad \text{...(i)} \ \textbf{1}$$

Also, $x + z = 10$, $x + y = 8$ and $y + z = 12$

Solving above eqn. (i),

$y = 5$, $z = 7$ and $x = 3$ ½ + ½ + ½

$\therefore$BL = 3 cm, CM = 5 cm and AN = 7 cm

31. Total numbers of outcomes = 6 **1**

(i) Prob. (getting a prime number (2, 3, 5) = $\dfrac{3}{6} = \dfrac{1}{2}$ **1**

(ii) Prob. (getting a number between 2 and 6 (3, 4, 5)

$$= \frac{3}{6} = \frac{1}{2} \qquad \textbf{1}$$

[CBSE Marking Scheme 2019]

Commonly Made Error

- Sometimes 1 is taken as prime number.

Answering Tip

- Students should be careful while choosing prime number and remember that 1 is not a prime number.

Section - D

32. Let the speed of stream be x km/h

$\therefore$ Speed of boat upstream $= (15 - x)$ km / h

and Speed of boat down stream

$= (15 + x)$ km/h **1**

$$\therefore \quad \frac{30}{15-x}+\frac{30}{15+x} = 4\frac{1}{2} = \frac{9}{2}$$ **1**

$$\Rightarrow \quad \frac{30(15+x+15-x)}{(15-x)(15+x)} = \frac{9}{2}$$

$\Rightarrow \quad 200 = 225 - x^2$ **1**

$\Rightarrow \quad x = 5$ (Rejecting -5) **1**

$\therefore \quad$ Speed of stream $= 5$ km/h **1**

[CBSE Marking Scheme 2019]

Commonly Made Error

- Speed of boat upstream and downstream are taken incorrectly.

Answering Tip

- Always remember that speed of boat upstream is obtained by subtracting the speed of stream from speed of boat in still water *i.e.,* $15 - x$ whereas speed of boat downstream is the sum of speed of boat in still water and speed of stream i.e., $15 + x$.

OR

Let one tap fills the tank in x h

Then, the other taps will fill the tank in $(x + 3)$ h

Given:

Time taken by both taps, running together, to fill the tank $= 3\frac{1}{13}\text{h} = \frac{40}{13}\text{h}$

Part of the tank filled by one tap in 1h $= \frac{1}{x}$

Part of the tank filled by other tap in 1 h $= \frac{1}{x+3}$

So, part of the tank filled by both taps, running together, in 1 h $= \frac{1}{x}+\frac{1}{x+3}$

$$\therefore \quad \frac{1}{x}+\frac{1}{x+3}=\frac{13}{40}$$ **1**

$$\Rightarrow \quad \frac{2x+3}{x^2+3x}=\frac{13}{40}$$

$\Rightarrow 13x^2 + 39x = 80x + 120$ **1**

$\Rightarrow 13x^2 - 41x - 120 = 0$

$\Rightarrow 13x^2 - 65x + 24x - 120 = 0$

$\Rightarrow 13x(x - 5) + 24(x - 5) = 0$

$\Rightarrow (x - 5)(13x + 24) = 0$

$x - 5 = 0$ or $13x + 24 = 0$

$\Rightarrow x = 5$ or $x = -\frac{24}{13}$ **1**

Since, time cannot be negative, so $x = 5$.

$\therefore$ Time taken by one tap to fill the tank = 5 h

Time taken by the other tap to fill the tank = 5 + 3 = 8 h. **1 + 1**

33. Given, $\Delta ABC \sim \Delta DEF$

Then according to question,

$$\frac{AB}{BC} = \frac{DE}{EF}$$ [From BPT] ½

$$\Rightarrow \quad \frac{2x-1}{2x+2} = \frac{18}{3x+9}$$ **1**

$\Rightarrow \quad (2x - 1)(3x + 9) = 18(2x + 2)$

$\Rightarrow \quad (2x - 1)(x + 3) = 6(2x + 2)$

$\Rightarrow \quad 2x^2 - x + 6x - 3 = 12x + 12$

$\Rightarrow \quad 2x^2 + 5x - 12x - 15 = 0$

$\Rightarrow \quad 2x^2 - 7x - 15 = 0$ **1**

$\Rightarrow \quad 2x^2 - 10x + 3x - 15 = 0$

$\Rightarrow \quad 2x(x - 5) + 3(x - 5) = 0$

$\Rightarrow \quad (x - 5)(2x + 3) = 0$

Either $x = 5$ or $x = \frac{-3}{2}$, which is not possible

So, $x = 5$ ½

Then in ΔABC, we have

$AB = 2x - 1 = 2 \times 5 - 1 = 9$ cm

$BC = 2x + 2 = 2 \times 5 + 2 = 12$ cm

$AC = 3x = 3 \times 5 = 15$ cm **1**

and in ΔDEF, we have

$DE = 18$ cm

$EF = 3x + 9 = 3 \times 5 + 9 = 24$ cm

$DE = 6x = 6 \times 5 = 30$ cm. **1**

34. Volume of rain water on the roof = volume of cylindrical tank **1**

i.e., $\quad 22 \times 20 \times h = \frac{22}{7}\times 1\times 3.5$ **1**

$\Rightarrow \quad h = \frac{1}{40}$ m **1**

$\Rightarrow \quad h = 2.5$ cm **1**

Water conservation must be encouraged. **1**

[CBSE Marking Scheme 2019]

Detailed Answer

Let length of roof, $l = 22$ m

breadth of roof, $b = 22$ m

and height of roof $= h$ m

Also, given, height of cylinder = 3.5 m **1**

and radius of cylinder $= \frac{\text{diameter}}{2} = \frac{2}{2} = 1$m **1**

Now, Volume of rain water on the roof = Volume of cylindrical tank

$\Rightarrow \quad l \times b \times h = \pi r^2 h$

$\Rightarrow \quad 22 \times 20 \times h = \frac{22}{7} \times (1)^2 \times 3.5$ **1**

$\Rightarrow \quad h = \frac{22 \times 3.5}{22 \times 20 \times 7}$

$\Rightarrow \quad h = \frac{1}{40}$ m $= 2.5$ cm **1**

Views on water conservation:

Water conservation reduces energy use and can even save household money. Water conservation must be encouraged. **1**

OR

Given Length = 8 m 50 cm= 850 cm

breadth = 6 m 25 cm = 625 cm

height = 4 m 75 cm= 475 cm

Since, the length of the longest rod is equal to *HCF* of 850, 625 and 475. **1**

Prime factor of 850 = $2 \times 5^2 \times 17$

Prime factor of 625 = 5^4

Prime factor of 475 = $5^2 \times 19$ **1**

Hence, *HCF* (625, 850, 475) = $5^2 = 25$ **1**

Thus, the longest rod that can measure the dimensions of the room exactly = 25 cm. **1**

[CBSE Marking Scheme 2016]

Commonly Made Errors

- Mostly candidates are unable to determine about what they have to find. Actually, mostly candidates don't get to know that the question is about HCF or LCM.
- Sometimes students calculate the longest length of rod that lies in the room by finding its diagonal.

Answering Tips

- Adequate practice is necessary for such type of questions and basic concept of HCF and LCM should be clear.
- Students should read the question properly.

35.

Daily Expenditure	x_i	Number of household (f_i)	$u_i = \frac{x_i - A}{50}$	$f_i u_i$
100 – 150	125	4	– 2	– 8
150 – 200	175	5	– 1	– 5
200– 250	225 = *A*	12	0	0
250 – 300	275	2	1	2
300 – 350	325	2	2	4
		$\Sigma f_i = 25$		$\Sigma f_i u_i = -7$

1

$$\text{Mean} = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h \qquad [\text{here}, A = 225, \Sigma f_i = 25, \Sigma f_i u_i = -7]$$ **2**

$$\text{Mean} = 225 + \left(\frac{-7}{25}\right) \times 50$$

$$= 221$$

Mean expenditure on food is ₹ 211. **2**

Section - E

36. (i) Area of square = $(\text{side})^2$

$= 15 \times 15$

$= 225 \text{ m}^2$ **1**

(ii) From the figure, it can be observe that the horse can graze a sector of 90° in a circle of 5 m radius.

Area that can be grazed by horse

= Area of sector

$= \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 5 \times 5$

$= 19.625 \text{ m}^2$ **1**

(iii) Area that can be grazed by the horse when length of rope is 10 m long

$= \frac{90°}{360°} \times \pi r^2$

$= \frac{1}{4} \times 3.14 \times 10 \times 10$

$= 78.5 \text{ m}^2$ **2**

OR

Increase in grazing area

$= (78.5 - 19.625)\ m^2$

$= 58.875\ m^2$ **1**

37. **(i)** Since,

Distance between two points (x_1, y_1) and (x_2, y_2).

$$d = \left|\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\right|$$

Now, distance between house and bank,

$$= \left|\sqrt{(5-2)^2 + (8-4)^2}\right|$$
$$= \left|\sqrt{(3)^2 + (4)^2}\right|$$
$$= \left|\sqrt{9+16}\right|$$
$$= \left|\sqrt{25}\right|$$
$$= 5 \text{ km}$$ 1

(ii) Distance between bank and daughter's school,

$$= \left|\sqrt{(13-5)^2 + (14-8)^2}\right|$$
$$= \left|\sqrt{(8)^2 + (6)^2}\right|$$
$$= \left|\sqrt{64+36}\right|$$
$$= \left|\sqrt{100}\right|$$
$$= 10 \text{ km}$$ **1**

OR

Distance between house to office,

$$= \left|\sqrt{(13-2)^2 + (26-4)^2}\right|$$
$$= \left|\sqrt{(11)^2 + (22)^2}\right|$$
$$= \left|\sqrt{121+484}\right|$$
$$= \left|\sqrt{605}\right|$$
$$= 24.59$$
$$= 24.6 \text{ km}$$ **1**

(iii) Distance between daughter's school and office,

$$= \left|\sqrt{(13-13)^2 + (26-14)^2}\right|$$
$$= \left|\sqrt{0 + (12)^2}\right|$$
$$= 12 \text{ km}$$ ½

Total distance (House + Bank + School + Office) travelled = 5 + 10 + 12 = 27 km ½

38. **(i)** We have, $\angle B = 30°$ and $AC = 4$ m

Then, $\sin 30° = \dfrac{AC}{AB}$

$\Rightarrow \dfrac{1}{2} = \dfrac{4}{AB}$

$\Rightarrow AB = 8$ m. **1**

(ii) $\sin^2 30° + \cos^2 60° = \left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2$

$= \dfrac{1}{4} + \dfrac{1}{4}$

$= \dfrac{2}{4} = \dfrac{1}{2}$. **2**

OR

Since, $\cos A = \dfrac{1}{2}$

$\Rightarrow \cos A = \cos 60°$

$\Rightarrow A = 60°$

Then $12 \cot^2 A - 2 = 12(\cot 60°) - 2$

$= 12\left(\dfrac{1}{\sqrt{3}}\right)^2 - 2$

$= 12 \times \dfrac{1}{3} - 2$

$= 4 - 2 = 2.$ **2**

(iii) Since, $AC \perp BC$,

then $\angle C = 90°$

$\sin C \times \cos A = \sin 90° \times \dfrac{AC}{AB}$

$= 1 \times \dfrac{4}{8}$

$= \dfrac{1}{2}$ **1**

■■■

SOLUTIONS

Self Assessment Paper-4

Mathematics Standard (041)

Section - A

1. Option (C) is correct.

Explanation: $p = 2^4 \times 3^3 \times 5^4 \times 9$
$= 16 \times 27 \times 625 \times 9$
$= 24{,}30{,}000$

Hence, number of zeroes in p are 4.

2. Option (C) is correct.

***Explanation*:** Let us assume, to the contrary, that $5-\sqrt{3}$ is rational,

i.e., $$5-\sqrt{3} = \frac{a}{b}(b \neq 0)$$

Therefore, $$5-\frac{a}{b} = \sqrt{3}$$

$$\Rightarrow \quad \frac{5b-a}{b} = \sqrt{3}$$

Since, a and b are integers, we get $\frac{5b-a}{b}$ is rational and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational.

Hence, $5-\sqrt{3}$ is irrational. **1**

3. Option (A) is correct.

Explanation: $f(x) = x^2 - 8x + k$

$$\text{Product of zeroes} = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

$$\Rightarrow \quad 20 = \frac{k}{1}$$

$$\Rightarrow \quad k = 20.$$

4. Option (D) is correct.

Explanation: We know that equation of the form $y = a$ is a line parallel to x-axis at a distance 'a' from it. $y = 0$ is the equation of the x-axis and $y = -7$ is the equation of the line parallel to the x-axis. So, these two equations represent two parallel lines. Therefore, there is no solution. **1**

5. Option (A) is correct.

***Explanation*:** Let the coordinates of points P and Q be $(0, b)$ and $(a, 0)$ respectively.

$$\therefore \quad \frac{a}{2} = 2 \Rightarrow a = 4$$

and $$\frac{b}{2} = -5 \Rightarrow b = -10$$

$\therefore$ P(0, – 10) and Q(4, 0) **1**

6. Option (D) is correct.

Explanation: $\because$ Points A and B lie on the circle and O is centre of the circle.

$\therefore \quad OA = OB$ (radii of the circle)

$$\Rightarrow \quad \sqrt{(4-2)^2+(3-3)^2} = \sqrt{(x-2)^2+(5-3)^2}$$

$$\Rightarrow \quad 2 = \sqrt{(x-2)^2+4}$$

Taking square on both sides,

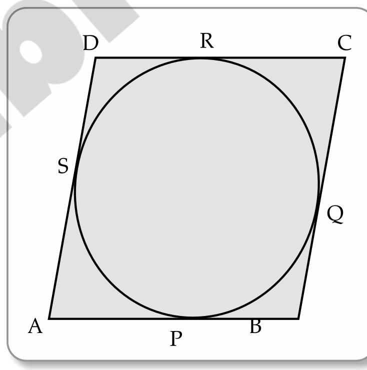
$\Rightarrow \quad 4 = (x-2)^2 + 4$

$\Rightarrow \quad (x-2)^2 = 0$

$\Rightarrow \quad x = 2$

7. Option (C) is correct.

***Explanation*:**



Here, $AP = AS$
$BP = BQ$
$CR = CQ$
$DR = DS$

(tangents drawn from external point of a circle)

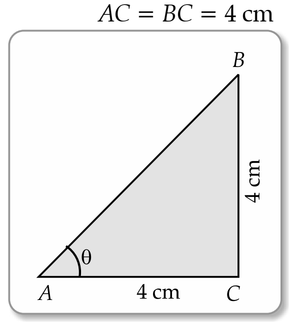
Adding (AP + PB) + (CR + RD)
$= (AS + SD) + (BP + QC)$

$\Rightarrow \quad AB + CD = AD + BC$ **1**

8. Option (A) is correct.

Explanation: ΔABC: is an isosceles triangle

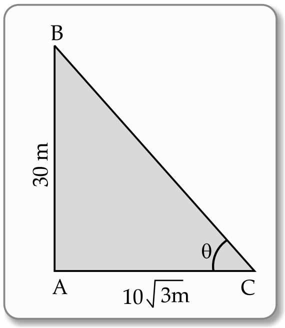
$\Rightarrow \quad AC = BC = 4$ cm ½



$\therefore \quad AB = \sqrt{4^2+4^2} = 4\sqrt{2}$ cm ½

9. **Option (A) is correct.**

Explanation:



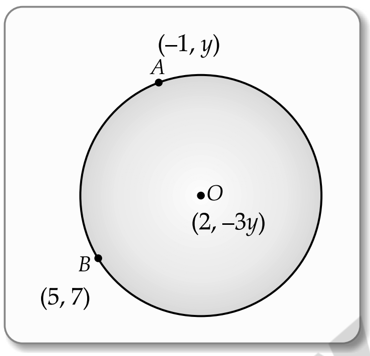
$$\tan\theta = \frac{30}{10\sqrt{3}} = \sqrt{3}$$

$$\Rightarrow \qquad \theta = 60^\circ \qquad \textbf{1}$$

10. **Option (B) is correct.**

Explanation: As points A and B lie on the circle and O is the centre.

AO and BO will be the radii of the circle.



So, $\qquad AO = BO$ radii of the circle

$$\Rightarrow \left|\sqrt{(2-(-1))^2+(-3y-y)^2}\right| = \left|\sqrt{(2-5)^2+(-3y-7)^2}\right|$$

(Applying distance formula on both AO and BO)

$$d = \left|\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\right|$$

$\Rightarrow \quad (3)^2 + (-4y)^2 = (-3)^2 + (-3y-7)^2$

$\Rightarrow \quad 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$

$\Rightarrow \quad 16y^2 - 9y^2 - 42y - 49 = 0$

$\Rightarrow \quad 7y^2 - 42y - 49 = 0$

$\Rightarrow \quad 7(y^2 - 6y - 49) = 0$

$\Rightarrow \quad y^2 - 7y + 1y - 49 = 0$

$\Rightarrow \quad y(y-7) + 1(y-7) = 0$

$\Rightarrow \quad (y-7)(y+1) = 0$

$\Rightarrow \quad y = 7, -1$

11. **Option (A) is correct.**

Explanation:

Area of circle $= \pi r^2$

$\therefore$ Area of the shaded region $= \pi(2)^2 - \pi(1)^2$

$4\pi - \pi = 3\pi$ sq cm

[CBSE Marking Scheme 2012] 1

12. **Option (C) is correct.**

Explanation: No. of balls in the bag = 3 + 5 = 8

P(that the drawn ball is not red) $= \dfrac{5}{8}$ **1**

13. **Option (A) is correct.**

Explanation: Area of first circular park whose diameter is 16 m,

$= \pi\left(\dfrac{16}{2}\right)^2$

$= \pi(8)^2$

$= 64\pi \text{ m}^2$

Area of second circular park whose diameter is 12 m,

$= \pi\left(\dfrac{12}{2}\right)^2$

$= \pi(6)^2$

$= 36\pi \text{ m}^2$

According to question,

Area of single circular park

= Area of first circular park

+ Area of second circular park

$\pi r^2 = 64\pi + 36\pi$

$\pi r^2 = 100\pi$

$r = 10$ m **1**

14. **Option (D) is correct.**

Explanation: All possible events are written below:

(1 1)	(1 2)	(1 3)	(1 4)	(1 5)	(1 6)
(2 1)	(2 2)	(2 3)	(2 4)	(2 5)	(2 6)
(3 1)	(3 2)	(3 3)	(3 4)	(3 5)	(3 6)
(4 1)	(4 2)	(4 3)	(4 4)	(4 5)	(4 6)
(5 1)	(5 2)	(5 3)	(5 4)	(5 5)	(5 6)
(6 1)	(6 2)	(6 3)	(6 4)	(6 5)	(6 6)

Total events = 36

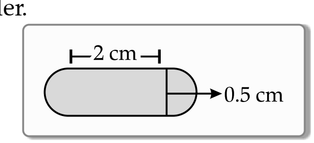
Out of the events in which 5 will not come up either time are (1, 1) (1, 2) (1, 3) (1, 4) (1, 6) (2, 1) (2, 2) (2, 3) (2, 4) (2, 6) (3, 1) (3, 2) (3, 3) (3, 4) (3, 6) (4, 1) (4, 2) (4, 3) (4, 4) (4, 6) (6, 1) (6, 2) (6, 3) (6, 4) (6, 6).

No. of required events in = 25

Required probability $= \dfrac{25}{36}$ **1**

15. **Option (A) is correct.**

Explanation: Capsule consists of 2 hemispheres and a cylinder.



$$r = \frac{0.5}{2} \text{ cm} = 0.25 \text{ cm}$$

$\Rightarrow \qquad r = 0.25$ cm

Total length of capsule $= r + h + r$

$\Rightarrow \qquad 2 = 2r + h$

$$\Rightarrow \qquad 2 = 2 \times 0.25 + h$$

$$\Rightarrow \qquad h = 2 - 0.5 = 1.5 \text{ cm}$$

Volume of capsule = Volume of two hemispheres + Volume of cylinder

$$= 2\times\left(\frac{4}{3}\pi r^3 \times \frac{1}{2}\right) + \pi r^2 h$$

$$= \frac{4}{3}\pi r^3 + \pi r^2 h$$

$$= \pi r^2\left(\frac{4}{3}r + h\right)$$

$$= \frac{22}{7}\times 0.25\times 0.25\left(\frac{4}{3}\times 0.25 + \frac{15}{10}\right)$$

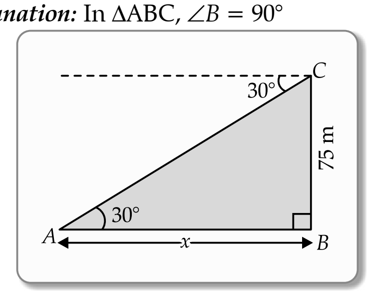
$$= \frac{22}{7}\times 0.25\times 0.25\left(\frac{1}{3} + \frac{3}{2}\right)$$

$$= \frac{22}{7}\times\frac{25}{100}\times\frac{25}{100}\times\frac{11}{6} = \frac{121}{336} = 0.3601$$

$\therefore$ Volume of capsule = 0.3601 cm^3 = 0.36 cm^3. **1**

16. **Option (C) is correct.**

Explanation: In ΔABC, $\angle B = 90°$



$$\tan\theta = \frac{CB}{AB}$$

$$\tan 30° = \frac{75}{x}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{x}$$

$$x = 75\sqrt{3} \text{ m}$$

17. **Option (B) is correct.**

Explanation: Let the number of ₹ 1 coins = x and the number of ₹ 2 coins = y

so, according to the question,

$$x + y = 50 \qquad ...(i)$$

$$x + 2y = 75 \qquad ...(ii)$$

Subtracting equation (i) from (ii),

$$y = 25$$

Substituting value of y in (i),

$$x = 25$$

So, y = 25 and x = 25 **1**

18. **Option (C) is correct.**

Explanation: The event of getting a total of atleast 10 i.e., 10, 11, 12.

$\therefore$ The elementary events are (6, 4), (4, 6), (5, 5), (6, 5), (5, 6) and (6, 6)

Favourable outcomes = 6

Total outcomes = 6 × 6 = 36

$$\therefore \text{ Required probability } = \frac{6}{36} = \frac{1}{6}$$

19. **Option (B) is correct.**

Explanation: In case of assertion:

In a right circular cone, if any cut is made parallel to its base, we get a circle.

$\therefore$ Assertion is correct.

In case of reason:

Let radius of sphere be r.

Given, volume of hemisphere = Surface area of hemisphere

$$\text{or,} \qquad \frac{2}{3}\pi r^3 = 3\pi r^2$$

$$\text{or,} \qquad r = \frac{9}{2} \text{ units}$$

$$\therefore \qquad \text{Diameter} = \frac{9}{2}\times 2 = 9 \text{ units}$$

$\therefore$ Reason is correct:

Hence, both assertion and reason are correct but reason is not the correct explanation for assertion.

20. **Option (A) is correct.**

Explanation: Let a, a^2 and a^3 be three numbers, then we have the smallest power of a^1, a^2 and a^3 is 1. So, HCF is a.

Now, let us consider the reason:

Prime factors of 12 = $2^2 \times 3$

Prime factors of 21 = 3×7

Prime factors of 15 = 3×5

$\therefore$ HCF of 12, 21 and 15 = 3, which is a common prime factor.

Thus both assertion and reason are correct and reason is the correct explanation for assertion.

Section - B

21. Given,

$$S_n = n^2$$

$$S_1 = a_1 = 1$$

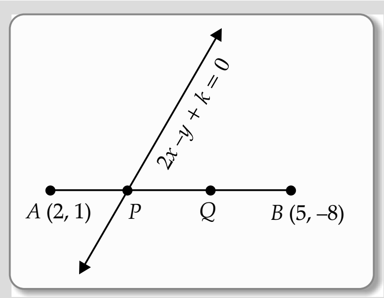
$$S_2 = a_1 + a_2 = 4 \qquad \textbf{1}$$

$$\Rightarrow \qquad a_2 = 3$$

$$\therefore \qquad d = a_2 - a_1 = 2$$

$$\text{Therefore,} \qquad a_{10} = 1 + 18 = 19 \qquad \textbf{1}$$

22.



½

$$AP : PB = 1 : 2$$

$$x = \frac{4+5}{3} = 3 \text{ and } y = \frac{2-8}{3} = -2$$

This, point p is (3, – 2) ½

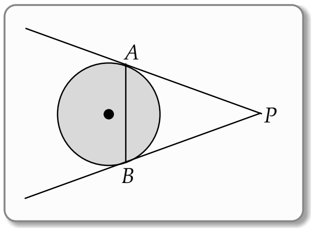
Point (3, – 2) lies on $2x - y + k = 0$ ½

$\Rightarrow \quad 6 + 2 + k = 0$ ½

$\Rightarrow \quad k = -8$

[CBSE Marking Scheme 2019]

23.



½

Case I: Since, PA = PB

Therefore, In ΔPAB, $\angle PAB = \angle PBA$ **1**

Case II: If the tangents at A and B are parallel then each angle between chord and tangent = 90º ½

24. Volume of sphere = Volume of cone

Let the radius of cone be R *cm*.

$\therefore \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 \times r$

or, $\quad 4r^3 = R^2 r$

or, $\quad R^2 = 4r^2$

or, $\quad R = 2r$ **1**

[CBSE Marking Scheme, 2012]

OR

$$\frac{\text{Voume of 1}^{\text{st}}\text{ cylinder}}{\text{Voume of 2}^{\text{nd}}\text{ cylinder}} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2}$$

$$= \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$$

$$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3}$$

$$= \frac{4}{9} \times \frac{5}{3} = \frac{20}{27}$$

$= 20 : 27$ **1**

[CBSE Marking Scheme, 2012]

25. $\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2 \tan 45° \cos^2 0° \sin 90°$

$$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times \left(\sqrt{3}\right)^2 - 2 \times 1 \times 1^2 \times 1$$

$$= \frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 1 \times 3 - 2 \times 1 \times 1 \times 1.$$

$$= \frac{1}{6} + \frac{3}{2} - 2$$

$$= \frac{1 + 9 - 12}{6}$$

$$= -\frac{2}{6} = -\frac{1}{3}$$

[CBSE Marking Scheme, 2015] 2

OR

Given,

$m^2 = a^2\cos^2 \theta + 2ab \sin \theta \cos \theta + b^2\sin^2 \theta$ (i)

and

$n^2 = a^2\sin^2 \theta - 2ab \sin \theta \cos \theta + b^2\cos^2 \theta$ (ii) **1**

Adding equations (i) and (ii),

$m^2 + n^2 = a^2(\cos^2 \theta + \sin^2 \theta) + b^2(\cos^2 \theta + \sin^2 \theta)$

$= a^2 (1) + b^2 (1)$ **1**

$= a^2 + b^2$ = RHS. **Hence proved.**

Section - C

26. Let $\sqrt{7}$ be a rational number.

$\therefore \quad \sqrt{7} = \frac{p}{q},$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get,

or $\quad 7 = \frac{p^2}{q^2}$ **1**

or $\quad p^2 = 7q^2$

$\because p^2$ is divisible by 7

$\therefore p$ is divisible by 7. (i) ½

Let $p = 7r$ for some positive integer r

or $\quad p^2 = 49r^2$

$\therefore \quad 7q^2 = 49r^2$

or $\quad q^2 = 7r^2$

or q^2 is divisible by 7 ½

$\therefore q$ is divisible by 7. (ii)

From (i) and (ii), p and q are divisible by 7, which contradicts the fact that p and q are co-primes.

Hence, our assumption is wrong.

$\therefore \sqrt{7}$ is an irrational number. **1**

27. Given, $\quad (1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$

For equal roots, $\quad b^2 - 4ac = 0$ ½

$\Rightarrow \quad 4m^2c^2 - 4(1 + m^2)(c^2 - a^2) = 0$ ½

$\Rightarrow \quad m^2c^2 - c^2 - m^2c^2 + a^2 + m^2a^2 = 0$ **1**

$\Rightarrow \quad c^2 = a^2(1 + m^2)$ **1**

[CBSE Marking Scheme, 2017]

28. $S_n = 180 = \frac{n}{2}[90 + (n-1)(-6)]$ **1**

$\Rightarrow \quad 360 = 90n - 6n^2 + 6n$

$\Rightarrow \quad 6n^2 - 96n + 360 = 0$

$\Rightarrow \quad 6[(n - 6)(n - 10)] = 0$

$\Rightarrow \quad n = 6$ or $n = 10$ **2**

[CBSE Marking Scheme, 2019]

OR

$a_m = \frac{1}{n} \Rightarrow a + (m - 1)d = \frac{1}{n}$ (i) ½

and $a_n = \frac{1}{m} \Rightarrow a + (n - 1)d = \frac{1}{m}$ (ii) ½

Solving (i) & (ii), we get

$$a = \frac{1}{mn}, d = \frac{1}{mn} \qquad \mathbf{1}$$

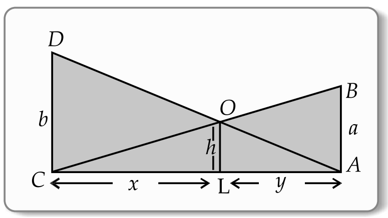
Now, $a_{mn} = a + (mn - 1)d$

$$\Rightarrow \quad a_{mn} = \frac{1}{mn} + (mn-1) \times \frac{1}{mn}$$

$$\Rightarrow \quad a_{mn} = 1 \qquad \mathbf{1}$$

[CBSE Marking Scheme, 2019]

29. **(i)** Let AB and CD be the two trees of height a and b metre such that the trees are p metre apart *i.e.,* $AC = p$. Let the lines AD and BC meet at O such that $OL = h$ m.



½

Let CL be x and LA be y,

then $x + y = p$

In ΔABC and ΔLOC,

$\angle CAB = \angle CLO$ (each 90°)

$\angle C = \angle C$ (Common)

$\therefore \Delta CAB \sim \Delta CLO$ (AA similarity)

or, $\dfrac{CA}{CL} = \dfrac{AB}{LO}$

or, $\dfrac{p}{x} = \dfrac{a}{h}$

or, $x = \dfrac{ph}{a}$ ...(i) ½

In ΔALO and ΔACD,

$\angle ALO = \angle ACD$ (each 90°)

$\angle A = \angle A$ (Common)

$\therefore \angle ALO \sim \angle ACD$ (AA similarity)

or, $\dfrac{AL}{AC} = \dfrac{OL}{DC}$

or, $\dfrac{y}{p} = \dfrac{h}{b}$

or, $y = \dfrac{ph}{b}$ (ii) ½

Adding eqns. (i) and (ii),

$$x + y = \frac{ph}{a} + \frac{ph}{b}$$

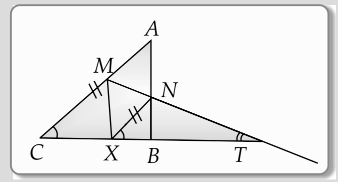
or, $p = ph\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

or, $\dfrac{1}{h} = \dfrac{1}{a} + \dfrac{1}{b} = \dfrac{a+b}{ab}$

$\therefore h = \dfrac{ab}{a+b}$ m. ½

(ii) Similar triangles. **1**

30. $\Delta TXN \sim \Delta TCM$



$$\Rightarrow \quad \frac{TX}{TC} = \frac{XN}{CM} = \frac{TN}{TM}$$

$\Rightarrow \quad TX \times TM = TC \times TN$ **(i) 1**

Again, $\Delta TBN \sim \Delta TXM$

$$\Rightarrow \quad \frac{TB}{TX} = \frac{BN}{XM} = \frac{TN}{TM}$$

$\Rightarrow \quad TM = \dfrac{TN \times TX}{TB}$ **(ii) 1**

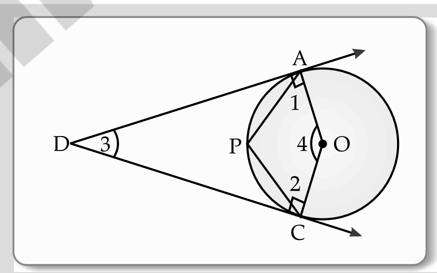
Using (ii) in (i), we get

$$\Rightarrow \quad TX^2 = \frac{TN}{TB} = TC \times TN$$

$\Rightarrow \quad TX^2 = TC \times TB$ **Hence Proved. 1**

[CBSE Marking Scheme, 2018]

OR



½

Given DA and DC are tangents from point D to a circle with centre O.

$$\angle 1 = \angle 2 = 90°$$

(radius $\perp$ tangent)

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$ **1**

or, $90° + 90° + 50° + \angle 4 = 360°$

or, $\angle 4 = 130°$

$\therefore$ Reflex $\angle 4 = 360° - 130° = 230°$ **1**

$$\angle APC = \frac{1}{2} \text{ reflex } \angle 4$$

(angle subtended at centre)

$\angle APC = \dfrac{1}{2} \times 230° = 115°$ ½

[CBSE Marking Scheme 2015]

Section - D

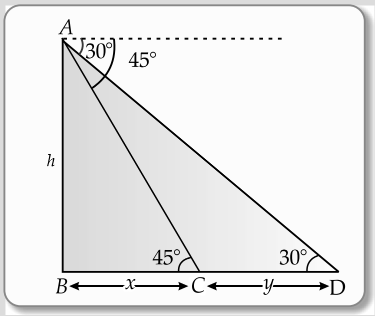
31. E_1: {(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)}

$\therefore$ P(5 will come at least once) = P $(E_1) = \dfrac{11}{36}$ 1

P (5 will not come either) $= 1 - \dfrac{11}{36} = \dfrac{25}{36}$ 2

[CBSE Marking Scheme, 2019]

32.



1

$$\frac{h}{x} = \tan 45^\circ = 1$$

$$h = x \qquad \text{(i) } \mathbf{1}$$

$$\frac{h}{x+y} = \tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h = x + y \qquad \text{(ii) } \mathbf{1}$$

Therefore from (i) & (ii),

$$\sqrt{3}x = x + y$$

$$\Rightarrow \quad y = x(\sqrt{3} - 1) \qquad \mathbf{1}$$

To cover a distance of $x(\sqrt{3}-1)$, car takes 12 min

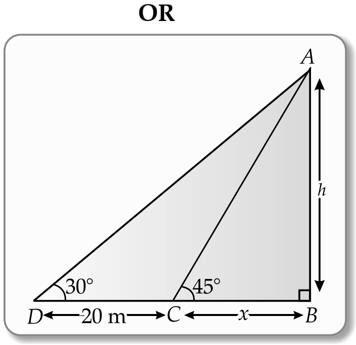
$\therefore$ Time taken by car to cover a distance of x units = $\dfrac{12}{\sqrt{3}-1}$ minutes

$$= 6(\sqrt{3}+1) \text{ min}$$

or 16.4 min (approx). **1**

[CBSE Marking Scheme 2019]

OR



1

$$\frac{h}{x} = \tan 45^\circ = 1$$

$$\Rightarrow \quad h = x \qquad \text{...(i) } \mathbf{1}$$

$$\frac{h}{x+20} = \tan 30^\circ$$

$$\Rightarrow \quad h\sqrt{3} = x + 20$$

$$\Rightarrow \quad h\sqrt{3} = h + 20 \qquad \text{[from (i)] } \mathbf{1}$$

$$\Rightarrow \quad h = \frac{20}{\sqrt{3}-1}$$

$$\Rightarrow \quad h = 10(\sqrt{3}+1) \qquad \mathbf{2}$$

$\therefore$ Height of the tower $= 10(\sqrt{3}+1)$m

33. $PA = PB = 4$ cm

[Tangents from external point] **1**

$\angle PAB = 180^\circ - 135^\circ = 45^\circ$

[Supplementary angles]

$\angle ABP = \angle PAB = 45^\circ$

[Opposite angles of equal sides] **1**

$\therefore \quad \angle APB = 180^\circ - 45^\circ - 45^\circ$

$= 90^\circ$ **1**

So, ΔABP is an isosceles right angled triangle.

$\Rightarrow \quad AB^2 = 2AP^2$ **1**

by pythagoras theorem

$\Rightarrow \quad AB^2 = 32$ **1**

Hence, $\quad AB = \sqrt{32} = 4\sqrt{2}$ cm

[CBSE Marking Scheme, 2017]

34. Volume of water in cylindrical tank

= Volume of water in park. **1**

$\Rightarrow \dfrac{22}{7} \times 1 \times 1 \times 5 = 20 \times 25 \times h$, where h is the height of standing water. **2**

$\Rightarrow \quad h = \dfrac{11}{350}$ m or $\dfrac{22}{7}$ cm **1**

Conservation of water **1**

[CBSE Marking Scheme 2017]

Detailed Answer

Let radius of cylindrical tank,

$$r = \frac{\text{diameter}}{2} = \frac{2}{2} = 1\text{m}$$

Height of cylindrical tank, H = 5 m

Length of the park of hospital, l = 25 m

breadth of the park of hospital, b = 20 m

and height of standing water in the park = h

Volume of water in cylindrical tank = Volume of water in park **1**

$$\Rightarrow \quad \pi r^2 H = l \times b \times h \qquad \mathbf{1}$$

$$\Rightarrow \frac{22}{7} \times (1)^2 \times (5) = 25 \times 20 \times h$$

$$\Rightarrow \quad h = \frac{22\times5}{7\times25\times20} \quad \mathbf{1}$$

$$\Rightarrow \quad h = \frac{11}{350}\text{m or } \frac{22}{7} \text{ m} \quad \mathbf{1}$$

Advantages of recycling water is that it reduces the need for water from natural habitats such as wetlands. **1**

OR

Area of minor Segment **½**

$$= \frac{22}{7}\times10\times10\times\frac{60°}{360°}-\frac{\sqrt{3}}{4}\times10\times10 \quad \mathbf{½}$$

$$= 10\times10\left[\frac{22}{7}\times\frac{1}{6}-\frac{\sqrt{3}}{4}\right]$$

$$= \frac{100}{84}(44-21\sqrt{3})\text{ cm}^2$$

$$= \frac{25}{21}(44-21\sqrt{3})\text{cm}^2 \quad \mathbf{2}$$

Area of major segment

$$= \left[\frac{22}{7}\times10\times10-\frac{25}{21}(44-21\sqrt{3})\right]\text{cm}^2 \quad \mathbf{1}$$

$$= \frac{100}{84}(220+21\sqrt{3})\text{cm}^2$$

$$= \frac{25}{21}(220+21\sqrt{3})\text{cm}^2 \quad \mathbf{1}$$

Commonly Made Error

- Incorrect/unable to use the formula for area of segment (minor) and equilateral triangle may lead to time consuming simplification.

Answering Tip

- The student should understand that section in major and use the area formula accordingly.

35.

Heights (in cm)	No. of students	Cumulative frequency
150-155	15	15
155-160	13	15 + 13 = 28
160-165	10	28 + 10 = 38
165-170	8	38 + 8 = 46
170-175	9	46 + 9 = 55
175-180	5	55 + 5 = 60

2

Since total frequency is 60.

$$\frac{N}{2} = 30 \quad \mathbf{1}$$

And cumulative frequency greater than or equal to 30 lies in class 160-165.

So, median class is 160-165.

∴ Upper limit of median class is 165. **2**

Section - E

36. (i) Parabola. **1**

(ii) General solution for $ax^2 + bx + c = 0$ is **1**

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2}$$

(iii) The number of zeroes of polynomial is the number of times the curve intersects the x-axis, i.e. attains the value 0. **2**

Here, the polynomial meets the X-axis at 3 points.

So, number of zeroes there.

OR

Since, the three zeroes = – 3, – 1, 2

Hence, the expression is $(x + 3)(x + 1)(x – 2)$

$= [x^2 + x + 3x + 3](x – 2)$

$= x^3 + 4x^2 + 3x – 2x^2 – 8x – 6$

$= x^3 + 2x^2 – 5x – 6$

37. (i) Given, area of triangle = 60 cm^2

$\Rightarrow \quad \frac{1}{2}\times AB\times BC = 60$

$\Rightarrow \quad (5x)(3x – 1) = 120$

$\Rightarrow \quad 3x^2 – x – 24 = 0$

$\Rightarrow \quad 3x^2 – 9x + 8x – 24 = 0$

$\Rightarrow \quad 3x(x – 3) + 8(x – 3) = 0$

$\Rightarrow \quad (x – 3)(3x + 8) = 0$

Either $x = 3$ or $x = -\frac{8}{3}$

Since length can't be negative, then $x = 3$. **1**

(ii) The length of $AB = 5x$ cm

$= 5 \times 3$ cm

$= 15$ cm **1**

OR

$\because \quad AB = 15$ cm and

$BC = (3x – 1)$ cm

$= (3 \times 3 – 1)$ cm

$= 8$ cm **½**

Now, in right angled ΔABC,

$AC^2 = AB^2 + BC^2$

(By using Pythagoras theorem)

$= (15)^2 + (8)^2 = 225 + 64$

$= 289 = (17)^2$

Hence, AC = 17 cm.

(iii) Here, $AB = 15$ cm, $BC = 8$ cm and $AC = 17$ cm.

Then, the perimeter of $\Delta ABC = (AB + BC + CA)$ cm

$= (15 + 8 + 17)$ cm

$= 40$ cm.

38. **(i)** Height of hemispherical dome = Radius of hemispherical dome = 21 m.

$$\text{Volume of dome} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 19{,}404 \text{ m}^3$$

(ii) The volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$$

$$= \frac{4 \times 22 \times 49}{3}$$

$$= 1437.33 \text{ m}^3$$

OR

Total surface Area of Combined figure

$= 2\pi r^2 + 2(lb + bh + hl) - lb$

$$= 2 \times \frac{22}{7} \times 14 \times 4 + 2(8 \times 6 + 6 \times 4 + 4 \times 8) - 8 \times 6]\text{m}^2$$

$= [1232 + 208 - 48] \text{ m}^2$

$= 1392 \text{ m}^2$

(iii) Volume of the cuboidal shaped top

$= l \times b \times h$

$= 8 \text{ m} \times 6 \text{ m} \times 4 \text{ m}$

$= 192 \text{ m}^3$.

■■■

SOLUTIONS

Self Assessment Paper-5

Mathematics Standard (041)

Section - A

1. **Option (D) is correct.**

Explanation: $18 = 2 \times 3 \times 3$

$21 = 3 \times 7$

and $27 = 3 \times 3 \times 3$

$\therefore$ HCF of 18, 21 and 27 = 3. **1**

2. **Option (C) is correct.**

Explanation: As we know that, the greatest exponents and raise each prime factor to the greatest exponent and multiply them to get LCM.

So, $n = 4$ **1**

3. **Option (C) is correct.**

Explanation: For zeroes, $(x - 1)(x + 2) = 0$

Either, $x - 1 = 0 \Rightarrow x = 1$

or, $x + 2 = 0 \Rightarrow x = -2$

So, we get two values of x i.e., $x = 1$ or -2

Hence, it is quadratic polynomial intersects x-axis at two points. **1**

4. **Option (B) is correct.**

Explanation:

$\frac{3}{2}x + \frac{5}{3}y = 7$

$9x + 10y = 14$

$\frac{a_1}{a_2} = \frac{\cancel{3}}{2} \times \frac{1}{\cancel{9}3} = \frac{1}{6}$

$\frac{b_1}{b_2} = \frac{\cancel{5}}{3} \times \frac{1}{\cancel{10}2} = \frac{1}{6}$

$\frac{c_1}{c_2} = \frac{\cancel{+}7}{\cancel{+}14} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Ans (B) Inconsistent

[Topper Answer, 2020] 1

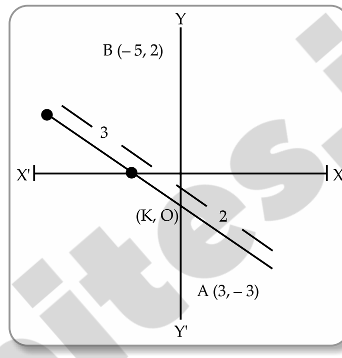
5. **Option (B) is correct.**

Explanation: Since y-coordinate of a given point is the distance of point from x-axis.

So, the distance is 5 units.

6. **Option (C) is correct.**

Explanation:



$$\therefore \quad k = \frac{2(-5)+3(3)}{2+3} \quad \left[\because x = \frac{mx_2 + nx_1}{m+n}\right]$$

$$= \frac{-10+9}{5} = -\frac{1}{5}$$

7. **Option (A) is correct.**

Explanation: Here, $x_1 = 0, y_1 = 4$ and $x_2 = 0, y_2 = -3$

By using distance formula,

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad ½$$

$$= \sqrt{(0-0)^2 + (-3-4)^2} = \sqrt{49}$$

$= 7$ units ½

8. **Option (D) is correct.**

Explanation: $\because$ $DE \parallel BC$ (given)

$$\therefore \quad \frac{AD}{BD} = \frac{AE}{CE} \quad \text{(from BPT) ½}$$

$$\Rightarrow \quad \frac{AD}{7.8} = \frac{0.8}{4.8}$$

$$\Rightarrow \quad AD = \frac{7.8 \times 0.8}{4.8}$$

$= 1.3$ cm ½

9. **Option (B) is correct.**

Explanation: In ΔABC and ΔDEF, $\angle B = \angle E$ and $\angle F = \angle C$. By AA similarity, we get $\Delta ABC \sim \Delta DEF$. Thus, the triangles are similar but not congruent.

10. Option (C) is correct.

Explanation: $\sin\theta = \frac{1}{\sqrt{2}}$ (given)

$\Rightarrow \sin\theta = \sin 45°$

$\Rightarrow \theta = 45°$

$\therefore \tan^2\theta + \cot^2\theta = (\tan 45°)^2 + (\cot 45°)^2$

$= (1)^2 + (1)^2 = 2$

11. Option (B) is correct.

Explanation:

P(winning the game) $= 0.03$

$\therefore P$(closing the game) $= 1 - 0.03$

$= 0.97.$

12. Option (A) is correct.

Explanation:

Volume of cylinder $= 2.2 \text{ dm}^3$

i.e., $\pi r^2 h = 2.2 \times 1000 \text{ cm}^3$

$\Rightarrow \frac{22}{7} \times 0.25 \times 0.25 \times h = 2200$ $\left[\because r = \frac{0.50}{2}\text{cm}\right]$

$\Rightarrow h = \frac{2200\times7}{0.25\times0.25} = 11200 \text{ cm}$

$\Rightarrow h = 112 \text{ m}$

13. Option (B) is correct.

Explanation:

Given: $\frac{r_1}{r_2} = \frac{2}{3}$ and $\frac{h_1}{h_2} = \frac{5}{3}$

$\therefore \frac{\text{volume of first cylinder}}{\text{volume of second cylinder}}$

$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \frac{h_1}{h_2}$

$= \left(\frac{2}{3}\right)^2 \times \frac{5}{3} = \frac{4}{9} \times \frac{5}{3}$

$= \frac{20}{27}$

Hence, the ratio of their values is 20 : 27.

14. Option (B) is correct.

Explanation:

Class mark $= \frac{70+80}{2} = \frac{150}{2}$

$= 75$

15. Option (A) is correct.

Explanation: We have, total surface area of a cone $= 90\pi \text{ cm}^2$

i.e., $\pi r(r + l) = 90\pi$

$\Rightarrow r(r + 13) = 90$ $(\because l = 13 \text{ cm, given})$

$\Rightarrow r^2 + 13r - 90 = 0$

$\Rightarrow (r - 5)(r + 18) = 0$

$\Rightarrow r = 5 \text{ cm}$

16. Option (B) is correct.

Explanation: Let $p(x) = x^2 + 5x + k$

$\because$ 3 is a zero of $p(x)$, then

$p(3) = 0$

$\Rightarrow (3)^2 + 5(3) + k = 0$

$\Rightarrow 9 + 15 + k = 0$

$\Rightarrow k = -24$ **1**

17. Option (D) is correct.

Explanation:

Here, $\frac{a_1}{a_2} = \frac{1}{-3} = -\frac{1}{3}, \frac{b_1}{b_2} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the system of linear equations has no solution. **1**

18. Option (A) is correct.

Explanation: $\therefore \sin 30° = \frac{1}{2}$, $\cot 45° = 1$ and $\cos 60° = \frac{1}{2}$

Putting these values in $\sin^2 30° + 2\cot 45° - \cos^2 60°$

$= \left(\frac{1}{2}\right)^2 + 2\times1 - \left(\frac{1}{2}\right)^2$

$= 2$

19. Option (B) is correct.

Explanation: In case of assertion:

From 1 to 100 numbers, there are 50 even and 50 odd numbers.

Total number of outcomes $T(E) = 100$

Number of outcomes favourable for event E (even numbers) $= F(E) = 50$

So, $P(E) = \frac{50}{100} = \frac{1}{2}$

Similarly, the probability of getting odd numbers $= \frac{1}{2}$.

Hence the probability of getting odd and even each $= \frac{1}{2}$.

Hence, the given statement is true.

$\therefore$ Assertion is correct.

In case of reason:

Since, there are two outcomes equal in all manners.

So, probability of both head and tail is equal to $\frac{1}{2}$ each.

Hence, the given statement is true.

$\therefore$ Reason is correct:

Hence, both assertion and reason are correct but reason is not correct explanation for assertion.

20. **Option (C) is correct.**

Explanation: In case of assertion:

$$3x - y = -18 \qquad ...(i)$$
$$6x - ky = -16 \qquad ...(ii)$$

For coincident lines,

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$
$$\Rightarrow \frac{3}{6} = \frac{-1}{-k} = \frac{-8}{-16}$$
$$\Rightarrow \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$$

So, $k = 2$.

$\therefore$ Assertion is correct.

In case of reason:

For parallel lines (or no solution)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
$$\Rightarrow \frac{3}{2} = \frac{2k}{5} \neq \frac{-2}{1}$$
$$\Rightarrow 4k = 15$$
$$\Rightarrow k = \frac{15}{4}$$

$\therefore$ Reason is incorrect.

Hence, assertion is correct but reason is incorrect.

Section - B

21.

$$\text{Sum of zeroes} = 7 - 2\sqrt{3} + 7 + 2\sqrt{3}$$
$$= 14 \qquad \tfrac{1}{2}$$
$$\text{Product of zeroes} = (7 - 2\sqrt{3})(7 + 2\sqrt{3})$$
$$= 49 - 12 = 37 \qquad \tfrac{1}{2}$$
$$\therefore \quad \text{Polynomial} = x^2 - (\text{sum of zeroes})x + \text{product of zeroes}$$
$$= x^2 - 14x + 37 \qquad 1$$

Commonly Made Error

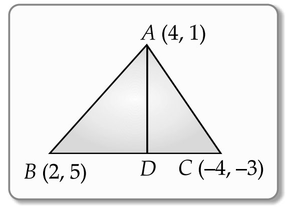
- Students often commit errors in finding a quadratic polynomial. Some students find sum of zeroes and product of zeroes but not find a complete polynomial.

Answering Tip

- Students should read the questions properly and solved step by step.

22.

$$\text{Coordinates of } D = \left(\frac{2-4}{2}, \frac{5-3}{2}\right) \qquad 1$$
$$= (-1, 1)$$
$$\text{Length of } AD = \sqrt{(4+1)^2 + (1-1)^2}$$
$$= \sqrt{(5)^2} = 5 \text{ units} \qquad 1$$



23. TP and TQ are given tangents.

$\therefore \quad \angle OPT = 90°$ (radius $\perp$ tangent)

and $\quad \angle OQT = 90°$ (radius $\perp$ tangent) **1**

In ☐PTQO,

$$90° + 90° + 112° + \angle PTQ = 360°$$
$$\Rightarrow \quad \angle PTQ = 68° \qquad 1$$

24.

$$\text{Volume of a cube} = 125 \text{ cm}^3$$
i.e., $$(\text{side})^3 = (5)^3$$
$$\Rightarrow \quad \text{side} = 5 \text{ cm} \qquad 1$$
$$\text{Now, surface area of a cube} = 6(\text{side})^2$$
$$= 6(5)^2 \text{ cm}^2$$
$$= 150 \text{ cm}^2 \qquad 1$$

OR

Given, Height (h) = 14 cm

and Base radius (r) = 6 cm

Volume of the remaining solid

= Volume of a right circular cylinder – Volume of a right circular cone

$$= \pi r^2 h - \frac{1}{3}\pi r^2 h \qquad 1$$
$$= \frac{2}{3}\pi r^2 h$$
$$= \frac{2}{3} \times \frac{22}{7} \times 6 \times 6 \times 14$$
$$= 1056 \text{ cm}^3 \qquad 1$$

25.

$$\because \quad \tan(5x + 30°) = 1$$
$$\therefore \quad \tan(5x + 30°) = \tan 45°$$
$$\Rightarrow \quad 5x + 30° = \tan 45° \qquad 1$$
$$\Rightarrow \quad 5x = 15$$
$$\Rightarrow \quad x = 3 \qquad 1$$

OR

$$\because \quad \sin(3x + 30°) = \frac{\sqrt{3}}{2}$$
$$\therefore \quad \sin(3x + 30°) = \sin 60°$$
$$\Rightarrow \quad 3x + 30 = 60° \qquad 1$$
$$\Rightarrow \quad 3x = 30$$
$$\Rightarrow \quad x = 10$$
$$\text{Now,} \quad 5x + 10 = 5 \times 10 + 10 = 60° \qquad 1$$

Section - C

26. Let us assume to the contrary that $3\sqrt{7}$ is rational, then $3\sqrt{7}$ is of the form $\frac{p}{q}$ where p and q are co-primes and $q \neq 0$.

$\therefore \qquad \frac{p}{q} = 3\sqrt{7}$

$\Rightarrow \qquad \frac{p}{3q} = \sqrt{7}$

$\frac{p}{3q}$ is rational as p and q are integers. **1**

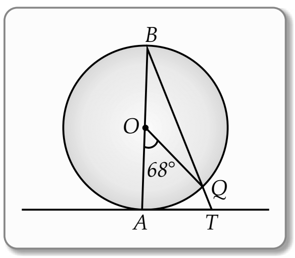
This contradicts the given fact that $\sqrt{7}$ is irrational

$\therefore$ Our assumption is wrong. **2**

Hence, $3\sqrt{7}$ is irrational. **Proved.**

27. $\angle AOQ = 68°$ (given)

$\therefore \qquad \angle ABQ = \frac{1}{2}\angle AOQ$ **1**



[Angle on the circumference of the circle by the same arc]

$= \frac{1}{2} \times 68$

$= 34°$ **1**

$\angle BAT = 90°$ [$\because OA \perp AT$]

$\therefore \qquad \angle ATQ = 90° - 34°$

$= 56°$ **1**

28. Given, $(x + 4)^2 = 3(7x - 4)$

$\Rightarrow \qquad x^2 + 16 + 8x = 21x - 12$

$\Rightarrow \qquad x^2 - 13x + 28 = 0$ **1**

Comparing with $ax^2 + bx + c = 0$, we get

$a = 1, b = -13$ and $c = 28$ **1**

Discriminant $= b^2 - 4ac$

$= (-13)^2 - 4 \times 1 \times 28$

$= 169 - 112 = 57$ **1**

OR

Let the number of wickets taken by Zahir be x.
Then, the number of wickets taken by Harbhajan = $2x - 3$ **1**

According to question, $x(2x - 3) = 20$ **1**

$\Rightarrow \qquad 2x^2 - 3x = 20$

$\therefore$ Required quadratic equation is,

$2x^2 - 3x - 20 = 0$ **1**

[CBSE Marking Scheme, 2015]

29. Given, $x = a \sec\alpha \cos\beta$, $y = b \sec\alpha \sin\beta$ and $z = c \tan\alpha$

$\therefore \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2}$

$= \frac{a^2\sec^2\alpha\cos^2\beta}{a^2} + \frac{b^2\sec^2\alpha\sin^2\beta}{b^2} - \frac{c^2\tan^2\alpha}{c^2}$ **1**

$= \sec^2\alpha\cos^2\beta + \sec^2\alpha\sin^2\beta - \tan^2\alpha$

$= \frac{1}{\cos^2\alpha}(1 - \sin^2\beta) + \frac{1}{\cos^2\alpha}.\sin^2\beta - \frac{\sin^2\alpha}{\cos^2\alpha}$

$= \frac{1 - \sin^2\beta + \sin^2\beta - \sin^2\alpha}{\cos^2\alpha}$ **1**

$= \frac{1 - \sin^2\alpha}{\cos^2\alpha} = \frac{\cos^2\alpha}{\cos^2\alpha}$

$= 1$ **1**

30. Here $a = 4$, $a_n = 94$ and $S_n = 980$

Therefore $\qquad 94 = 4 + (n-1)d$

$\Rightarrow \qquad (n-1)d = 90$ ...(i)

Also $\qquad 980 = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}(8 + 90)$

$= 49n$ (using (i)) **1+½**

$\Rightarrow \qquad n = 20$

and $\qquad d = \frac{90}{19}$ **1+½**

[CBSE Marking Scheme, 2019]

Detailed Solution:

Given:

First term, $a = 4$

Last term, $a_n = 94$

Sum of n terms, $S_n = 980$

Since, $\qquad S_n = \frac{n}{2}(a + a_n)$

$\Rightarrow \qquad 980 = \frac{n}{2}(4 + 94)$

$\Rightarrow \qquad 1960 = 98n$

$\Rightarrow \qquad n = \frac{1960}{98}$

$\Rightarrow \qquad n = 20$ **1**

Now, $\qquad S_n = \frac{n}{2}[2a + (n-1)d]$ **1**

$\Rightarrow \qquad 980 = \frac{20}{2}[2 \times 4 + (20-1)d]$

$\Rightarrow \qquad 1960 = 20(8 + 19d)$

$\Rightarrow \qquad 8 + 19d = 98$

$\Rightarrow \qquad 19d = 90$

$$\Rightarrow \qquad d = \frac{90}{19}$$

Hence, the common difference is $\frac{90}{19}$. **1**

Commonly Made Error

- Some students fail to find the value of n as they get confused between the nth term and last term.

Answering Tip

- Understand the formulae related to given condition and use them to solve the problems.

OR

Let first term be a and common difference be d.

Here, $a_{14} = 2a_8$

or, $a + 13d = 2(a + 7d)$

$a + 13d = 2a + 14d$

$a = -d$ (i)

Again, $a_6 = -8$

or, $a + 5d = -8$ (ii)

Solving (i) and (ii), we get

$a = 2, d = -2$ **1**

$$S_{20} = \frac{20}{2}[2\times 2 + (20-1)(-2)] \quad \mathbf{1}$$

$= 10[4 + 19 \times (-2)]$

$= 10(4 - 38)$

$= 10 \times (-34) = -340$ **1**

[CBSE Marking Scheme, 2015]

31. Let there be x red balls and 20 white ball. Total no. of balls $= n(s) = 20 + x$

$\therefore$ P(drawing red ball) $= 4 \times P$(drawing white ball)

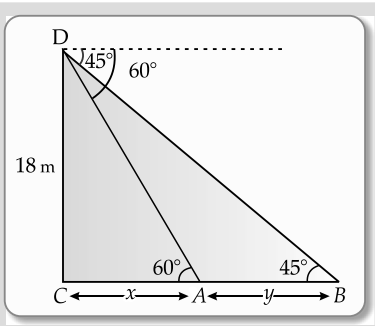
$$\frac{x}{20+x} = 4\times\frac{20}{20+x} \quad \mathbf{1}$$

$\Rightarrow \qquad x = 80$

Hence, there are 80 red balls in the bag. **2**

Section - D

32.



1

In ΔDCA, $\frac{DC}{CA} = \tan 60°$

$\Rightarrow \quad \frac{18}{x} = \sqrt{3}$ **1**

$\Rightarrow \quad x = \frac{18}{\sqrt{3}}$

$\Rightarrow \quad x = 6\sqrt{3}$ **1**

In ΔDCB, $\frac{DC}{CB} = \tan 45° = \frac{18}{x+y} = 1$ **1**

$\Rightarrow \quad x + y = 18$

$\Rightarrow \quad 6\sqrt{3} + y = 18$

$\Rightarrow \quad y = 18 - 6\sqrt{3}$

$= 6(3-\sqrt{3})$ m

Hence, the distance between the points

$= 6(3-\sqrt{3})$ m.

[CBSE Marking Scheme, 2017]

OR

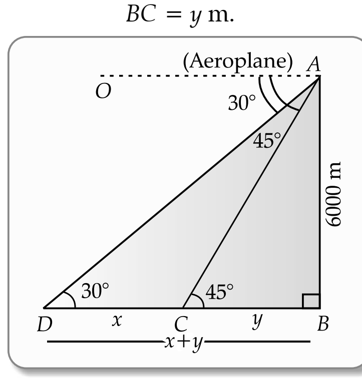
Let AB be the height of the aeroplane, then AB = 6000 m.

Also let D and C be the positions of two ships on the same line. From the point A of an aeroplane, the angles of depression of two ships D and C are $\angle OAD = 30°$ i.e., $\angle BDA = 30°$ and $\angle OAC = 45°$ i.e., $\angle BCA = 45°$

Let distance between two ships

$DC = x$ m

and $BC = y$ m.



In ΔABC, $\frac{AB}{BC} = \tan 45°$

$\Rightarrow \quad \frac{6000}{y} = 1$

$\Rightarrow \quad y = 6000$

In ΔABD, $\frac{AB}{BD} = \tan 30°$

$$\frac{6000}{x+y} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x + y = 6000\sqrt{3}$$

$$x + 6000 = 6000\sqrt{3}$$

$$x = 6000\sqrt{3} - 6000$$

$$= 6000(\sqrt{3} - 1)$$

$$= 6000(1.732 - 1)$$

$$= 6000 \times 0.732$$

$$= 4392 \text{ m}$$

Hence, the distance between two ships

$= 4392$ m

33. In ΔAPE and ΔABD,

$\angle APE = \angle ABD$

(corresponding angles) ½

$\angle PAE = \angle BAD$ (common) ½

$\therefore$ $\Delta APE \sim \Delta ABD$ (By AA criterian)

$\therefore$ $\frac{AE}{AD} = \frac{PE}{BD}$ (i) **1**

In ΔAQE and ΔACD,

$\angle AQE = \angle ACD$

(corresponding angles)

$\angle QAE = \angle CAD$ (common) ½

$\therefore$ $\Delta AQE \sim \Delta ACD$ (By AA criterian) ½

$\therefore$ $\frac{PE}{BD} = \frac{EQ}{DC}$ (ii) ½

From eq. (i) and (ii), we get

$\frac{PE}{BD} = \frac{EQ}{DC}$ ½

$\because$ $BD = DC$ (AD is the median)

$\therefore$ $PE = EQ$ **1**

Hence AD bisects PQ. **Proved.**

34. Quantity of water flowing through pipe in 1 h

$= \pi \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ m}^3$ **2½**

Required time $= \left(50 \times 44 \times \frac{21}{100}\right) \div \left(\pi \times \frac{7}{100} \times \frac{7}{100} \times 15000\right)$

$= 2$ hours **2½**

[CBSE Marking Scheme, 2020]

Detailed Solution:

Speed of water flowing through the pipe

$= 15$ km/hr $= 15000$ m/hr

Volume of water flowing in 1hr $= \pi R^2 H$

$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000 \text{ m}^3$

$= 231 \text{ m}^3$

Volume of water in the tank when the depth is 21 cm

$= lbh$

$= 50 \times 44 \times \frac{21}{100} \text{ m}^3$

$= 462 \text{ m}^3$

Time taken to fill 462 m^3 $= \frac{462}{231} = 2$ hrs.

OR

(i) $\cos(90° - \theta) = \cos(3\theta - 30°)$

$\Rightarrow$ $90° - \theta = 3\theta - 30°$

$\Rightarrow$ $\theta = 30°$ **2½**

(ii) $\frac{AB}{AC} = \sin 30°$

$\frac{200}{AC} = \frac{1}{2}$

$\therefore$ Length of rope $= AC = 400$ m **2½**

[CBSE SQP Marking Scheme, 2020]

35. Here, modal class $= 30 - 35$

$\therefore l = 30, f_0 = 9, f_1 = 12, f_2 = 3$ and $h = 5$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h \quad \mathbf{1}$$

$$= 30 + \left(\frac{12 - 9}{2 \times 12 - 9 - 3}\right) \times 5$$

$$= 30 + \frac{3}{12} = 30 + 0.25$$

$= 30.25.$ **2 + 1**

Section - E

36. **(i)** Let the no. of marbles, John and Jivanti have, be x and y respectively.

According to the given information,

$x + y = 45$ (i)

and $x - y = 15$ (ii)

Solving eqs. (i) and (ii), we get

$x = 30$ and $y = 15$

So, Jivanti has 15 marbles

(ii) According to given problem,

$x + y = 55$ (i)

and $x - y = 15$ (ii)

Solving eqs. (i) and (ii), we get

$x = 35$ and $y = 20.$

So, Jivanti has 20 marbles

(iii) From the given passage, we get $x = 30.$

Hence, John had 30 marbles.

OR

The given problem is based on pair of linear equations.

37. **(i)** For cuboid

$l = 15$ cm, $b = 10$ cm and $h = 3.5$ cm

$$\begin{aligned} \text{Volume of the cuboid} &= l \times b \times h \\ &= 15 \times 10 \times 3.5 \\ &= 525 \text{ cm}^3 \end{aligned}$$

(ii) For conical depression:

$$r = 0.5 \text{ cm}, \quad h = 1.4 \text{ cm}$$

Volume of conical depression

$$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$$

$$= \frac{11}{30} \text{ cm}^3$$

OR

Volume of four conical depressions

$$= 4 \times \frac{11}{30} = 1.47 \text{ cm}^3$$

(iii) Volume of the wood in the entire stand

$$\begin{aligned} &= \text{Volume of cuboid} \\ &\quad - \text{Volume of 4 conical depressions} \\ &= 525 - 1.47 \\ &= 523.53 \text{ cm}^3 \end{aligned}$$

38. **(i)** No. of cards of king of red colour = 2

Total no. of cards = 52

Probability of getting king of red colour

$$= \frac{\text{No. of king of red colour}}{\text{Total number of cards}}$$

$$= \frac{2}{52} = \frac{1}{26}$$

(ii) No. of cards (Jack heart) = 1

No. of total card = 52

$$\text{The required probability} = \frac{\text{No. of Jack-Heart}}{\text{Total card}}$$

$$= \mathbf{\frac{1}{52}}$$

OR

No. of red face cards = 6

No. of face card = 13

Total no of cards = 52

Probability of getting red face card

$$= \frac{\text{No. of red face cards}}{\text{Total no. of cards}}$$

$$= \frac{6}{52} = \frac{3}{26}$$

(iii) No. of spade cards = 13

Total no. of cards = 52

$$\text{Required Probability} = \frac{\text{No. of spade card}}{\text{Total no of card}}$$

$$= \frac{13}{52} = \frac{1}{4}$$

■■■